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Pressure Group Size and the Politics of Income Redistribution

D. Wittman

University of California, Santa Cruz, CA 95064, USA

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Abstract. Numerous authors have argued that small groups with concentrated interests have an advantage over larger groups with more diffuse interests. We argue that the reverse situation is more likely. In addition, we extend the one candidate analysis to situations where there are two contestants for any one legislative seat.

The economic approach to politics, at present, consists of two divergent and contradictory approaches – spatial models of candidate strategies and the theory of pressure groups.

Spatial models typically have 2 candidates, N types of voters and K issues and confront the issue of intransivity head on. While some models demonstrate the lack of transitivity (see McKelvey 1976 for an example), those other models which do yield an equilibrium outcome typically show that the majority, via the median voter, has its way in the political system (see for example, Plott 1967; Coughlin 1984).[1]

In contrast, the pressure group literature finds that the political system caters to the interests of the minority. This literature has avoided the issue of intransitivity altogether by assuming that an equilibrium exists[2] and can be found by solving a maximization problem (the formal models having only one candidate and no more than two groups of voters). For example, Peltzman (1976) has a model of the regulator who chooses the tax scheme which maximizes political support, while Becker (1983) has a model of political pressure on the government (a single entity). Less formal arguments have also been made by Friedman and Friedman (1980) and

[1] There are some variants of the basic model. In some models the candidates may converge to the mean (see Hinich 1978) while in others there is a convergence to the Pareto optimal set (for example, Ferejohn et al. (1984) have a model where the limiting distribution under majority rule will tend to concentrate near the Pareto set while Coughlin (1982) and Coughlin and Palfrey (1985) have models where the candidates converge to the Pareto optimal set).

[2] Peltzman (1980, p 222) "brush[es] past the rather formidable problems connected with ... stability of equilibrium" by "assuming that competition among politicians will lead them to converge" to "a politically 'dominant' redistributive program".

Demsetz (1982). In one way or another, these authors argue that the majority with their low per capita stakes (diffuse interests) are at the mercy of the minorities with their high per capita stakes (concentrated interests).

In this paper I combine these two strands of inquiry and develop a two candidate, k group model of political pressure for redistribution.[3] I identify conditions where, contrary to previous arguments, the diffuse interests of the majority will override the concentrated interests of the minority. Thus in my model, even in the presence of income redistribution the median voter result still holds. I also show the conditions whereby transitivity does and does not hold.

In order to facilitate the exposition, the model is developed in three parts. In Sect. 1, I consider the basic two group – one candidate model and show that, under reasonable assumptions, the previous work demonstrating the power of concentrated minority interests over the diffuse interest of the majority is incorrect. Indeed I demonstrate that the minority is taxed in order to subsidize the majority. In Sect. 2, I demonstrate the existence of an equilibrium when there are k groups and two candidates. Again the diffuse majority taxes the concentrated minorities. In Sect. 3, I consider an alternative assumption regarding the form of the political pressure function and demonstrate that the outcome is independent of the relative size of pressure groups.

1. Concentrated Versus Diffuse

"The steel industry and its workers ... are willing to act because the benefits from protection are concentrated on the relatively few who invest and work in the industry. Their incomes are significantly affected. The larger costs of their protection are borne in dispersed fashion by the much more numerous population of taxpayers and consumers. The dilution of costs renders its bearers politically ineffective". Demsetz (1982).

"... a [democratic] system tends to give undue political power to small groups that have highly concentrated interests.". ... Consider the government program of favoring the merchant marine by subsidies for shipbuilding ... The estimated cost ... is 15,000 dollars per year for each of the 40,000 people actively engaged in the industry. Ship owners, operators and their employees have a strong incentive to get and keep these measures. ... On the other hand [these subsidies] only come to about 3 dollars a person per year. Which of us will vote against a candidate because he imposed that cost on us?" Friedman and Friedman (1980).

"Politically successful groups tend to be small relative to the size of the groups taxed to pay their subsidies". Becker (1983).

In this section we present a model from the pressure group literature which yields results diametrically opposite to these statements. We present the model in terms of an election where a politician maximizes expected vote support, M, and

[3] Related two candidate k group models have previously been developed in Enelow and Hinich (1984, Chap. 5), Coughlin (1986), and Morton (1987).

each voter votes for the politician with some probability, f.[4] This probability is solely a function of the voter's tax or subsidy. Alternative interpretations for the same set of equations are possible. For example, one could use the same notation to signify contributions or political pressure.

Let

$$M=\frac{N_1}{2}\left[1-f(T_1)\right]+\frac{N_2}{2}\left[1+f\left(\frac{N_1T_1}{N_2}\right)\right]. \tag{1}$$

Each of the N_1 members of group 1 is taxed $T_1 \geqq 0$. Each of the N_2 members of group 2 receives a subsidy of $\frac{N_1T_1}{N_2}$. The probability that a voter in group 1 votes for the politician is $\frac{1}{2}\left[1-f(T_1)\right]$, and the probability that a voter in group 2 votes for the politician is $\frac{1}{2}\left[1+f\left(\frac{T_1N_1}{N_2}\right)\right]$. We assume that $f(0)=0$, $f'(x)>0$, $f(\infty)\leqq 1$, and $f''(x)<0$ for $x>0$. The probability function is drawn in Fig. 1. The function characterizes the fact that diffuse costs have a smaller effect on any individual voter in the large group than concentrated benefits have on the individual voter in the small group.[5] For example, if $N_1<N_2$, then $f(T_1)>f\left(\frac{T_1N_1}{N_2}\right)$. A negative second derivative is consistent with psychological studies of perception [the correct choice of the longer line increases at a decreasing rate as the longer line becomes longer – see Marks (1974)]. The assumption that $f''<0$ is also in accordance with the standard assumption in economic models that marginal productivity is decreasing. Furthermore, the 0–1 bounds on f require that $f''<0$ for large values of T_1. We will assume that the maximum value of T_1 is T^* (an individual has only a limited amount of wealth). M can be characterized as the expected political support for the politician. The implicit alternative is a policy of no income redistribution.

Proposition 1. *Assume $N_1 \neq N_2$. Members of group 1 will be taxed if and only if $N_1<N_2$. That is, the minority will subsidize the majority.*

Proof. Letting subscripts of M stand for the partial derivative,

$$M_{T_1}=-\frac{N_1}{2}f'(T_1)+\frac{N_1}{2}f'\left(\frac{N_1T_1}{N_2}\right). \tag{2}$$

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4 The objective function, M, corresponds to the model in Peltzman (1976, p 213–222). It is especially appropriate to focus on the Peltzman article because his argument is based on the "same" type of model that I use here in contradicting his conclusion that the diffuse interests of the majority are at the mercy of the concentrated interests of minorities. That is, I overturn a tenet of the pressure group literature using one of its own models.

5 In this paper most of our analysis is concerned with concentrated and diffuse benefits and costs – per capita stakes. Some authors have also used the words "concentrated" and "diffuse" to refer to the physical dispersion of the members of the pressure group. For example, brussels sprout growers are geographically concentrated in one county, while the location of dairy farmers is diffuse. Information can be spread among the brussels sprout farmers at lower cost. We will discuss these costs later in this section. At note 7 we will discuss a double S function which captures the problem of fixed costs of political pressure and in Sect. C we will discuss concave probability functions.

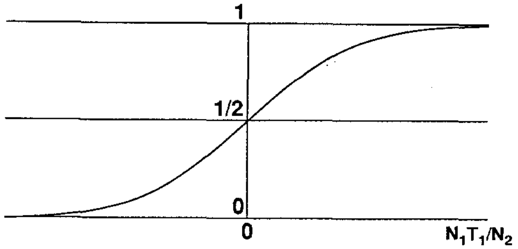

Fig. 1. The probability that a voter in group 2 votes for the politician when the voter receives a subsidy $N_1 T_1 / N_2$

If $N_1 < N_2$ and $T_1 > 0$, then $\frac{N_1 T_1}{N_2} < T_1$ and $f'(T_1) < f'\left(\frac{N_1 T_1}{N_2}\right)$ because $f'' < 0$. Hence, $M_{T_1} > 0$. Thus, the politician will increase the tax on the concentrated group 1 as much as possible ($T_1 = T^*$). If group 1 is the larger group, then the tax would be as small as possible ($T_1 = 0$). Thus the minority with concentrated interests subsidizes the majority with diffuse interests. q.e.d.

In this model there are two factors that go in opposite directions. Large groups have more voters while small groups (with greater per capita stakes) have more intensity. We have shown that the size effect outweights the intensity effect when the response function is the one drawn in Fig. 1.

It should be noted that this is not the only special case where this occurs. We will now show that it may hold even when individuals in the large group are less sensitive to taxes and subsidies than individuals in the small group.

Let the probability that a voter in the small group (1) votes for the politician be

$$\frac{1}{2}\left[1 - (f(T_1)\right] = \frac{1}{2}\left[1 - (T_1)^b\right]$$

and the probability that a voter in the large group votes for the politician be

$$\frac{1}{2}\left[1 + f\left(\frac{CN_1 T_1}{N_2}\right)\right] = \frac{1}{2}\left[1 + \left(\frac{CN_1 T_1}{N_2}\right)^b\right]$$

$$\text{where} \quad 0 < b < 1, \; C > 0, \quad \text{and} \quad T^* < 1 \; . \tag{3}$$

$0 < b < 1$ insures the appropriate shape while $T^* < 1$ insures that the probability remains within $(0, 1)$. C is a relative sensitivity factor. If $C < 1$, then an individual in the large group is less sensitive to policy than an individual in the small group. For example if $C = 1/2$, then the politician would have to give two dollars to a person in the large group in order to achieve the equivalent response from giving a person in the small group one dollar. C might be less than one if the small group was concentrated geographically or was organized for some other purpose. The specific functional form implies constant elasticity of $f - 1/2$.

Proposition 2. *If* $C > \left(\frac{N_1}{N_2}\right)^{\frac{1}{b} - 1}$*, then redistribution will be toward the larger group (2).*

Proof.

$$M = \frac{N_1}{2}\left[1 - T_1^b\right] + \frac{N_2}{2}\left[1 + \left(\frac{CT_1 N_1}{N_2}\right)^b\right], \tag{4}$$

$$M_{t_1} = -\frac{N_1}{2} bT_1^{b-1} + \frac{N_2}{2} bT_1^{b-1} \left(\frac{CN_1}{N_2}\right)^b = -\frac{N_1}{2} bT_1^{b-1} + \frac{N_1}{2} bT_1^{b-1} C^b \left(\frac{N_1}{N_2}\right)^{b-1}$$

$$= \frac{N_1}{2} bT_1^{b-1} \left[C^b \left(\frac{N_1}{N_2}\right)^{b-1} - 1 \right]. \tag{5}$$

We now investigate the conditions for this expression to be greater than zero. Since B and T_1 are positive, (5) is positive if and only if the following holds:

$$C^b \left(\frac{N_1}{N_2}\right)^{b-1} > 1 \quad \text{equivalently} , \quad C > \left(\frac{N_1}{N_2}\right)^{\frac{1-b}{b}} . \tag{6}$$

q.e.d.

Hence, whether distribution is from the members of the small group to the members in the large group depends critically upon the ratio of the size of the two groups: the smaller group 1 is to group 2 and the smaller the elasticity (b), the smaller C need be in order for the redistribution to go from the concentrated interests to the diffuse interests of the larger group. For $b = 1/2$, $C > N_1/N_2$ is sufficient for the redistribution towards the larger group. Thus even if individuals in the larger group are less sensitive ($C < 1$), our initial results are likely to hold.

However, if anything, C is likely to be greater than 1. To illustrate the reason for C being greater than 1, consider the possibility that group 1 has only 1 member. Defining the *net* probability by $|f/2|$ (i.e., the probability differential from 0.5), we have implicitly assumed that the net probability of a voter in group 1 voting against the politician if he pays T is equal to the net probability of any individual in group 2 voting for the politician if she receives T.[6] But, certainly, there are many reasons to expect that the voters in the larger group would have a higher net probability of voting for the politician when the per capita subsidy is T than the probability of a member in the small group voting against when the per capita tax is T. One member of the larger group might inform other members or some political entrepreneur might provide the information in return for some rewards (since the total gain is greater for the larger group even though the per capita gain (or loss) is the same). Thus, in general, we would expect C to be somewhere between 1 and N_2/N_1. In other words, even though the diffuse interest voter has a lower net probability of voting in favor of a subsidy for the diffuse interest voter than the net probability of a concentrated interest voter voting against the subsidy for the diffuse interest voter $\left(\text{i.e., } f\left(\frac{CN_1 T_1}{N_2}\right) < |f(T_1)| \text{ for } N_1 < N_2\right)$, the individual in the large group is likely to be more responsive to an *equivalent* amount (i.e., $f(CT) > f(T)$ is still likely).

2. Two Candidates and k Pressure Groups

We next turn our attention toward the existence of a local equilibrium when there are two candidates, X and Y, and k interest groups.

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6 This must be in two different states of the world since the member of group 1 could not pay T and the members of group 2 (if more than one member) could not each receive T at the same time.

The spatial literature has typically assumed that the probability of any voter voting for candidate X is a concave function of X's position, x, and a convex function of Y's position, y (see for example, Hinich et al. (1972), Denzau and Kats (1977), and Wittman (1983, 1984, 1988)). Although the assumption of concavity is problematic, it does have one very redeeming quality – the sum of concave functions is also concave. Hence if X and Y are maximizing their expected vote, they are maximizing concave functions. An author can then readily apply any one of the numerous existence of equilibrium theorems requiring concave utility functions. Here, however, the individual probability functions (in Fig. 1) are quasiconcave. Unfortunately, the sum of quasiconcave functions need not be quasiconcave. Luckily, the inherent symmetry of the model and the thrust towards corner solutions allows us a relatively easy demonstration of existence without having to confront the issue of quasiconcavity.[7]

Let the expected vote for X be M and the expected vote for Y be $1-M$.

$$M=\frac{N_1}{2}\,[1+f(T_1^y-T_1^x)]+\frac{N_2}{2}\,[1+f(T_2^y-T_2^x)]\ldots$$

$$+\frac{N_k}{2}\left[1+f\left(\frac{\sum_{i=1}^{k-1} N_i[T_i^x-T_i^y]}{N_k}\right)\right], \tag{7}$$

where T_i^j is the tax (or subsidy if $T_i^j<0$) that candidate j will impose on each member of group i if j wins the election. A tax on a member of group 1 (multiplied by N_1/N_k) is a subsidy to a member of group k. $T_i^j \leqq T^*$. $f(z)=-f(-z)$, $f(0)=0$, $f'(z)>0$, $f''(z)>0$, $=0$, <0 for $z<0$, $=0$, and >0, respectively. We will make extensive use of the fact that $f'(z)=f'(-z)$.

In the one candidate model, T_i^y was implicitly equal to 0 for all i. Note that we have redefined f so that it can be a function of negative values; thus, z in $f(z)$ can now take on negative values, but when

$$z<0\ ,\quad f(z)=-f(|z|)<0\ .$$

Definition. Let T^x be the $k-1$ dimensional allocation vector $(T_1^x, T_2^x, \ldots)$ by X, and T^y be the $k-1$ dimensional allocation vector by Y.[8]

Proposition 3. *Assume that* $N_k>\sum_{i=1}^{k-1} N_i$. T^x, T^y *is an equilibrium if and only if* $T_i^j=T^*$ *for* $i=1,2,\ldots k-1$ *and* $j=1,2$. *In other words, there is one and only one equilibrium strategy – the minority groups are taxed to the maximum possible by both candidates.*

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[7] A third possibility (beyond the concave function and the single S-shaped quasiconcave function) is a double S quasiconcave symmetric function: starting at a zero subsidy the probability of voting for the politician would be one half; as the subsidy increased there would be at first increasing returns to scale (convexity) and then decreasing returns to scale (concavity with a positive slope). Again starting at the zero subsidy/tax point but now moving left an increasing tax would at first decrease the probability of voting for at an increasing rate and then at a decreasing rate. This possibility might arise if very small differentials do not result in political pressure either because they are too small to be detected or because they are not sufficient to overcome the fixed transaction costs of organizing. Unfortunately, such a characterization may lead to intransitivity.

[8] There are k groups but only $k-1$ degrees of freedom.

Proof. The outline of the proof is as follows. We first derive the first order conditions. We then demonstrate that these conditions are satisfied if and only if $T^x = T^y$. We then show that when the platforms are identical, T^x (T^y) is at an inflection point and $X(Y)$ prefers to increase the subsidy to group k, the large group. Therefore, an equilibrium can only take place at a corner solution where both X and Y have taxed the small groups as much as possible.

In order to facilitate the exposition, we will assume that T_i^x, $T_i^y \geqq 0$. It will be apparent from our proof that the reverse strict inequality is incompatible with an equilibrium (and thus we will not bother to explore all the possible distributions of subsidies and taxes).

Candidate X maximizes M subject to the constraint that each $T_i^x \leqq T^*$; and candidate Y maximizes $[1-M]$ subject to the constraint that each $T_i^y \leqq T^*$. The first order conditions of the constrained maximization problem for X and Y are

For X:

$$M_{T_i^x} - \Lambda_i^x = -\frac{N_i}{2} f'(T_i^y - T_i^x) + \frac{N_i}{2} f'\left(\sum_{i=1}^{k-1} N_i[T_i^x - T_i^y]/N_k\right) - \Lambda_i^x \leqq 0$$

$$T_i^x \geqq 0 \quad [M_{T_i^x} - \Lambda_i^x] T_i^x = 0$$

$$T_i^* - T_i^x \geqq 0 \quad \Lambda_i^x \geqq 0 \quad [T_i^* - T_i^x]\Lambda_i^x = 0 \ .$$

For Y:

$$-M_{T_i^x} - \Lambda_i^y = +\frac{N_i}{2} f'(T_i^y - T_i^x) - \frac{N_i}{2} f'\left(\sum_{i=1}^{k-1} N_i[T_i^x - T_i^y]/N_k\right) - \Lambda_i^y \leqq 0$$

$$T_i^y \geqq 0 \quad [M_{T_i^y} - \Lambda_i^y] T_i^y = 0 \tag{8}$$

$$T_i^* - T_i^y \geqq 0 \quad \Lambda_i^y \geqq 0 \quad [T_i^* - T_i^y]\Lambda_i^y = 0 \quad \text{for } i = 1, 2, \ldots k-1.$$

When the platforms are identical and Λ_i^x, $\Lambda_i^y = 0$, these conditions are satisfied since $\frac{2}{N_i} M_{T_i^x} = -f'(0) + f'(0) = 0$, and Λ_i^x, $\Lambda_i^y \geqq 0$.

We next show that when there is a discrepancy between the platforms (that is, $|D_i| = |T_i^x - T_i^y| \neq 0$ for some i), the first order conditions cannot be satisfied for both X and Y. When $T_i^x = T_i^y = T^*$ does not hold, then at least one of the three following possibilities must be true.

(A) $\quad T^* > T_i^x > 0$.

These inequalities imply $\Lambda_i^x = 0$ (by complementary slack) and $f'(T_i^y - T_i^x) = f'\left(\sum_{i=1}^{k-1} N_i[T_i^x - T_i^y]/N_k\right)$ (by the first equality and complementary slack). Hence $|D_i| = \left|\sum_{i=1}^{k-1} N_i D_i\right| \Big/ N_k$.

(B) $\quad T^* > T_i^y > 0$.

These inequalities imply $f'(T_i^y - T_i^x) = f'\left(\sum_{i=1}^{k-1} N_i[T_i^x - T_i^y]/N_k\right)$. Hence, again $|D_i| = \left|\sum_{i=1}^{k-1} N_i D_i\right| \Big/ N_k$.

(C) $T_i^x=0$ and/or $T_i^y=0$.

For $T_i^x=0$, $-\frac{N_i}{2}f'(T_i^y-T_i^x)+\frac{N_i}{2}f'\left(\sum_{i=1}^{k-1}N_i[T_i^x-T_i^y]/N_k\right)\leqq 0$ holds with strict equality, since T_i^x can be negative (i.e., it can be a subsidy) in the more general formulation (we are looking at only "one half" of the programming pair – the other half has a non-negative subsidy). Therefore, a strictly negative inequality at 0 is not consistent with maximization.[9] Hence, again $|D_i|=\left|\sum_{i=1}^{k-1}N_iD_i\right|\Big/N_k$.

For identical reasons, when $T_i^y=0, \frac{N_i}{2}f'(T_i^y-T_i^x)-\frac{N_i}{2}f'\left(\sum_{i=1}^{k-1}N_i[T_i^x-T_i^y]/N_k\right)$ must also equal zero and $|D_i|=\left|\sum_{i=1}^{k-1}N_iD_i\right|\Big/N_k$.

Together *A*, *B* and *C* imply that for all *i*

$$|D_i|=\frac{\left|\sum_{i=1}^{k-1}N_iD_i\right|}{N_k}\leqq\frac{\left|\sum_{i=1}^{k-1}N_iD_i\right|}{\sum_{i=1}^{k-1}N_i}\,. \tag{9}$$

Since the numerators differ, the second relationship only holds with equality if $\left|\sum_{i=1}^{k-1}N_iD_i\right|=0$. But then by the first relationship all D_i must equal 0 and the platforms are identical. Hence if any $|D_i|\neq 0$, the last relationship must hold with strict inequality. This implies that the absolute value of each observation is strictly less than the absolute value of the average. This is impossible. Hence, for the first order conditions to hold $D_i=0$ for all *i*.

We now show that when $T_i^x=T_i^y$, *X* will want to increase T_i^x. That is, we have an inflection point: an equilibrium existing only if we are at a corner solution (taxing the small groups to the limit).

Looking at (7) when $T_i^x=T_i^y$, $M=\sum_{i=1}^{k}\frac{N_i}{2}$. Consider another allocation where $T_i^x\geqq T_i^y$ with strict inequality for some $i<k$. We will now show that *M* is larger. Making use of the symmetry relations (7) can be rewritten as follows.

$$\begin{aligned}M&=\sum_{i=1}^{k-1}\frac{N_i}{2}[1-f(T_i^x-T_i^y)]+\frac{N_k}{2}\left[1+f\left(\frac{\sum_{i=1}^{k-1}N_i[T_i^x-T_i^y]}{N_k}\right)\right]\\&=\sum_{i=1}^{k}\frac{N_i}{2}-\sum_{i=1}^{k-1}\frac{N_i}{2}f(T_i^x-T_i^y)-\frac{N_mf(0)}{2}+\frac{N_k}{2}f\left(\frac{\sum_{i=1}^{k-1}N_i[T_i^x-T_i^y]+N_m0}{N_k}\right),\end{aligned} \tag{10}$$

[9] Later it will be apparent that the other corner, taxing members of group *k* to the limit, cannot be a maximum.

where $N_m = N_k - \sum_{h=1}^{k-1} N_h$. Since $f(0)=0$, we have not changed M. We will now demonstrate that the net of the last three terms is positive and therefore M is increased when X increases the tax burden on the small groups. Since all the expressions are functions of positive terms we are dealing with a concave function.

Hence

$$\sum_{i=1}^{k-1} \frac{N_i}{N_k} f(T_i^x - T_i^y) + \frac{N_m f(0)}{N_k} < f\left(\frac{\sum_{i=1}^{k-1} N_i [T_i^x - T_i^y] + N_m 0}{N_k}\right)$$

by strict concavity. q.e.d.

These results are analogous to those in the spatial literature. There too, when candidates with symmetric payoff functions maximize their expected plurality, they converge to the preferences of the median voter and present identical platforms.

We next turn our attention toward the case where the largest group does not have a majority. Unfortunately, in such situations, intransitivity appears. Since no one group has a majority, any collection of groups which together make up a majority of voters can be treated as a super group, k'. Candidate X will then subsidize the members of super group k' by taxing all the other groups to the limit. But then there exists another collection of groups, k'', which include a majority of voters (including some groups from super group k'). Candidate Y will then subsidize the members of super group k'' by taxing all the voters not in group k'' to the limit; etc. Thus an equilibrium does not exist. When the largest group has less than a majority, the notion of a median voter is no longer defined. Hence it should not be surprising that there is no equilibrium.

3. Concave Probability Functions and the Existence of an Equilibrium

In this section we analyze the outcome when the probability (or pressure) function is concave, as in Fig. 2. A plausible explanation for such a form is that utility may be a concave function of after tax/subsidy income. If probability of political pressure (e.g., voting) for X is a linear function of the differential in i's utility between what X offers i and Y offers i, then the probability function is itself concave with respect to X's strategy choice.

Here zero income is the baseline in contrast to our earlier models where the tax or subsidy provided by the other candidate was the baseline. Because our focus has changed, we will define new variables and functions.

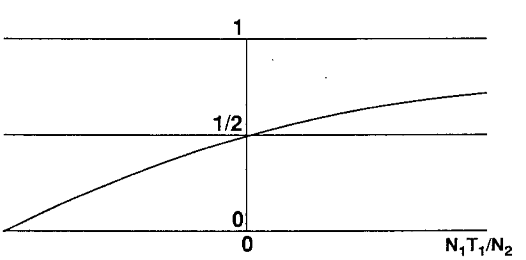

Fig. 2. The probability that a voter in group 2 votes for the politician when the voter receives a subsidy $N_1 T_1 / N_2$

Let Z_i^j be the after tax/subsidy income to each member of group i if j wins the election $\left(\sum_{i=1}^{k} N_i Z_i^j = Z\right)$. Let $U(Z_i^j)$ be i's utility if j wins ($U' > 0$). Utility is a strictly concave function of after tax/subsidy income. We will scale U so that $0 \leqq U \leqq 1$. P_i, the probability that i votes for X, equals

$$\frac{1}{2}+\frac{U(Z_i^x)-U(Z_i^y)}{2} .$$

Proposition 4. *Given our assumptions about U and P_i the following holds: (a) a unique equilibrium exists; (b) after tax/subsidy income is identical for all voters (hence taxes and after tax/subsidy income is independent of group size); and (c) if pretax/subsidy income is distributed equally, then there is no political income redistribution.*

Proof of Existence. P_i is a concave function of Z_i^x and a convex function of Z_i^y. The sum of concave (convex) functions are concave (convex). X maximizes and Y minimizes the sum of these functions. The possible set of strategies are defined on a convex compact set. Therefore we can apply a theorem of Aubin (1981) which generalizes the influential existence theorem derived in Nash (1950, 1951) which demonstrates the existence of an equilibrium under such conditions.[10]

Proof that $Z_i^x = \dfrac{Z}{N}$.

X solves the following constrained maximization problem

$$M=\frac{N_1}{2}\,[1+U(Z_1^x)-U(Z_1^y)]+\frac{N_2}{2}\,[1+U(Z_2^x)-U(Z_2^y)]$$
$$\ldots+\frac{N_k}{2}\,[1+U(Z_k^x)-U(Z_k^y)]+\Lambda\left(Z-\sum_{i=1}^{k} N_i Z_i^x\right) . \tag{11}$$

The Kuhn-Tucker conditions are

$$\begin{aligned} &M_{Z_i^x}=N_i U'(Z_i^x)/2-N_i\Lambda\leqq 0 \quad Z_i^x\geqq 0 \quad M_{Z_i^x}[Z_i^x]=0\\ &M_\Lambda=Z-\textstyle\sum N_i Z_i^x\geqq 0 \quad \Lambda\geqq 0 \quad M_\Lambda[\Lambda]=0 \ . \end{aligned} \tag{12}$$

Since $U'(Z_i^x)>0$, Λ must be strictly greater than zero in order for the first inequality to hold. Since $\Lambda>0$, $Z-\sum N_i Z_i^x=0$ by complementary slack. Hence, at least one $Z_i^x>0$. Label this group 1. Therefore, by complementary slack the corresponding condition, $U'(Z_1^x)/2-\Lambda$ must equal zero. By strict concavity $U'(Z_1^x)<U'(0)$. Therefore, in order for the other j inequalities to hold, $Z_j^x>0$, for all j. But this in turn requires that $U'(Z_j^x)/2-\Lambda=0$ for all j. Hence all $Z_j^x=\dfrac{Z}{\sum N_i}$. q.e.d.

Thus political income redistribution is independent of the relative size of the pressure groups, and there is political pressure for income equality. If there were

[10] Coughlin (1986) assumes probability is a linear function of the upper half of two logit functions and demonstrates the existence of an equilibrium. This section can be seen as an extension of his work to different issues. See also Lindbeck and Weibull (1987).

perfect equality before taxes, there would be no political income redistribution. Here, as in our earlier models, we have a positive explanation for government efficiency ($\sum Z_i^x = Z$): it is a direct result of competition for office. In addition this model provides a positive explanation for redistribution towards income equality which does not rely on individual's desire for insurance against bad times.

4. Concluding Remarks

In this paper we have provided new insights into the nature of the Peltzman-Becker pressure group models, spatial voting models and the relationship between these two (previously) separate literatures by extending and showing some of the limits of redistributive models of politics. The models that we considered showed that there are some interesting circumstances where there is no political advantage to being a small group with concentrated per capita interests.

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