ARMS CONTROL VERIFICATION AND OTHER GAMES INVOLVING IMPERFECT DETECTION

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This article presents an analysis of the strategic behavior of countries when there is imperfect verification of an arms control agreement. It provides a framework for determining whether an arms control agreement is desirable, shows which factors are needed for the agreement to be maintained in the absence of third-party enforcers, and develops propositions relating changes in verification capabilities to changes in the likelihood of cheating and the use of verification technology. These propositions yield several paradoxes of information (for example, the better the verification technology, the less often it will be employed). Since the analysis incorporates both simultaneous and sequential moves by the players, it provides new insights into other applied areas as well as game theory.

> No issue has loomed larger in Soviet-American arms control negotiations than verification.
>
> —Martin Einhorn, Gordon Kane, and Miroslav Nincic

> Verification stands as the litmus test by which arms control agreements are assessed.
>
> —William Potter

An arms control agreement involves the risk that one side's cheating may go undetected by the other side; a situation of no arms control involves a different type of risk. I provide a game theory analysis of how each side acts in the presence of such risks.

What issues determine whether an arms control agreement is possible? While much of the extensive literature on arms control suggests that the answer lies in the technological characteristics of verification, I argue that the technology of verification is just one part of a much more inclusive analysis that starts with the objective of each side. Clearly cheating on the arms control agreement is not the objective of the other side. Rather the likelihood of cheating depends (among other things) on the costs to the potential cheater of being caught, the benefits of not being caught, and the probability of being caught.

Even without the game theory analysis, important insights can be gained by putting the cost-benefit assumptions in a formal framework. By explicitly laying one's cards on the table, the limits of one's assumptions can be more readily understood than in a less formal analysis. For example, by exploring the possibility that the other side may want to exaggerate its offensive and defensive capabilities, I challenge the assumption implied in much published work that it tends to underreport its strength. Furthermore I show that it is not just a simple set of costs and benefits of cheating that must be considered but a matrix array (at least five payoff pairs). Hence, any empirical or intuitive discussion must discuss the whole range of issues presented here. For example, in discussing the feasibility of an arms control agreement it is necessary to estimate the cost to each side should one side incorrectly believe that the other side has cheated (an issue heretofore neglected).

The discussion of verification capabilities has often been the domain of natural scientists. Here I bring politics back into the analysis. I show that verification is not a fixed pair of probabilities—that of mistakenly believing the other side has been honest and that of mistakenly believing the other side has cheated (type 2 and type 1 errors)—but rather a continuum of probability pairs. Even if the verification technology cannot be altered to provide different type 1 and type 2 error combinations, the response to the output from the monitoring equipment can be. The detector (the country being cheated against) may choose different levels of evidence in which to respond. Thus the likelihood of the country making a type 1 or type 2 error is not merely a technological characteristic of the equipment but instead depends critically on the political response.

An important part of the cost-benefit calculations is the determination of the likelihood of cheating and the probability that cheating will be observed. But these probabilities are determined by the joint behavior of both sides: the likelihood of cheating depends on the probability of being observed, which depends on the likelihood of cheating. The game theory analysis allows us to disentangle these interwoven strategies.

Since an arms control agreement is not enforceable by a third party, one needs to ask why an agreement to go back to earlier levels would be self-enforcing if the previous level was not self-enforcing in the absence of an agreement. I provide the framework of analysis to answer this fundamental question.

More generally, I provide an analysis of behavior when information (regarding performance) is imperfect and both sides can act strategically. Hence, this theory may also be usefully applied to other areas, such as analyzing agreements between OPEC countries, determining the optimal punishment for crimes, and predicting strategic behavior between Congress and the presidency.

The analysis not only provides new insights into applied areas such as arms control but also yields new perspectives on game theory as theory. In the context of a game, if one side (the detector) monitors the other side's postrandomized choice, the game is turned from a simultaneous choice game to a sequential game, with the detector making its move after observing (perhaps imperfectly) the other player's move. The reward structure is considerably different in sequential and simultaneous games. I develop a game-theoretic model incorporating both sequential and simultaneous moves by the players.[1]

I introduce the concept of a simultaneous sequential game in the context of a zero-sum game. I demonstrate that if the detection equipment is less than perfect, the detector's optimal strategy may be to ignore the detection equipment.[2] I show that this strategy is equivalent to a simultaneous game where the detector sometimes chooses to act contrary to one of the signals. In turn, this is equivalent to the detector's trading off a higher rate of type 2 error for a lower rate of type 1 error. Then I model an arms control agreement as a nonconstant sum game. I give mathematical precision to some of the implicit assumptions made in the literature regarding the costs and benefits of arms control. I derive propositions relating the likelihood of cheating and the optimal use of the monitoring equipment to the accuracy of the monitor and show that the greater the accuracy of the monitor, the less likely it will be used. I also determine the condi-

Table 1. The Simultaneous-Sequential Game

	Simultaneous Game (C Does Not Look at Detector)		Sequential Game (C Looks at Detector)	
Row	C Assumes R Chose Row 1 (C_1, C_1)	C Assumes R Chose Row 2 (C_2, C_2)	Pure Strategy (C_1, C_2)	Mixed Strategy $(VC_1 + [1 - V]C_2, WC_2 + [1 - W]C_1)$
1	X_{11}	X_{12}	$PX_{11} + (1 - P)X_{12}$	$P[VX_{11} + (1 - V)X_{12}] + (1 - P)[WX_{12} + (1 - W)X_{11}]$
2	X_{21}	X_{22}	$QX_{22} + (1 - Q)X_{21}$	$Q[WX_{22} + (1 - W)X_{21}] + (1 - Q)[VX_{21} + (1 - V)X_{22}]$

Note: R maximizes. $X_{12}, X_{21} > X_{22}, X_{11}$. C minimizes.

tions for an arms control agreement to be feasible. Then I consider the conditions whereby an arms control agreement can produce an equilibrium even though the reduced level of armaments is not a Cournot equilibrium in the absence of an agreement. Essentially, the answer lies in the creation of a new matrix of outcomes generated by the agreement. In particular, I show the important role of verification. Finally, I extend the results to other areas including the punishment of criminals and the monitoring of agents.

Beyond these specific results, I create a framework of analysis for investigating arms control. I suggest some important issues to consider and appropriate questions to ask and delineate some key parameters that scientists of arms control should estimate and policy makers must determine. Thus the theory presented here creates a research agenda for future empirical work and a list of critical issues for future commentary on the topic of arms control.[3]

The Simultaneous-Sequential Methodology in the Context of Constant-Sum Detector Games

I present the simultaneous-sequential game, demonstrate that imperfect detection leads to mixed strategies, and show the equivalence between the simultaneous sequential approach and choosing a point on the type 1–type 2 trade-off curve. Because this section is devoted to constant-sum detector games, it is not appropriate for analyzing arms control, which is more accurately characterized as a non-constant-sum game. However, the analysis here serves as an important stepping-stone to the more complex non-constant-sum game and is valuable on its own terms for bringing insight to issues of detection in constant-sum games.

In order to make the logic as clear as possible I first consider the simultaneous game (no detection). The normal form representation of the simultaneous game comprises the first two columns in Table 1. I assume that R (row) maximizes, C (detector) minimizes, and $X_{12}, X_{21} > X_{11}, X_{22}$. Thus, no one row (or column) in the *simultaneous* game dominates another row (column).[4] The extensive form is considered in Appendix A.

We will now turn our attention to the sequential game. If R chooses R_i (possibly after a randomized draw from a mixed strategy between rows 1 and 2), there is some probability that C will detect that R has made this choice before C makes its choice (or while C can still alter its choice). Letting σ_j be the signal that R_j has been chosen, the probability of the monitoring equipment's signaling that R_j has been chosen when R has chosen R_i will be denoted by $\text{Prob}(\sigma_j \mid R_i)$. The probabilities of correct and incorrect detection given differing strategy choices (R_i) by R are then defined as follows:[5]

$$\text{Prob}(\sigma_1|R_1) = P; \text{Prob}(\sigma_2|R_1) = 1 - P; \text{Prob}(\sigma_2|R_2) = Q; \text{Prob}(\sigma_1|R_2) = 1 - Q.$$

I assume that $1 > P, Q > 1/2$. Since there are only two choices, the detector is better than a blind guess. It is possible that P (the probability of correct detection when row 1 is chosen) and Q (the probability of correct detection when row 2 is chosen) differ. For example, row 1 may be a maintenance in armaments, while row 2 may be an increase in the level of armaments. It may be hard to detect an increase in arms, in which case the detector has a high likelihood of mistakenly attributing row 1 (arms maintenance) when row 2 (arms increase) is true. However when row 1 (arms maintenance) is the case, attributing row 2 (arms increase) may be very unlikely.[6]

The sequential game, as the name implies, involves a sequence. R chooses a row (R_i). C is not able to observe this choice; however, the detection equipment emits a noisy signal (σ_1 or σ_2), which C is able to observe. C then chooses a strategy, C_j (in the sequential game the C_j are conditional on the observation of the signal and stand for the columns in the simultaneous game). The resulting outcomes are denoted by X_{ij}.

We are now ready to incorporate the possibility of detection (with error) into an expanded game matrix (cols. 3 and 4 in Table 1). Column 3 represents payoffs of the sequential game when C (the detector) undertakes the following pure strategy: if the signal indicates that R_1 has been chosen (that is, σ_1 is observed), C chooses C_1; if the signal indicates that R_2 has been chosen, C chooses C_2. At the top of column 3 the vector C_1, C_2 represents C's response to signals σ_1 and σ_2, respectively.[7] It should be noted that C_i is C's best response to R_i since $X_{12}, X_{21} > X_{22}, X_{11}$.[8] When R chooses R_1, $100P\%$ of the time the detector will yield the correct signal, σ_1, that R has indeed chosen R_1. If C chooses C_i in response to σ_i, $100P\%$ of the time the outcome will be X_{11}, and $100(1 - P)\%$ of the time the outcome will be X_{12}. Hence, the outcome is that represented in the first row of column 3. A similar analysis will yield the second row of column 3. Note that column 3 involves sequential moves by C after R has made its postrandomized choice but that the decision whether to use column 3 in the first place is made simultaneously with—or in the absence of information about—R's decision. Thus, the decision to act sequentially is made simultaneously. The first two columns in Table 1 represent the simultaneous game, while the third column represents the sequential game. Hence, the first 3 columns represent a simultaneous-sequential game.

Column 4 represents the outcome of the sequential game when C (the detector) may act contrary to the detector by undertaking a mixed strategy in response to a signal. V is the probability of C's choosing C_1 and $1 - V$ is the probability of C's choosing C_2 when σ_1 is observed; while W is the probability of C's choosing C_2 and $1 - W$ is the probability of C's choosing C_1 when σ_2 is observed. Looking at row 1, column 4, when R chooses R_1, C will observe σ_1 with probability P; and given that σ_1 has been observed, C will choose C_1 (yielding outcome X_{11}) with probability V and will choose C_2 (yielding outcome X_{12}) with probability $1 - V$. There is also a probability of $1 - P$ that C will observe σ_2 when R chooses R_1. Given that σ_2 has been observed, the outcome will be X_{12} with probability W and X_{11} with probability $1 - W$. At the top of column 4, the vector $VC_1 + (1 - V)C_2$, $WC_2 + (1 - W)C_1$ represents C's response to signals σ_1 and σ_2, respectively.

Each of the first three columns in Table

1 are special cases of column 4. If $V, W = 1$, column 4 is equivalent to column 3; if $V = 0$ and $W = 1$, column 4 is equivalent to column 2; and if $V = 1$ and $W = 0$, column 4 is equivalent to column 1. Thus, column 4, the sequential game where C may act contrary to the signal, spans the same space as columns 1, 2 and 3, the simultaneous-sequential game (the simultaneous part is that C may ignore the signal entirely; the sequential part that if C makes use of the signal, it will never act contrary).

PROPOSITION 1. *C's optimal strategy is a mixed strategy.*

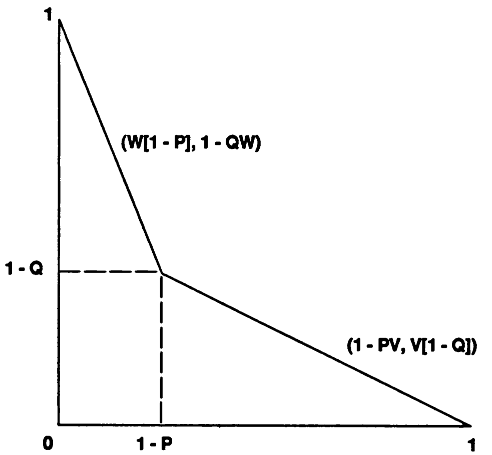
Proof. According to Nash (1951) an equilibrium exists. If either $0 < V < 1$ or $0 < W < 1$, column 4 is a mixed strategy. Note that in columns 1, 2, and 3 and rows 1 and 2 no pure strategy equilibrium exists. Therefore, there must be a mixed strategy equilibrium. Thus, if column 3 is used (as part of C's mixed strategy), it must be used with probability less than 1; and if column 4 is used, column 4 itself involves mixed strategies. QED.

PROPOSITION 2. *A mixed strategy between columns 1 and 3 yields an equivalent outcome to the mixed strategy within column 4 (when $V = 1$); while a mixed strategy between columns 2 and 3 yields an equivalent outcome to the mixed strategy within column 4 (when $W = 1$). These mixed strategies are superior to any other mixed strategy. The optimal strategy by row also involves a mixed strategy.*

The proof is in Appendix B. Here I will provide the intuition. Proposition 2 states that an optimal strategy by C requires that some percentage of the time, either C acts contrary to one (and only one) of the signals emanating from its detection equipment or (equivalently) C ignores its detection equipment entirely (and acts without any knowledge of the signal).[9] The intuition for the equivalence result can be established via an example. Looking at Table 1, consider the case where C chooses column 1 with probability .1 and column 3 with probability .9. If R chooses R_1, X_{11} will occur with probability $.1 + .9P$; if R chooses R_2, X_{22} will occur with probability $.9Q$. I will now compare this to the result where C chooses column 4, $V = 1$, and $W = .9$. Looking at column 4, if R chooses R_1, X_{11} will occur with probability $P + (1 - P)(1 - .9) = .1 + .9P$; if R chooses R_2, X_{22} will occur with probability $.9Q$. Hence, both methods lead to the same result. The decision to not use the monitor has nothing to do with its being costly to use in the conventional sense, for I have not assumed that there is a cost (in man-hours, for example) to using it.

The idea of a mixed strategy equilibrium may be a hard pill to swallow. However, I will now show that it is equivalent to a pure strategy equilibrium when the monitoring technology is allowed to vary in a specified way. I have characterized the error possibilities as being $(1 - P)$ and $(1 - Q)$. These can be seen as parameters for a family of type 1 and type 2 errors, respectively. For example, consider a smoke alarm, where smoke is row 2 and no smoke is row 1. The probability of the alarm going off when there is no smoke (type 1 error) is $1 - P$; while the probability of the alarm not going off when there is smoke (type 2 error) is $1 - Q$. A smoke alarm can be adjusted so that it is less sensitive to particles in the air. In this way, the probability of a type 1 error (the likelihood of the alarm going off when there is no fire) is reduced. However, this adjustment will increase the probability of a type 2 error. Hence an adjustable smoke alarm is a trade-off curve between type 1 and type 2 errors rather than a point $(1 - P, 1 - Q)$. Figure 1 is a kinked linear type 1-type 2 error trade-off curve, the set of points being $(W[1 - P], 1 - QW)$ for $0 \leq W \leq 1$ and $(1 - PV, V[1 - Q])$ for 0

Figure 1. The Type 1–Type 2 Error Trade-off Curve



Note: The vertical axis is the probability of a type 2 error; the horizontal axis is the probability of a type 1 error.

$\leq V \leq 1$. If $W = 1$, the probability of type 1 error (given that R has chosen r_1) is $1 - P$, while the probability of a type 2 error (given that R has chosen r_2) is $1 - Q$; that is, we are at the kink. If $W = 0$ (that is, C chooses a monitoring technology that always calls for C_1 to be chosen), the probability of a type 1 error is zero and the probability of a type 2 error (given R_2) is 1 (upper left-hand corner of the graph). If the trade-off curve is the one depicted in Figure 1, we have the same set of options as in column 4, Table 2 (for $V = 1$, the probability of a type 1 error, X_{12}, is $W[1 - P]$ and the probability of a type 2 error, X_{21}, is $Q[1 - W] + 1 - Q = 1 - QW$). However, W is no longer a randomized strategy but instead represents a particular technology choice along the type 1–type 2 trade-off curve.[10]

I have considered three alternative viewpoints that on closer inspection are equivalent. These are (1) ignoring the detection equipment, (2) going contrary to the detection equipment, and (3) choosing a particular point on the type 1–type 2 (kinked line) trade-off curve. I will now concentrate on the simultaneous-sequential approach where C may at times make choices in the absence of knowledge concerning the detector. While my analysis will be in terms of "ignoring the detector," the reader may choose to substitute "go contrary to the detector" or "choose a higher rate of type 2 error."

The Matrix of Outcomes in Non-constant-Sum Arms Control Detector Game

The simultaneous-sequential framework can be used to analyze any game. I concentrate on arms control. It is not easy to model arms control as a game. The ordinal relationships I posit are reasonable, but arguments can be made for other rankings. Consider the expressed fear that the other side may cheat in the presence of an arms control agreement. Each side may try to convince the other that it has reduced its weaponry when it has in fact not done so. In particular, a side that intends to attack will underreport its armaments, hoping to lull the other side into a false sense of security. However, arguments can be made for lying in the other direction as well. A country that fears attack will exaggerate its armaments, thereby hoping to dissuade the other side from attacking.[11] Exaggeration may also help an aggressor country get its way without resorting to war. I will rule out exaggeration because underreporting of armaments is the perceived concern in arms control. In particular, if there is no arms control agreement, I assume that neither R nor C has surreptitiously reduced its armaments.

The players have two games: (1) no arms control and (2) arms control, encompassing a possible return to a situa-

Table 2. The Arms Control Game

Arms Control Game	Simultaneous Game (C Does Not Look at Detector): C Assumes R Chose Row 1 (C_1, C_1)	Simultaneous Game (C Does Not Look at Detector): C Assumes R Chose Row 2 (C_2, C_2)	Sequential Game (C Looks at Detector): Pure Strategy (C_1, C_2)	Sequential Game (C Looks at Detector): Mixed Strategy $(VC_1 + [1 - V]C_2, WC_2 + [1 - W]C_1)$
Row 1: Lives up to Arms Control Agreement	X_{11}, Y_{11}	$X_{\bar{3}\bar{3}}, Y_{\bar{3}\bar{3}}$	$PX_{11} + (1 - P)X_{\bar{3}\bar{3}}$, $Py_{11} + (1 - P)Y_{\bar{3}\bar{3}}$	$P[VX_{11} + (1 - V)X_{\bar{3}\bar{3}}] + (1 - P)[WX_{\bar{3}\bar{3}} + (1 - W)X_{11}]$, $P[VY_{11} + (1 - V)Y_{\bar{3}\bar{3}}] + (1 - P)[WY_{33} + (1 - W)Y_{11}]$
Row 2: Cheats on Arms Control Agreement	X_{21}, Y_{21}	$X_{\bar{3}\bar{3}}, Y_{\bar{3}\bar{3}}$	$PX_{33} + (1 - P)X_{21}$, $PY_{\bar{3}\bar{3}} + (1 - P)Y_{21}$	$P[VX_{\bar{3}3} + (1 - V)X_{21}] + (1 - P)[WX_{21} + (1 - W)X_{\bar{3}3}]$, $P[VY_{\bar{3}\bar{3}} + (1 - V)Y_{21}] + (1 - P)[WY_{21} + (1 - W)Y_{\bar{3}\bar{3}}]$

Note: $X_{21} > X_{11} > X_{33} > X_{\bar{3}3} > X_{\bar{3}\bar{3}}$; $Y_{11} > Y_{33} > Y_{\bar{3}\bar{3}} > Y_{21}$.

tion of no arms control (see Tables 2 and 3). If R announces that it has not reduced its armaments (that is, there is no arms control agreement), C knows that the no-arms-control-agreement game is being played. If R announces that it has reduced armaments, C knows that R has chosen either row 1 or row 2 in the arms control game. The arms control game is played only if both sides have a higher expected payoff under the arms control game than that under a situation of no arms control. Within the arms control game, R has two pure strategies available, and C has three (columns 1, 2 and 3).

To simplify, I assume that C is honest with regard to arms control (perhaps R has a perfect detector, and any violation of the agreement by C will be detected). Thus, C's decision regarding arms control is implicitly built into both matrices (see Table 2). If R has not agreed to arms control, neither has C, and the outcome is X_{33}, Y_{33} in the no-arms-control game matrix. If R has agreed to arms control, C has also agreed to arms control, and there is no cheating on the agreement by C in the arms control game matrix. In Appendix C the completely symmetrical case,

Table 3. No Arms Control Game

No Arms Control Game	C
R	X_{33}, Y_{33}

with both sides detecting and having the option of cheating, is investigated.

I assume that in the arms control game if C chooses C_2 (C "believes" that R has cheated), the situation reverts to that of no arms control.[12] To simplify the discussion I assume that $P = Q$.

Looking mainly at the first two columns (the simultaneous part) of the arms control game in Table 2, I make the following additional assumptions (concentrating at first mainly on R).

1. $X_{11} > X_{33}$ and $Y_{11} > Y_{33}$; (1)

   that is, there exists some arms reduction agreement that if fully enforced and maintained, would make both sides better off.[13]

2. $X_{21} > X_{11}$. R prefers X_{21} (not reducing arms at all or not to the level stipulated in the agreement but having C believe it has reduced arms to the stipulated level) over X_{11} (reducing armaments and having C believe it). I make this assumption since this is the perceived problem in arms control—that the country gains by cheating on the agreement if it can avoid detection.[14]

3. $X_{\bar{3}3} < X_{33}$. When C detects a violation before R can benefit from it (by employing strategy C_1, C_2 of the sequential game and observing σ_2) or acts as if a violation has occurred (by employing strategy C_2, C_2 in the simultaneous game), the outcome is very similar to the outcome when there is no arms control agreement, X_{33}, except for an initial difference.[15] For example, if arms control means destruction of easily detected armaments by R but R cheats and fails to reduce arms that cannot be easily detected, going back to a situation of no arms control implies that R will be replacing the recently destroyed arms. Under this characterization R's cheating and being caught before it can gain any benefit would be worse for R than never having had arms control in the first place, although for the most part the outcome would be similar to the outcome if there had never been arms control. Therefore, to characterize this stream of events I will use the notation $X_{\bar{3}3}$ ($< X_{33}$).

4. $X_{\bar{3}3} > X_{\bar{3}\bar{3}}$. If R has indeed lived up to the arms control agreement but C mistakenly believes that R has violated the agreement, once again the outcome is basically similar to that of no arms control. However, the difference in level of armaments between the two states of the world is greater than if R had actually cheated and been caught. Therefore, I connote the outcome by $X_{\bar{3}\bar{3}}$ ($< X_{\bar{3}3}$).[16]

5. $0 < Y_{21} < Y_{ij}$ (for $ij \neq 21$). This concerns C's preferences. The worst outcome for C is to believe that (or act as if) R has lived up to an arms control agreement when R has actually cheated on the agreement. Once again (but now looking from the viewpoint of C), this is the perceived fear.[17]

6. $Y_{33} > Y_{\bar{3}\bar{3}}$. C prefers no arms control over the following stream of events: an initial arms control agreement, believing that R has cheated, and a resulting dissolution of the agreement. The line of reasoning follows that for R. Thus, for example, destruction and replacement of arms is more costly to C than is not destroying them in the first place.[18]

The arms control game has four columns. Columns 1 and 2 are the simultaneous game; columns 3 and 4 are the sequential game. Column 4 is the mixed strategy response to signals σ_i. In column 3, $V = W = 1$; hence, column 3 is a pure strategy response to signals σ_i and a special case of column 4.[19]

Equilibrium in Arms Control Detector Games

The Optimal Strategy

I now determine each side's optimal strategy under an arms control agreement. I will forego the tedious proof that column 4 (with $V = 1$) is equivalent to a mixed strategy between columns 1 and 3; and instead ignore column 4 in the analysis. The reader is free to substitute the words "C acts contrary to the signal that R has cheated" or "C chooses a higher type 1 error rate" for the phrase "C acts in the absence of knowledge regarding the detection equipment and assumes that R has not cheated."

In the following proposition I show that any arms control agreement involves some chance that R will cheat and that C will choose to increase the probability of a type 1 error by at times ignoring evidence of cheating by R. I also show that an arms control agreement is impossible if R can improve its welfare by cheating when C is using the monitor.

PROPOSITION 3. *If the detection equipment is less than perfect ($1/2 < P = Q < 1$), it holds that (1) the arms control agreement game has a unique pure strategy equilibrium, and R and C prefer the no-arms-control outcome over this pure strategy equilibrium; (2) there exists a mixed strategy equilibrium under an arms control agreement if and only if $P(X_{11}) + (1 - P)X_{\bar{3}\bar{3}} > P(X_{\bar{3}3}) + (1 - P)X_{21}$, that is, when C uses the detector, R's expected utility is higher when R is honest; (3) C prefers no arms control to any equilibrium that includes (C_2, C_2).*

Proof of Part 1. Row 2, column 2 is a pure strategy equilibrium. If R chooses row 2, C's best strategy is to choose column 2 since $Y_{\bar{3}\bar{3}} > Y_{21}$ by assumption. If C chooses column 2, R's best strategy is to choose row 2, since $X_{\bar{3}3} > X_{\bar{3}\bar{3}}$. $X_{\bar{3}3} < X_{33}$ and $Y_{\bar{3}\bar{3}} < Y_{33}$. Therefore, both R and C would prefer no arms control agreement to such a pure strategy equilibrium. It is quite easy to establish that no other pure strategy Nash equilibrium exists. The only other pure strategy available to R is to choose row 1. C's best choice, given row 1, is to choose column 1; but R's best choice, given column 1, is to choose row 2. Thus no other pure strategy equilibrium exists. Since R, as well as C, prefers no arms control to the outcome of an arms control equilibrium in pure strategies, if an arms control agreement exists, a mixed strategy equilibrium must be possible.[20]

Proof of Part 2.[21] A necessary condition for a mixed strategy equilibrium is that row 2 does not dominate row 1. Since $X_{21} > X_{11}$ and $X_{\bar{3}3} > X_{\bar{3}\bar{3}}$, a necessary condition for a randomized equilibrium is that the reverse relationship holds for column 3: $P(X_{11}) + (1 - P)X_{\bar{3}\bar{3}} > P(X_{\bar{3}3}) + (1 - P)X_{21}$; that is, when C uses (only) the imperfect monitor (column 3), the expected utility to R is greater when R is honest than when it cheats on the arms control agreement. From now on I will assume that these necessary and sufficient conditions exist. Note that for $P = .5$ these conditions are not satisfied and that as P increases, these conditions are more likely to be satisfied. Thus the quality of the monitoring may play an important role in the feasibility of an arms control agreement.[22]

Proof of Part 3. I will provide the underlying intuition for the proof (see Wittman 1987 for the details). In a mixed strategy equilibrium, randomization by R makes C indifferent between the two columns it has chosen for its mixed strategy. If column 2 is one of C's choices, C can expect $Y_{\bar{3}\bar{3}}$, which is less than the outcome if there is no arms control agreement (Y_{33}). Therefore, Y would not enter into this agreement in the first place. QED

Thus, if there is an arms control agreement, it involves a mixed strategy equilibrium with C sometimes not observing its monitoring equipment and assuming that R has been honest (C is choosing a greater rate of type 1 errors). Proposition 3 may provide an explanation for why one side may choose to ignore information that appears to show that the other side has violated the agreement.

The Effect of Monitoring Quality on C and R's Behavior

So far the focus has been on the existence or nonexistence of particular pure and mixed strategy equilibria. We now turn our attention towards comparative statics in order to discover the relationships between the quality of the monitoring device, the costs and benefits to the players, and the equilibrium strategies.

$$\text{Let } K \equiv (Y_{\overline{33}} - Y_{21})/(Y_{11} - Y_{\overline{33}}). \quad (2)$$

Note that in the first two columns in Table 2, K is the ratio of C's loss from mistakenly believing that R has been honest to C's loss from mistakenly believing that R has been dishonest. With honesty by R as the null hypothesis, K is the ratio of the loss from a type 2 error to that from a type 1 error.

Let c_1 stand for the probability of C's choosing column 1 and r_1 stand for the probability of R's choosing row 1.

PROPOSITION 4. *(1a) As K increases, so does r_1 (the probability of R's choosing row 1, not cheating); (b) As $K \to 0$, $r_1 \to 0$; (c) At $K = 1$, $r_1 = P$ (the probability that the detector is correct); (d) As $K \to \infty$, $r_1 \to 1$; (2) As P increases, so does r_1.*

The proof is in Appendix D. Here I will provide the explanation and the intuition.

I first consider part 1. At a Cournot-Nash equilibrium, C's optimizing strategy makes R indifferent between the rows, and R's optimizing strategy makes C indifferent between the columns. C is choosing between columns 1 and 3 and in equilibrium is indifferent between them (see Table 2). If Y_{21} decreases (thereby increasing K) and R does not change its equilibrium strategy, C will prefer column 3 (since C's expected utility is reduced only by $r_2(1 - P)$ times the change in Y_{21} when C chooses column 3 instead of being reduced by r_2-times-the-change-in-Y_{21} when C chooses column 1). Therefore, in order to make C once again indifferent, R increases its probability of choosing row 1, thereby increasing C's expected utility from choosing column 1 (relative to column 3). Hence, in equilibrium, R acts like an altruist—as the relative cost to C from R's cheating increases (i.e., K increases), the probability of R's cheating decreases in order to maintain C's indifference as to the choice of column.

When $K < 1$ (that is, when the cost to C of mistakenly believing that R has been honest is relatively low), the probability of R's cheating $(1 - r_1)$ will be greater than the probability of C's monitor being incorrect $(1 - P)$; and when K is greater than 1, the probability of R's being honest (r_1) will be greater than the probability of C's monitor being correct (P).[23] When Y_{33} is approximately equal to $Y_{\overline{33}}$, an approximate rule of thumb is that $r_1 > P$ if the outcome under a situation of no arms control is closer to that when R is honest and C believes R has been honest than to that when R cheats on an arms control agreement contrary to C's belief.

Now I turn to part 2. When the quality of C's detector improves, R will be more likely to be honest. The logic is best understood by again starting with the equilibrium outcome where C is initially indifferent between columns 1 and 3. When the quality of the detector improves, the expected payoff to C from using the detector (column 3) increases. If R did not alter its strategy, C would prefer column 3. Therefore, R makes C indifferent by increasing the probability of R's not cheating (that is, increasing r_1).

I next consider the relationship between P and c_1.

PROPOSITION 5. *As P (the quality of the monitor) increases, $1 - c_1$ (the likelihood of C's using the monitor) decreases; (2) The smaller X_{33} and $X_{\bar{3}\bar{3}}$ are, the less likely C will use its detector and the more likely it will assume that R has been honest, that is, when a termination of the arms control agreement significantly reduces R's utility, C is more likely to assume that R has been honest—c_1 increases; (3) The greater the payoff to R from cheating and not getting caught, the greater the probability that C will use its monitor; that is, as X_{21} increases, c_1 decreases.*

The proofs are in Appendix E. Part 1 states that the better the detection equipment, the less likely the detector will use it and the more likely the detector will implicitly assume that the other side is honest by undertaking strategy C_1, C_1. The explanation for this counterintuitive result is that when C's detection equipment improves, R's optimal strategy is to be honest more often (as demonstrated in proposition 4); thus C can partially take advantage of this increased honesty by assuming that R has been honest.

The game theory logic for part 2 is that if R has more to lose from a failure of the arms control agreement, C makes R indifferent between the two strategies by assuming more often that R is honest. For part 3, a gain to R from cheating and getting away with it results in C's lowering the probability of R's assuming that C has not cheated.

The Conditions for an Arms Control Agreement To Be Possible

Now I turn to the most critical issue: Is an arms control agreement possible? A necessary condition for an arms control agreement is that both sides be better off than under no agreement.[24] The conditions for an arms control agreement yielding greater expected utility to R than no arms control are contained in the following proposition.

PROPOSITION 6. *If $PX_{11} + (1 - P)X_{\bar{3}\bar{3}} > X_{33}$, R prefers the arms control equilibrium to the outcome when there is no arms control.*

The proof is in Appendix F. Since I have assumed that $P > 1/2$ and that $X_{\bar{3}\bar{3}}$ is relatively close to X_{33}, this condition is likely to be met.

Not surprisingly, the conditions for C's entering into an arms control agreement are more complex, as can be seen in proposition 7.

PROPOSITION 7. *For an arms control equilibrium to be preferred by C to the equilibrium under no arms control, the following inequality must hold:*

$$\frac{Y_{\bar{3}\bar{3}}(KP + P) + (1 - 2P)Y_{21}}{1 - P + PK} \geq Y_{33}.$$

The proof is found in Appendix G. In order to get a feel for the conditions implied by this inequality I will consider some polar cases. It is easy to establish that an arms control agreement is impossible when the monitoring equipment is no better than a blind guess (i.e., $P = 1/2$). At $P = 1/2$, the left side of the inequality in proposition 7 reduces to $Y_{\bar{3}\bar{3}}$. $Y_{\bar{3}\bar{3}}$ is less than Y_{33} by assumption. At $P = 1$, an arms control agreement is always possible. Letting $P = 1$ and making use of the definition for K (equation 2), the left side of the inequality reduces to

$$Y_{\bar{3}\bar{3}} + \frac{Y_{\bar{3}\bar{3}} - Y_{21}}{K} = Y_{\bar{3}\bar{3}} + (Y_{\bar{3}\bar{3}} - Y_{21})\left(\frac{Y_{11} - Y_{\bar{3}\bar{3}}}{Y_{\bar{3}\bar{3}} - Y_{21}}\right)$$

$$= Y_{11} > Y_{33},$$

by assumption.

Finally, at

$$K \equiv \frac{Y_{\bar{3}\bar{3}} - Y_{21}}{Y_{11} - Y_{\bar{3}\bar{3}}} = 1,$$

the left side of the inequality reduces to

$$Y_{\bar{3}\bar{3}} + (2P - 1)(Y_{\bar{3}\bar{3}} - Y_{21}) = Y_{\bar{3}\bar{3}} + (2P - 1)(Y_{11} - Y_{\bar{3}\bar{3}}).$$

Thus, for a $P = .75$, any differential between Y_{11} and $Y_{\bar{3}\bar{3}}$ greater than twice the differential between Y_{33} and $Y_{\bar{3}\bar{3}}$ will mean that C's expected utility under an arms control agreement equilibrium is greater than C's expected utility when there is no arms control.

The inequality relationship in proposition 7 may also provide insight into the effect of increased armament reduction. With fewer armaments remaining, Y_{21} (the cost to C of mistakenly believing that R has been honest) is likely to increase. In turn, this reduces the left-hand side of the inequality in proposition 7 as Y_{21} is multiplied by $(1 - 2P)$, a negative number. Hence the need for improved verification. Yet this is not the complete story as the left-hand side of the inequality also suggests that we must consider other variables, such as the costs of a failed disarmament treaty, in deciding whether an arms control agreement is possible.

It has been demonstrated that any arms control agreement in which detection is less than perfect involves some strictly positive probability of R's cheating and some strictly positive probability of C's not paying attention to its detection equipment (C_1, C_1 is chosen with probability greater than 0). Even though the probability of undetected cheating's occurring is greater than zero, an arms control agreement is appropriate for R and C if the expected utility is greater than under a situation of no arms control. The possibility of undetected cheating is not a sufficient reason for not having an arms control agreement. As in other areas, the expected costs and benefits must be weighed. To quote President Kennedy's message to the Senate (1963) regarding a nuclear test ban: "The risks of detection outweigh the potential gains from violation, and the risk to the United States from such violation is outweighed by the risk of a continued unlimited nuclear arms race."[25] An example of this thinking can be found in the U.S.-Soviet treaty prohibiting nuclear underground tests above 150 kilotons. It is impossible to distinguish between 153 kilotons and 147, but such minor cheating would have little effect on the balance of power.[26]

Self-enforcing Agreements

An arms control agreement between two superpowers is not enforced by a third party. Why, then, should there be an arms control agreement equilibrium if there is no equilibrium at that level of armaments in the absence of an arms control agreement? To put the problem in concrete terms, If an arms control agreement puts both parties back to the level of armaments that each had two years previously, what forces exist now (which did not exist previously) to maintain the equilibrium level of armaments at the two-year-old level? Here I will provide some of the possible explanations.

The arms control equilibrium may be unstable, or one of many possible equilibria. Thus, if newly developed monitoring capabilities or new information about the greater costs of war create a new, low level-of-armaments equilibrium, simple Cournot behavior may not lead to it. Simultaneous behavior (i.e., non-Cournot behavior) by both parties by means of an explicit arms control agreement may be necessary to get back to the equilibrium encompassing low levels of expenditures on armaments.[27]

The arms control agreement itself may change the payoff matrix, thereby creating equilibria that were not possible in the absence of an agreement. Thus, by agreement, each side may allow the other side to have on-sight surveillance (thereby increasing P). As shown earlier, the probability of detecting increased armaments must be of a certain size before arms control is better than no arms control. Both sides may require improved monitoring, and without simultaneous improvements in monitoring the equilibrium may not be achieved by Cournot behavior. Thus, once again, verification plays an important role. The recent International Nuclear Forces treaty may have been implemented because of the increased Soviet willingness for on-sight inspection.

The agreement may also involve the simultaneous exchange of "hostages."[28] Providing hostages without a *quid pro quo* will typically hurt the hostage-giving country. So Cournot behavior would not result in an exchange of hostages. Only an agreement involving the simultaneous exchange of hostages by both parties may be possible. The matrix of payoffs is then changed (e.g., X_{21} is reduced), thereby reducing the benefit of increased arms.

Extensions to Other Areas

The framework developed here can be applied to other areas, such as the punishment of criminals and the accountability of politicians. The main alteration is to the elements of the two-by-four matrix. Clearly, different objective functions and different levels of detection are likely to alter the payoff structures and the optimal strategies by the players. In particular, for some applications, pure strategies might dominate mixed strategies.

It is insightful to contrast the approach used in this paper with previous work outside of the arms control literature. I first consider research concerned with the optimal punishment of criminals. Becker (1968) and, more recently, Polinsky and Shavell (1979) consider the possibility that not all criminals are detected and convicted; but they have not considered, as I have here, the other type of error—that innocent people may be convicted. However, from society's (the detector's) view, the possibility of punishing an innocent person should be a very important consideration.

In principal-agent relationships there may be imperfect monitoring. An incumbent (agent) who voters mistakenly believe has shirked (not acted in the interests of the majority) may lose an election, and a shirking incumbent who voters mistakenly believe has implemented the interests of the majority may be reelected. The principal-agent literature assumes that the principal always acts according to its monitoring equipment. It does not consider the possibility that the principal might choose to ignore the information, nor does it consider the role of type 1 and type 2 errors on the principal's or the agent's optimal strategies.[29]

As another example, consider OPEC agreements to limit production. There is considerable potential for undetected cheating. The failure to hold prices in line suggests that quotas are not being rigidly adhered to. If there is imperfect monitoring, verification of OPEC agreements has obvious similarities to the verification problem in arms control.

From a more general perspective, theorists have developed two separate models of behavior, one involving simultaneous play games and the other involving sequential play. As shown here, imperfect monitoring (in comparison to no monitoring) alters the simultaneous game into a hybrid simultaneous-sequential game; similarly, imperfect monitoring (in comparison to perfect monitoring) by the second player in a sequential game alters the pure sequential game into a hybrid simultaneous-sequential one. The methodology used here integrates these two separate strands into a more general approach where one or both players choose (if both, simultaneously) whether to play the simultaneous or the sequential game.

Conclusion

All actions, including arms control agreements, involve some risk. I have argued that requiring a risk-free arms control agreement is to make the wrong calculations.[30] Rather, one must compare expected utilities with and without arms control. I have shown how to make the appropriate calculations by developing a game theory model incorporating both simultaneous and sequential behavior. The monitored country (*R*) knows that the monitoring country (*C*) may detect the monitored country's choice, albeit imperfectly. The strategy of each country takes this possibility into account as well as the knowledge that each side can replicate the other's thinking. I derive results concerning optimal levels of verification and cheating by each country and demonstrate the inherent randomness of optimal strategic behavior.

I have provided two paradoxes of information that may occur when the evidence is imperfect and both sides engage in strategic behavior: (1) a player should purposely ignore the evidence on occasion, even though the evidence is more likely to be correct than incorrect, and (2) the better the information, the more the player should ignore it. For arms control in particular, this latter paradox implies that improvements in verification technology will lead to a greater willingness to assume that the other side has not cheated. Intuitively, the paradox is partially resolved in light of the realization that the objective of the country is not to maximize the probability of making the correct choice but rather to maximize its expected utility.

Since an arms control agreement is not enforceable by a third party, why would an agreement to go back to earlier levels be self-enforcing if the previous level was not self-enforcing in the absence of an agreement? The explanation is not found in a change in attitude but rather in a change in the payoff matrix. In particular, improved verification capabilities created by the agreement (such as on-sight inspections) may significantly alter the incentives to cheat. In contrast, partial unilateral disarmament adjustments may take place in the absence of an agreement if there is an improvement in monitoring technology not requiring joint action by the countries.

In calculating each country's expected payoff from the arms control game a matrix array of costs and benefits must be determined. These include the cost to country *C* and the benefit to country *R* when *R* cheats (both with and without detection), the cost to each country when one side is falsely accused of cheating, and the benefit to each country when there is no cheating (and there is no accusation of cheating), as well as the probabilities of failing to detect cheating and mistakenly believing that cheating has occurred. These costs, benefits, and probabilities must be considered even by those who undertake a less formal analysis.

Although a matrix of cost-benefit calculations is more complex than a simple pair, this does not mean that empirical hypotheses are impossible. In generating empirical predictions, one can often rely on *ceteris paribus* clauses. For example, other things being equal, the less benefit that a country gains from cheating and the more accurate the monitor, the greater the likelihood that the particular weapon will be subject to an agreement. Since different weapon systems vary in their ability to be detected and their effect on each country's expected payoff, there is a great potential to test the theoretical relationships developed here.

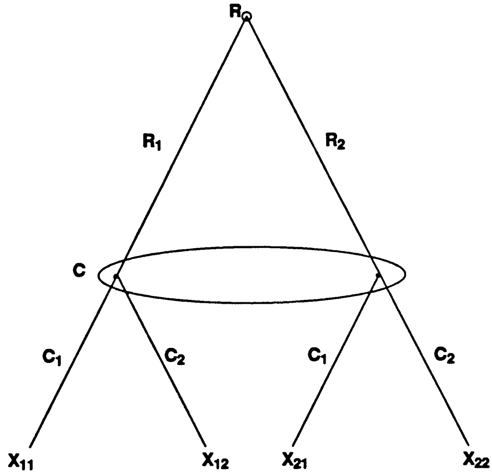
Although the probability of the monitoring equipment's giving a false signal is exogenous, the actual error rate is shown to be endogenous. The detector can improve its welfare by intentionally ignoring the equipment. Equivalently, the detector has a choice of which point on the type 1–type 2 trade-off curve it will choose. To choose the monitoring technology that minimizes type 2 error would itself be in error, since the appropriate monitoring technology maximizes expected utility of the detector. Indeed, by observing the chosen technology with its type 1 and type 2 errors, future researchers may be able to infer something about the utility functions.

The endogeneity of behavior has yielded numerous qualitative relationships. For example, improvement in monitoring will result in a monitored country's being honest more often and the monitoring country's choosing a higher rate of type 2 error (assuming the other side is honest when it is not). This latter proposition may also be tested.

Other interesting extensions and refinements of this argument are possible. For example, the probability of detection and the cost to country C of cheating by country R may be a function of time or of the degree of cheating by R. Also, the level of armaments may be known within a range rather than in terms of an honest-dishonest dichotomy. As a final example, the analysis of type 1 and type 2 errors can be extended to analyze the symmetric case. All of these issues can be incorporated into the basic framework provided here.

The large literature on arms control verification has focused on the technical aspects of monitoring. I have treated the monitoring capabilities as given and instead concentrated on the optimal strategy, given certain levels of risk. I have thus turned what appears to be merely a technical problem into a question of political choice.

Figure A-1. The Extensive Form of the Simultaneous Game



Appendix A: The Game in Extensive Form

In Figure A-1, I have drawn an extensive form version of the simultaneous game. Starting at the root of the tree, R chooses a row; C chooses a column without knowledge of R's choice (hence the balloon around both of R's choices, indicating that C's information set cannot distinguish between the two); at the end of the game tree the X_{ij} outcomes (when R chooses i and C chooses j) are listed.

I next consider the extensive form for the sequential game. Note that in Figure A-2, R chooses a row, C is not able to observe this choice, a noisy signal (σ_i) is emitted, C observes this signal, and C chooses a strategy, C_i (in the sequential game the C_i are conditional on the observation of the signal and stand for the columns in the simultaneous game). The resulting outcomes are denoted by X_{ij}.

The extensive form game presented in Figure A-2 is only a partial characterization of this simultaneous-sequential game. The complete characterization involves both Figures A-1 and A-2 and is drawn in

Figure A-2. The Extensive Form of a Sequential Game

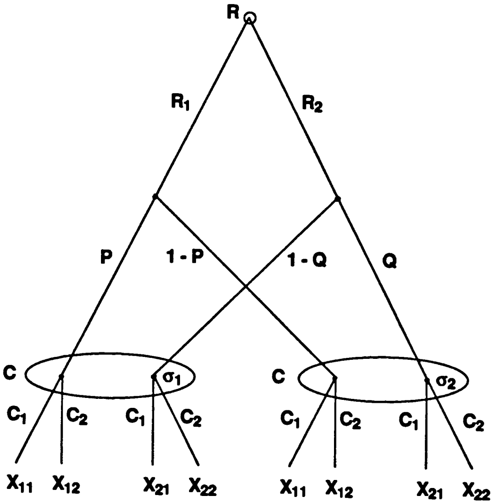


Figure A-3. Essentially, C has the option of choosing the sequential or the simultaneous game without R's knowledge.

Appendix B

Proof of Proposition 2. I first look at the expected outcome when C chooses column 4 and $V = 1$. The expected outcome, when R chooses R_1 with probability r_1 and R_2 with probability r_2, is r_1 times the element in row 1, column 4 plus r_2 times the element in row 2, column 4; that is,

$$r_1\{PX_{11} + (1 - P)[WX_{12} + (1 - W)X_{11}]\} + r_2\{Q[WX_{22} + (1 - W)X_{21}] + (1 - Q)X_{21}\}$$
$$= r_1PX_{11} + r_1WX_{12} - r_1PWX_{12}$$

Figure A-3. Simultaneous-Sequential Game

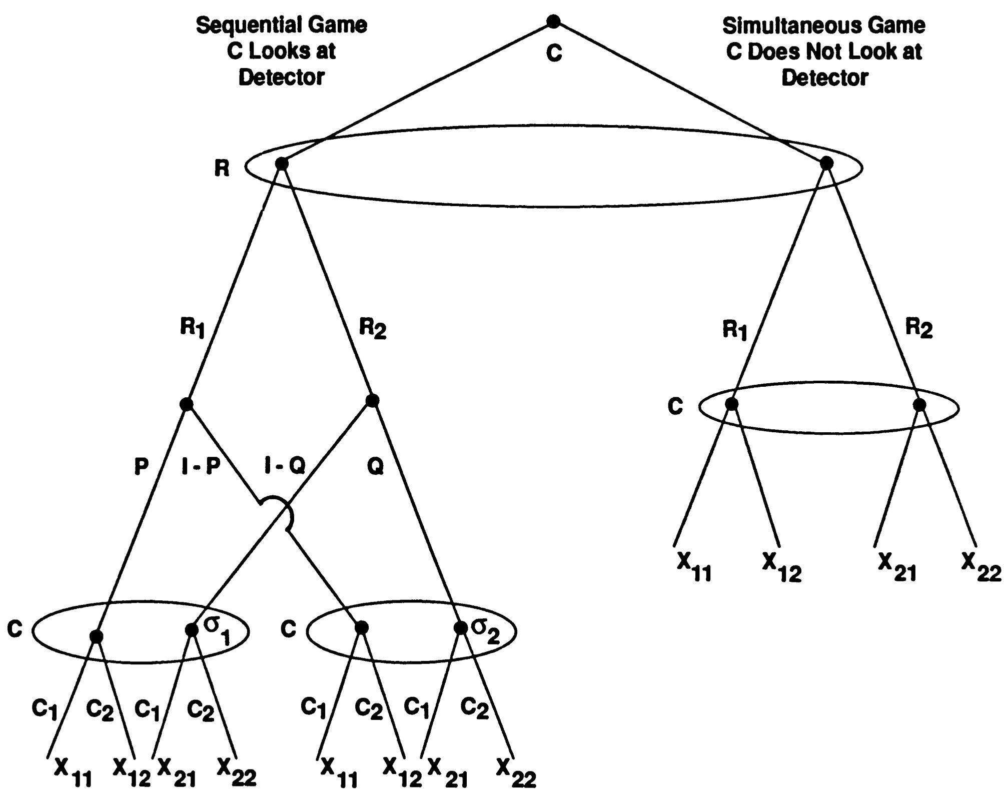


Table A-1. The Symmetric Game

Arms Control Agreement	C Honest: C Does Not Look at Detector: C Assumes R Honest (C_1, C_1)	C Honest: C Does Not Look at Detector: C Assumes R Cheated (C_2, C_2)	C Honest: C Uses P% Detector (C_1, C_2)	C Cheats
R honest				
R assumes *C* honest (R_1, R_1)	X_{11}, Y_{11}	X_{33}, Y_{33}	$PX_{11} + (1 - P)X_{33}$, $PY_{11} + (1 - P)Y_{33}$	X_{12}, Y_{12}
R assumes *C* cheated (R_2, R_2)	X_{33}, Y_{33}	X_{33}, Y_{33}	X_{33}, Y_{33}	X_{33}, Y_{33}
R uses *P*% detector (R_1, R_2)	$PX_{11}+ (1 - P)X_{33}$, $PY_{11} + (1 - P)Y_{33}$	X_{33}, Y_{33}	$P^2X_{11} + (1 - P^2)X_{33}$, $P^2Y_{11} + (1 - P^2)Y_{33}$	$PX_{33} + (1 - P)X_{12}$, $PY_{33} + (1 - P)Y_{12}$
R cheats	X_{21}, Y_{21}	X_{33}, Y_{33}	$PX_{33} + (1 - P)X_{21}$, $PY_{33} + (1 - P)Y_{21}$	X_{33}, Y_{33}

Note: $X_{21} > X_{11} > X_{33} > X_{12}$; $Y_{12} > Y_{11} > Y_{33} > Y_{21}$.

$$+ r_1X_{11} - r_1WX_{11} - r_1PX_{11} + r_1PWX_{11} + r_2QWX_{22} + r_2QX_{21} - r_2QWX_{21} + r_2X_{21} - r_2QX_{21}. \quad \text{(A-1)}$$

I will compare this outcome to the expected outcome when C chooses column 3 with probability W' and column 1 with probability $1 - W'$, and R choose R_1 with probability r'_1 *and* R_2 with probability r'_2:

$$r'_1(1 - W')X_{11} + r'_1W'[PX_{11} + (1 - P)X_{12}] + r'_2(1 - W')X_{21} + r'_2W'[QX_{22} + (1 - Q)X_{21}]$$
$$= r'_1X_{11} - r'_1W'X_{11} + r'_1W'PX_{11} + r'_1W'X_{12} - r'_1W'PX_{12} + r'_2X_{21} - r'_2W'X_{21} + r'_2W'QX_{22} + r'_2W'X_{21} - r'_2W'QX_{21}. \quad \text{(A-2)}$$

Setting $r'_i = r_i$ and $W = W'$, it is readily established that equations A-1 and A-2 are equivalent. A similar process demonstrates that a mixed strategy between columns 2 and 3 is equivalent to choosing column 4 with $W = 1$ and $0 < V < 1$. These mixed strategies are superior to a mixed strategy between columns 1 and 2. I will provide the intuition, which should make the result obvious. Assume that C chooses column 1 with probability P and column 2 with probability $1 - P$. If R chooses r_1, C's expected payout is the same as it would be if C had chosen column 3 with probability 1. However, if R chooses r_2, C's expected payment is $PX_{21} + (1 - P)X_{22}$. This is greater than $QX_{22} + (1 - Q)X_{21}$, C's expected payment when C chooses column 3 with certainty (and R has chosen r_2), since $P, Q > 1/2$ and $X_{21} > X_{22}$. Since C wants to minimize payments, column 3 dominates this particular mixed strategy between columns 1 and 2. Hence, any other mixed strategy between columns 1 and 2 is dominated by another mixed strategy between columns 3 and 1 or 2.[31] The rationale for R's taking a mixed strategy is most apparent when C chooses a mixed strategy between columns 1 (or 2) and 3. No one row dominates. Therefore R chooses a mixed strategy. QED

Appendix C: The Symmetric Game of Detection with Imperfect Detection

I allow for imperfect monitoring of C by R, thereby creating a symmetric relationship between the two players. Looking at Table A-1, I make the following additional notational conventions and assumptions:

1. If either side is honest with regard to arms control but believes, either by detection (possibly mistaken) or by assumption, that the other side has cheated before it is too late to do anything about it, the arms control agreement is broken and the outcome reverts to that of no arms control (X_{33}, Y_{33}). Note that I assume that the outcome of a failed agreement will be X_{33}, Y_{33}; that is, I do not distinguish between the minor variations in X_{33}, Y_{33} as I did in the previous section. Ignoring these minor variations allows us to concentrate on the more complex issues (however, the reader can readily extend the framework of the analysis to include these variations).

2. If one side has successfully avoided detection of its violation until it is too late for the other side to do anything about it, the violator will present the other side with the facts of its violation and make demands on the other side based on its superior position.[32] (a) If the other side has also cheated, it will provide information that it too has cheated, and the outcome reverts to no arms control (X_{33}, Y_{33}); (b) If the other side has not cheated, the outcome reflects the relative advantage of the one side over the other (X_{21}, Y_{21} if R has cheated and not C; and X_{12}, Y_{12} if C has cheated and not R). Note that a and b mean that there is only one strategy choice by a player if it has cheated, as opposed to three strategy choices if the player has not cheated.

3. $X_{12} < X_{ij}$ and $Y_{12} > Y_{ij}$ for all $ij \neq 12$. This is the symmetric relationship to X_{21}, Y_{21}.

From Table A-1, it is apparent that column 4 dominates column 2 and row 4 dominates row 2; that is, an optimal strategy will never involve assuming that the other side has cheated (without looking at the detection equipment). Thus the matrix can be simplified as in Table A-2. For $P < 1$, once again one can expect that randomized strategies will be undertaken with there being some positive probability of each side's cheating and some positive probability of each side's ignoring its detection equipment.

Appendix D: The Effect of Monitoring Quality on C and R's Behavior

Letting c_1 stand for the probability of C's choosing column 1 and r_1 stand for the probability of R's choosing row 1, C's expected utility is

$$W^C = c_1 r_1 Y_{11} + (1 - c_1) r_1 [P Y_{11} + (1 - P) Y_{\overline{33}}] + c_1 (1 - r_1) Y_{21} + (1 - c_1)(1 - r_1)[P Y_{\overline{33}} + (1 - P) Y_{21}]. \quad \text{(A-3)}$$

First-order interior conditions for C are[33]

$$W^C_{c_1} = r_1 Y_{11} - r_1 [P Y_{11} + (1 - P) Y_{\overline{33}}] + (1 - r_1) Y_{21} - (1 - r_1)[P Y_{\overline{33}} + (1 - P) Y_{21}]$$

$$= r_1 (1 - P)(Y_{11} - Y_{\overline{33}}) + (1 - r_1) P (Y_{21} - Y_{\overline{33}}) = 0. \quad \text{(A-4)}$$

I will make use of these first-order conditions (equation A-4) in establishing the

Table A-2. The Symmetric Game in a Concise Form

Arms Control Agreement	C Honest		
	C Assumes *R* Honest	*C* Uses *P*% Detector	*C* Cheats
R honest			
R assumes *C* honest	X_{11}, Y_{11}	$PX_{11} + (1 - P)X_{33}$, $PY_{11} + (1 - P)Y_{33}$	X_{12}, Y_{12}
R uses *P*% detector	$PX_{11} + (1 - P)X_{33}$, $PY_{11} + (1 - P)Y_{33}$	$P^2X_{11} + (1 - P^2)X_{33}$, $P^2Y_{11} + (1 - P^2)Y_{33}$	$PX_{33} + (1 - P)X_{12}$, $PY_{33} + (1 - P)Y_{12}$
R cheats	X_{21}, Y_{21}	$PX_{33} + (1 - P)X_{21}$, $PY_{33} + (1 - P)Y_{21}$	X_{33}, Y_{33}

Note: $X_{21} > X_{11} > X_{33} > X_{12}$; $Y_{12} > Y_{11} > Y_{33} > Y_{21}$.

following equivalence and definitional relationships:

$$\frac{r_1(1 - P)}{P(1 - r_1)} = \frac{Y_{\bar{3}\bar{3}} - Y_{21}}{Y_{11} - Y_{\bar{3}\bar{3}}} \equiv K. \quad \text{(A-5)}$$

We now turn our attention to the relationship of r_1 to K and P.

Proof of Proposition 4, Part 1. From equation A-5 we know that $[r_1(1 - P)]/[(1 - r_1)P] = K$. This relationship can be solved for r_1 by the following steps:

$$r_1 = \frac{PK - r_1PK}{1 - P} = \frac{PK}{1 - P} - \frac{r_1PK}{1 - P}.$$

Equivalently,

$$r_1\left(1 + \frac{PK}{1 - P}\right) = r_1\left(\frac{1 - P + PK}{1 - P}\right) = \frac{PK}{1 - P},$$

or

$$r_1 = \frac{PK}{1 - P + PK} = \frac{P}{[(1 - P)/K] + P}. \quad \text{(A-6)}$$

From the last equality it is clear that (1) as K increases, so does r_1, (2) as K decreases toward 0, r_1 approaches 0, (3) when $K = 1$, $r_1 = P$, and (4) as K increases toward infinity, r_1 approaches 1. QED

Appendix E

Proof of Proposition 5, Part 1. R's expected utility is

$$W^R = c_1r_1X_{11} + (1 - c_1)r_1[PX_{11} + (1 - P)X_{\bar{3}\bar{3}}] + c_1(1 - r_1)X_{21} + (1 - c_1)(1 - r_1)[PX_{\bar{3}3} + (1 - P)X_{21}].$$

First-order interior conditions for R are:

$$W^R_{r_1} = c_1X_{11} + (1 - c_1)[PX_{11} + (1 - P)X_{\bar{3}\bar{3}}] - c_1X_{21} - (1 - c_1)[PX_{\bar{3}3} + (1 - P)X_{21}] = 0. \quad \text{(A-7)}$$

In order to find the effect of P on c_1 I take the total differential of equation A-7.

$$[X_{11} - PX_{11} - (1 - P)X_{\bar{3}\bar{3}} - X_{21} + PX_{\bar{3}3} + (1 - P)X_{21}]dc_1 + (1 - c_1)(X_{11} - X_{\bar{3}\bar{3}} - X_{\bar{3}3} + X_{21})dP = 0. \quad \text{(A-8)}$$

Rearranging equation A-8, I get the following relationship:

$$[(1 - P)(X_{11} - X_{\bar{3}\bar{3}}) + P(X_{\bar{3}3} - X_{21})]dc_1 = -(1 - c_1)(X_{11} - X_{\bar{3}\bar{3}} - X_{\bar{3}3} + X_{21})dP. \quad \textbf{(A-9)}$$

On the right-hand side of the equality, it is clear that the sign of the expression is negative, since X_{11}, $X_{21} > X_{\bar{3}3}$, $X_{\bar{3}\bar{3}}$ by assumption. I will show that the left-hand side of the equality is also negative so that an increase in P results in an increase in c_1. Earlier (in the conditional statement of proposition 3, part 2) I assumed that

$$P(X_{11}) + (1 - P)X_{\bar{3}\bar{3}} > P(X_{\bar{3}3}) + (1 - P)X_{21}. \quad \textbf{(A-10)}$$

Rearranging equation A-10 I obtain the following relationships:

$$0 > -PX_{11} + PX_{\bar{3}3} + (1 - P)(X_{21} - X_{\bar{3}\bar{3}}) = (1 - P)(X_{11} - X_{\bar{3}\bar{3}}) - X_{11} + P(X_{\bar{3}3} - X_{21}) + X_{21}.$$

$$> (1 - P)(X_{11} - X_{\bar{3}\bar{3}}) + P(X_{\bar{3}3} - X_{21})$$

= the bracketed term on the left-hand side of equation A-9.

The last inequality holds because $X_{21} > X_{11}$ by assumption. Thus, the left-hand side of equation A-9 < 0, and P and c_1 move in the same direction.

Proof of Part 2. In order to find the effect of a change in both $X_{\bar{3}3}$ and $X_{\bar{3}\bar{3}}$ on c_1 I take the total differential of equation A-7, treating the change in $X_{\bar{3}3}$ as equivalent to the change in $X_{\bar{3}\bar{3}}$.

$$[(1 - P)(X_{11} - X_{\bar{3}\bar{3}}) + P(X_{\bar{3}3} - X_{21})]dc_1 = -(1 - c_1)(1 - 2P)dX_{\bar{3}3}.$$

Once again the left-hand side is negative. The right-hand side is positive since $P >$ 1/2 by assumption. Therefore as $X_{\bar{3}3}$ and $X_{\bar{3}\bar{3}}$ increase (decrease), c_1 decreases (increases). QED

Proof of Part 3. This follows the same logic and is left to the reader.

Appendix F

Proof of Proposition 6. From proposition 3, if an arms control equilibrium exists, it must involve mixed strategies by C (between columns 1 and 3) and by R (between rows 1 and 2). At equilibrium, R is indifferent between rows 1 and 2. From row 1 in Table 2, R strictly prefers X_{11} to $PX_{11} + (1 - P)X_{\bar{3}\bar{3}}$. Therefore, the greater the probability of C's choosing column 3 in equilibrium, the lower R's expected utility. If the probability of C's choosing column 3 were 1 (or very close to it), R's expected utility would only be $PX_{11} + (1 - P)X_{\bar{3}\bar{3}}$. But even under these circumstances, R's expected utility would be greater than X_{33} by assumption. QED

Appendix G

Proof of Proposition 7. At equilibrium, C is indifferent between columns 1 and 3. This indifference is an immediate consequence of rearranging the first-order conditions for C (equation A-3) as follows:

$$r_1Y_{11} + (1 - r_1)Y_{21} = r_1[PY_{11} + (1 - P)Y_{\bar{3}\bar{3}}] + (1 - r_1)[PY_{\bar{3}3} + (1 - P)Y_{21}]. \quad \textbf{(A-11)}$$

In order to calculate C's expected utility, I substitute $PK/(1 - P + PK)$ for r_1 (from equation A-6) into the right-hand side of equation A-1.

$$W^C = [PK/(1 - P + PK)][PY_{11} + (1 - P)Y_{\bar{3}\bar{3}}] + [(1 - P)/(1 - P + PK)][PY_{\bar{3}3} + (1 - P)Y_{21}]$$

$$= [P^2KY_{11} + PKY_{\bar{3}\bar{3}} - P^2KY_{\bar{3}\bar{3}} + PY_{\bar{3}\bar{3}} - P^2Y_{\bar{3}\bar{3}} + (1-P)^2Y_{21}]/(1-P+PK). \quad \text{(A-12)}$$

$$K \equiv \frac{Y_{\bar{3}\bar{3}} - Y_{21}}{Y_{11} - Y_{\bar{3}\bar{3}}} .$$

Substituting this expression into the first and third terms in equation A-12, I get

$$[P^2Y_{\bar{3}\bar{3}} - P^2Y_{21} + Y_{\bar{3}\bar{3}}(KP + P - P^2) + (1 + P^2 - 2P)Y_{21}]/(1 - P + PK) = [Y_{\bar{3}\bar{3}}(KP + P) + (1-2P)Y_{21}]/(1-P+PK). \quad \text{(A-13)}$$

For an arms control equilibrium to be preferred by C to the equilibrium under no arms control, the right-hand side of equation A-13 must be greater than Y_{33}. QED

Notes

1. See Hirshleifer 1987 for an interesting comparison of sequential move games to simultaneous move games. He does not incorporate both games into one general game, as I do here.

2. While the words *signal* and *detection* have been used in statistics, the analysis here is quite different, since I analyze game playing by both participants.

3. Many theoretical results depend upon the relative size of the various parameters, the relative size being determined by empirical studies.

4. If one row (or column) dominated another row (column), there would be no need for detection equipment, as the other side's optimal choice would always be known.

5. These signals are not under any control by R; rather, any action by R has some potential of being observed.

6. The differing error rates, $1 - P$ and $1 - Q$, correspond to type 1 and type 2 errors in statistics. See Richelson 1979 for a discussion of detection vs. false alarms.

7. At the top of column 1, vector C_1, C_1 indicates that C chooses C_1 whether σ_1 or σ_2 has occurred.

8. However, as will be shown, the best response to σ_i is not choosing C_i with certainty.

9. The effect of exogenous changes on the strategies by R and C, such as the quality of the monitoring equipment and the size of X_{ii}, can be derived. For example, it can be shown that a reduction in the probability of a type 1 error has the same qualitative change on the strategies of the players as a reduction in the probability of a type 2 error (see Wittman 1987).

10. This trade-off curve is closely related to the receiver operating characteristic curve (used by O'Neill [1988]) which treats the probability of X_{12} as a function of the probability of X_{22}. One can also view row's strategy as being deterministic rather than random. For example, row 1 might be sending 10 planes north, while row 2 might be sending 10 planes south. A mixed strategy might be to choose row 1 with probability .6 and row 2 with probability .4. The deterministic equivalent is to have 6 planes flying north and 4 planes flying south (assuming that there are constant returns to scale).

11. In the post–World War II time period, the Soviet Union did not publish truthful population statistics because it feared that other countries would realize how weak it was in military manpower. Some believe that the Soviets were against on-sight inspection in the past because it would have revealed that they were militarily very weak.

12. The phrase "C believes that R has cheated" stands for the phrase that C undertakes strategy C_2, which I assume is the best strategy given that R cheats (R_2). C_2 may be the strategy choice arising from either the simultaneous game or the sequential game, thus "C believes" means C acts as if C believed that R has cheated.

13. X_{11} and Y_{11} are functions of the economic costs and political power benefits from armaments. For a discussion of these factors and the appropriate functional forms see Wittman 1987. An agreement to eliminate nuclear weapons may actually make both sides worse off if the probability of war increases substantially when the threat of a nuclear second strike is absent. I will not discuss this possibility in greater detail, since my attention is to cases in which arms reduction is desired by both sides if fully enforceable and perhaps desired even if not fully enforceable.

14. While this is the perceived problem, arguments could be made to the contrary if the extra military advantage from cheating on an arms control agreement were outweighed by the extra cost of the armaments.

15. I assume that the detection, if it takes place, occurs before R can gain any advantage by cheating. A more general model might make the probability of detection and the cost to C from R's cheating a function of time. I will not consider these extensions here, in order to prevent the analysis from getting out of control by considering too many alternatives.

16. Once again, I do not distinguish how this "belief" arose. If C chooses C_i, the outcome is the same whether or not C looked at the detection equipment before making the decision.

17. If R were not intending to make use of its arms, R's cheating would not hurt C. However, I have assumed that C is honest. Therefore, there

would be no gain to R in cheating unless R were intending to use its arms against C. Some authors have argued that the issue of verification (protection against cheating) is just a smoke screen. However, for certain kinds of potential limitations and certain types of weapon systems, the ability to monitor at all accurately is extremely problematical. For example, a cruise missile (both in the testing stage and in deployment) is harder to detect than an ICBM silo; research on "Star Wars" weapons is harder to monitor than is their testing; and monitoring the production of nuclear warheads is much more difficult than detecting nuclear explosions in space. Indeed, Earle (1986) argued that 85%–95% of the verbiage in SALT II was devoted to the issue of verification. Furthermore, as I shall show, an arms control agreement may not achieve an equilibrium different from the outcome encompassing no agreement unless there is a difference in verification.

18. It is possible that for propaganda purposes C may engage in an arms control agreement even though it intends to claim that R is cheating soon after the agreement is made. This line of argument suggests that assumption 3 is incorrect and that the reverse is true: $Y_{33} < Y_{\bar{3}\bar{3}}$. The intention also suggests that $Y_{33} > Y_{11}$. Since I am investigating situations in which an honestly held arms control agreement is preferred to a situation of no arms control, I will not consider this possibility further.

19. The approach used here is considerably different from previous work on verification and arms control. Brams and Davis (1987) and Brams and Kilgour (1986) assume that R's primary goal is to hide the truth and C's primary goal is to discover the truth. I posit different goals for R and C. Under my formulation, if R has been honest with regard to arms control, it does not want to hide the truth. These authors do not consider the possibility that detection is a sequential game with the detector making its move after observing (possibly with error) R's choice. Brams and Kilgour (1988, chap. 8) view detection as a type of deterrence and the possibility of using verification as a threat, inducing a pure strategy response. Fichtner (1986) has a model where the detector works perfectly but is costly to use. Maschler's 1963 model of a test ban agreement does not consider the choice between using the detector and not using the detector, and he specifically avoids probabilistic choice by the participants. Thus, his model does not deal with columns 1, 2, or 4 of Table 2 but is similar to column 3 (although the probabilities and payoffs I use are only implicit in his model; thus, he assumes that the United States will not cheat because it does not want to take the *risk* of being caught). Furthermore he makes different assumptions. For example, to translate into my model, he assumes that column prefers an arms control agreement with cheating by row (but not by column) to the outcome in which there is no arms control agreement. Most of the other literature on inspection is cast as a game in which one side (with a fixed number of inspections available to it) maximizes the probability of catching the other side cheating and the other side minimizes the probability of being caught, given that it is going to cheat. Work by Kuhn (1963) and Davis and Kuhn (1963) fall into this genre. Thus, the objective functions and the game itself are considerably different from that presented here. Harsanyi (1967) applies the power index to arms control and no arms control, respectively, and is not interested in verification. McGuire (1965) shows that in an uncertain world, increased armaments are a substitute for increased knowledge about the other side. Under arms control, information about the other side's arms is provided, thereby allowing for reduced armaments. My work is thus only tangentially related to his.

20. I assume that $X_{\bar{3}3} > X_{33}$. With the inequalities reversed, there would be no pure strategy equilibrium.

21. See Wittman 1987 for the proof of the sufficiency conditions. Note that a mixed strategy equilibrium never involves the choice of having no arms control. No arms control is a different game. Both sides know whether they are in the arms control game (with possible cheating).

22. Improved verification capabilities (higher P) may have been the reason why SALT I was implemented in 1972 rather than earlier.

23. This is restating Proposition 4, part 1 in a slightly different way. Since $r_1 = P$ when $K = 1$ and increases as K increases, $r_1 < P$ for $K < 1$. Equivalently, $1 - r_1 > 1 - P$ for $K < 1$.

24. Note that this is a necessary but not sufficient condition for an arms control agreement. Bargaining over the terms of the agreement may take a very long time and in some cases may prevent an agreement from being reached. See Wittman (1979) for a discussion of necessary conditions for an agreement to end a war.

25. Aspin and Kaplan (1980) made similar statements regarding SALT: " 'Adequate verification' means verification sufficient to assure that the uncertainties in verifying an arms control treaty creates fewer risks than those facing the United States in a world without the treaty."

26. Of course, when detection is difficult and the cost of not detecting violations is high, an agreement is less likely.

27. If simultaneous behavior is not necessary, technological changes in monitoring capabilities may result in partial unilateral disarmament. An analysis of the choice between unilateral and bilateral agreements is a topic that deserves more attention.

28. The hostages need not be people. See Williamson 1983.

29. Stiglitz and Weiss (1983) considered the possibility of (job) terminations, the analog to the dissolution of an arms control agreement. However,

they did not consider the possibility of any error in detection.

30. In fact, I have shown that an optimal strategy increases certain types of risk.

31. Since R has only two pure strategies, an optimal strategy by C would not involve a mixed strategy between column 4 (which is, itself, a mixed strategy) and another column.

32. See Katz (1980) for a depiction of such a scenario after the USSR has violated the SALT agreement. See also Tsipis, Hafemeister, and Janeway 1986.

33. The pure strategy solutions are corner solutions.

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