Worksheet 5.5.
Net or Total Change as the Integral of a Rate

1. Find the displacement over the time interval \([1, 6]\) of a helicopter whose vertical velocity at time \(t\) is

\[
v(t) = 0.02t^2 + t \text{ ft/s}
\]

2. A particle is moving along a straight line with velocity \(v(t) = \cos t \text{ m/s}\). Find (a) the total displacement and (b) the total distance traveled over the time interval \([0, 4\pi]\).
3. The velocity (ft/s) of a car is recorded at half-second intervals:

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>v(t)</td>
<td>0</td>
<td>12</td>
<td>20</td>
<td>29</td>
<td>38</td>
<td>44</td>
<td>32</td>
<td>35</td>
<td>30</td>
</tr>
</tbody>
</table>

Use the average of the left- and right-endpoint approximations as illustrated below to estimate the total distance traveled over the time interval [0, 4].
Solutions to Worksheet 5.5

1. Find the displacement over the time interval \([1, 6]\) of a helicopter whose vertical velocity at time \(t\) is 
\[ v(t) = 0.02t^2 + t \text{ ft/s}. \]

Given \(v(t) = \frac{1}{50}t^2 + t \text{ ft/s}\), the change in height over \([1, 6]\) is 
\[
\int_1^6 v(t) \, dt = \int_1^6 \left( \frac{1}{50}t^2 + t \right) \, dt = \left[ \frac{1}{150}t^3 + \frac{1}{2}t^2 \right]_1^6
\]
\[
= \left( \frac{1}{150}(6)^3 + \frac{1}{2}(6)^2 \right) - \left( \frac{1}{150}(1)^3 + \frac{1}{2}(1)^2 \right) = \frac{284}{15} \approx 18.93 \text{ ft}. 
\]

2. A particle is moving along a straight line with velocity \(v(t) = \cos t \text{ m/s}\). Find (a) the total displacement and (b) the total distance traveled over the time interval \([0, 4\pi]\).

Total displacement is given by 
\[
\int_0^{4\pi} \cos t \, dt = \sin t\bigg|_0^{4\pi} = 0 \text{ m}, 
\]
while total distance is given by 
\[
\int_0^{4\pi} |\cos t| \, dt = \int_0^{\pi/2} \cos t \, dt + \int_{\pi/2}^{3\pi/2} -\cos t \, dt + \int_{3\pi/2}^{5\pi/2} \cos t \, dt + \int_{5\pi/2}^{7\pi/2} -\cos t \, dt + \int_{7\pi/2}^{4\pi} \cos t \, dt
\]
\[
= \sin t\bigg|_0^{\pi/2} - \sin t\bigg|_{3\pi/2}^{5\pi/2} + \sin t\bigg|_{5\pi/2}^{7\pi/2} - \sin t\bigg|_{7\pi/2}^{4\pi} = 8 \text{ m}. 
\]

3. The velocity (ft/s) of a car is recorded at half-second intervals below.

<table>
<thead>
<tr>
<th>(t)</th>
<th>(0)</th>
<th>.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v(t))</td>
<td>0</td>
<td>12</td>
<td>20</td>
<td>29</td>
<td>38</td>
<td>44</td>
<td>32</td>
<td>35</td>
<td>30</td>
</tr>
</tbody>
</table>

Use the average of the left- and right-endpoint approximations as illustrated below to estimate the total distance traveled over the time interval \([0, 4]\).
Let $\Delta x = .5$. Then

$$R_N = .5 \cdot (12 + 20 + 29 + 38 + 44 + 32 + 35 + 30) = 120 \text{ ft}.$$  
$$L_N = .5 \cdot (0 + 12 + 20 + 29 + 38 + 44 + 32 + 35) = 105 \text{ ft}.$$  

The average of $R_N$ and $L_N$ is 112.5 ft