Two people are running on opposite sides of a street towards each. Person X is running at a rate of 3m/s and Person Y is running at a rate of 4m/s. If the street is 10m wide, at what rate is the distance between the runners changing when they pass each other and then 5s after they pass?

Solution: The situation can be pictured with a right triangle where one leg is the vertical distance across the street (10m), the other leg is the horizontal distance along the street between the two runners (label it $x$), and the hypotenuse is the actual distance between the two runners (label it $z$). By the Pythagorean Theorem, $z^2 = x^2 + 10^2$. If we let $t$ denote the time variable, then differentiating with respect to $t$ yields

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 0.$$ 

When the two runners pass each other, their horizontal distance apart is zero, hence $x = 0$. So we see that at this point, $\frac{dz}{dt} = 0$ m/s. After they pass each other, the horizontal distance apart is changing at a rate of $3 + 4 = 7$ m/s since the two runners are going in opposite directions at constant rates. This means that after the runners pass each other $\frac{dz}{dt} = \frac{7x}{z}$. Now five seconds after they pass, Person X has traveled $(3)(5) = 15$ m, and Person Y has traveled $(4)(5) = 20$ m. Hence $x = 15 + 20 = 35$ m. Then $z = \sqrt{35^2 + 10^2} = 5\sqrt{53}$ m. So altogether, we see that 5s after the runners pass each other, their distance apart is changing at a rate of

$$\frac{dz}{dt} = \frac{7(35)}{5\sqrt{53}} = 49 \frac{1}{\sqrt{53}} \approx 6.73$$ m/s.