

# Chapter 9, section 2 from the 3rd edition: The Obstfeld-Rogoff Two-Country Model\*

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## 1 Introduction

The analysis in earlier chapters was conducted within the context of a closed economy. Many useful insights into monetary phenomena can be obtained while still abstracting from the linkages that tie different economies together, but clearly many issues do require an open-economy framework if they are to be adequately addressed. New channels through which monetary factors can influence the economy arise in open economies. Exchange-rate movements, for example, play an important role in the transmission process that links monetary disturbances to output and inflation movements. Open economies face the possibility of economic disturbances that originate in other countries, and this raises questions of monetary policy design that are absent in a closed-economy environment: Should policy respond to exchange-rate movements? Should monetary policy be used to stabilize exchange rates? Should national monetary policies be coordinated?

This chapter begins with a two-country model based on [Obstfeld and Rogoff \(1995\)](#) and [Obstfeld and Rogoff \(1996\)](#). The two-country model has the advantage of capturing some of the important linkages between economies while still maintaining a degree of simplicity and tractability. Because an open economy is linked to other economies, policy actions in one economy have the potential to affect other economies. Spillovers can occur. And policy actions in one country will depend on the response of monetary policy in the other. Often, because of these spillovers, countries attempt to coordinate their policy actions. The role of policy coordination is examined in [section 3](#).

[Section 4](#) considers the case of a *small open economy*. In the open-economy literature, a small open economy denotes an economy that is too small to affect world prices, interest rates, or economic activity. Since many countries really are small relative to the world economy, the small-open-economy model provides a framework that is relevant for studying many policy issues.

The analyses of policy coordination and the small open economy are conducted using models in which behavioral relationships are specified directly rather than derived from underlying assumptions about the behavior of individuals and firms. As a result, the frameworks are of limited use for conducting normative analysis as they are unable to make predictions about the welfare of the agents in the model. This is one reason for beginning the discussion of the open economy with the Obstfeld-Rogoff model; it is based explicitly on the assumption of optimizing agents and therefore offers a natural metric—in the form of the utility of the representative agent—for addressing normative policy questions. This chapter ends by returning, in [section ??](#), to a class of models based on optimizing agents and nominal rigidities. These models are the open-economy counterparts of the new Keynesian models of [Chapter 8](#).

## 2 The Obstfeld-Rogoff Two-Country Model

Obstfeld and Rogoff (1995) and Obstfeld and Rogoff (1996) examine the linkages between two economies within a framework that combines three fundamental building blocks. The first is an emphasis on intertemporal decisions by individual agents; foreign trade and asset exchange open up avenues for transferring resources over time that are not available in a closed economy. A temporary positive productivity shock that raises current output relative to future output induces individuals to increase consumption both now and in the future as they try to smooth the path of consumption. Since domestic consumption rises less than domestic output, the economy increases its net exports, thereby accumulating claims against future foreign output. These claims can be used to maintain higher consumption in the future after the temporary productivity increase has ended. The trade balance therefore plays an important role in facilitating the intertemporal transfer of resources.

Monopolistic competition in the goods market is the second building block of the Obstfeld-Rogoff model. This by itself has no implications for the effects of monetary disturbances, but it does set the stage for the third aspect of their model—sticky prices. These basic building blocks have already been discussed, so the focus here is on the new aspects introduced by open-economy considerations. Detailed derivations of the various components of the model are provided in the appendix to this chapter. It will simplify the exposition to deal with a nonstochastic model in order to highlight the new considerations that arise in the open-economy context.

Each of the two countries is populated by a continuum of agents, indexed by  $z \in [0, 1]$ , who are monopolistic producers of differentiated goods. Agents  $z \in [0, n]$  reside in the home country, while agents  $z \in (n, 1]$  reside in the foreign country. Thus,  $n$  provides an index of the relative sizes of the two countries. If the countries are of equal size,  $n = \frac{1}{2}$ . Foreign variables are denoted by a superscript asterisk (\*).

The present discounted value of lifetime utility of a domestic resident  $j$  is

$$U^j = \sum_{t=0}^{\infty} \beta^t \left[ \log C_t^j + b \log \frac{M_t^j}{P_t} - \frac{k}{2} y_t(j)^2 \right], \quad (1)$$

where  $C_t^j$  is agent  $j$ 's period- $t$  consumption of the composite consumption good, defined by

$$C_t^j = \left[ \int_0^1 c_t^j(z)^q dz \right]^{\frac{1}{q}}, \quad 0 < q < 1, \quad (2)$$

and consumption by agent  $j$  of good  $z$  is  $c^j(z)$ ,  $z \in [0, 1]$ . The aggregate domestic price deflator  $P$  is defined as

$$P_t = \left[ \int_0^1 P_t(z)^{\frac{q}{q-1}} dz \right]^{\frac{q-1}{q}}. \quad (3)$$

This price index  $P$  depends on the prices of all goods consumed by domestic residents (the limits of integration run from 0 to 1). It incorporates prices of both domestically produced goods  $\{P(z) \text{ for } z \in [0, n]\}$  and foreign-produced goods  $\{P(z) \text{ for } z \in (n, 1]\}$ . Thus,  $P$  corresponds to a consumer price index concept of the price level, not a GDP price deflator that would include only the prices of domestically produced goods.

Utility also depends on the agent's holdings of real money balances. Agents are assumed to hold only their domestic currency, so  $M_t^j/P_t$  appears in the utility function (1). Since agent  $j$  is the producer of good  $j$ , the effort of producing output  $y_t(j)$  generates disutility. A similar utility function is assumed for residents of the foreign country:

$$U^{*j} = \sum_{t=0}^{\infty} \beta^t \left[ \log C_t^{*j} + b \log \frac{M_t^{*j}}{P_t^*} - \frac{k}{2} y_t^*(j)^2 \right],$$

where  $C^{*j}$  and  $P^*$  are defined analogously to  $C^j$  and  $P$ .

Agent  $j$  will pick consumption, money holdings, holdings of internationally traded bonds, and output of good  $j$  to maximize utility subject to the budget constraint

$$P_t C_t^j + M_t^j + P_t T_t + P_t B_t^j \leq P_t(j) y_t(j) + R_{t-1} P_t B_{t-1}^j + M_{t-1}^j.$$

The gross real rate of interest is denoted  $R$ , and  $T$  represents real taxes minus transfers. Bonds purchased at time  $t-1$ ,  $B_{t-1}^j$ , yield a gross real return  $R_{t-1}$ . As in the analysis of chapter 2, the role of  $T$  will be to allow for variations in the nominal supply of money, with  $P_t T_t = (M_t - M_{t-1})$ . Dividing the budget constraint by  $P_t$ , one obtains

$$C_t^j + \frac{M_t^j}{P_t} + T_t + B_t^j \leq \left[ \frac{P_t(j)}{P_t} \right] y_t(j) + R_{t-1} B_{t-1}^j + \left( \frac{1}{1 + \pi_t} \right) \frac{M_{t-1}^j}{P_{t-1}}, \quad (4)$$

where  $\pi_t$  is the inflation rate from  $t-1$  to  $t$ . To complete the description of the agent's decision problem, one needs to specify the demand for the good the agent produces. This specification is provided in chapter appendix section 5.1, where it is shown that the following necessary first-order conditions can be derived from the individual consumer/producer's decision problem:

$$C_{t+1}^j = \beta R_t C_t^j \quad (5)$$

$$k y_t^j = q \left( \frac{1}{C_t^j} \right) \left( \frac{y_t^j}{C_t^w} \right)^{q-1} \quad (6)$$

$$\frac{M_t^j}{P_t} = b C_t^j \left( \frac{1 + i_t}{i_t} \right), \quad (7)$$

together with the budget constraint (4) and the transversality condition

$$\lim_{i \rightarrow \infty} \prod_{s=0}^i R_{t+s-1}^{-1} \left( B_{t+i}^j + \frac{M_{t+i}^j}{P_{t+i}} \right) = 0.$$

In these expressions,  $i_t$  is the nominal rate of interest, defined as  $R_t(1 + \pi_{t+1}) - 1$ .

In (6),  $C_t^w \equiv nC_t + (1 - n)C_t^*$  is world consumption, where  $C_t = \int_0^n C_t^j dj$  and  $C_t^* = \int_n^1 C_t^j dj$  equal total home and foreign consumption. Equation (5) is a standard Euler condition for the optimal consumption path. Equation (6) states that the ratio of the marginal disutility of work to the marginal utility of consumption must equal the marginal product of labor.<sup>1</sup> Equation (7) is the familiar condition for the demand for real balances of the domestic currency, requiring that the ratio of the marginal utility of money to the marginal utility of consumption equal  $i_t/(1 + i_t)$ . Similar expressions hold for the foreign consumer/producer.

Let  $S_t$  denote the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. A rise in  $S_t$  means that the price of foreign currency has risen in terms of domestic currency; consequently, a unit of domestic currency buys fewer units of foreign currency. So a rise in  $S_t$  corresponds to a fall in the value of the domestic currency.

The exchange rate between goods produced domestically and goods produced in the foreign economy will also play an important role. The law of one price requires that good  $z$  sell for the same price in both the home and foreign countries when expressed in a common currency.<sup>2</sup> This requires

$$P_t(z) = SP_t^*(z).$$

It follows from the definitions of the home and foreign price levels that

$$P_t = S_t P_t^*. \tag{8}$$

Any equilibrium must satisfy the first-order conditions for the agent's decision problem, the law-of-one-price condition, and the following additional market-clearing conditions:

$$\text{Goods market clearing: } C_t^w = n \left[ \frac{P_t(h)}{P_t} \right] y_t(h) + (1 - n) \left[ \frac{P_t^*(f)}{P_t^*} \right] y_t^*(f) \equiv Y_t^w,$$

where  $P(h)$  and  $y(h)$  are the price and output of the representative home good (and similarly for

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<sup>1</sup>See (72) in the appendix.

<sup>2</sup>While the law of one price is intuitively appealing and provides a convenient means of linking the prices  $p(j)$  and  $p^*(j)$  to the nominal exchange rate, it may be a poor empirical approximation. In a study of prices in different U.S. cities, Parsley and Wei (1996) found rates of price convergence to be faster than in cross-country comparisons, and they conclude that tradable-goods prices converge quickly. Even so, the half-life of a price difference among U.S. cities for tradables is estimated to be on the order of 12-15 months.

$P^*(f)$  and  $y^*(f)$ ), and

$$\text{Bond market clearing: } nB_t + (1 - n)B_t^* = 0.$$

From the structure of the model, it should be clear that one-time proportional changes in the nominal home money supply, all domestic prices, and the nominal exchange rate leave the equilibrium for all real variables unaffected—the model displays monetary neutrality. An increase in  $M$  accompanied by a proportional decline in the value of home money in terms of goods (i.e., a proportional rise in all  $P(j)$ ) and a decline in the value of  $M$  in terms of  $M^*$  (i.e., a proportional rise in  $S$ ) leaves equilibrium consumption and output in both countries, together with prices in the foreign country, unchanged.

In the steady state, the budget constraint (4) becomes

$$C = \frac{P(h)}{P}y(h) + (R - 1)B, \quad (9)$$

where  $B$  is the steady-state real stock of bonds held by the home country. For the foreign country,

$$C^* = \frac{P(f)}{P}y(f) - (R - 1)\left(\frac{n}{1 - n}\right)B. \quad (10)$$

These two equations imply that real consumption equals real income (the real value of output plus income from net asset holdings) in the steady state.

## 2.1 The Linear Approximation

It will be helpful to develop a linear approximation to the basic Obstfeld-Rogoff model in terms of percentage deviations around the steady state. This serves to make the linkages between the two economies clear, and provides a base of comparison when, in the following section, a more traditional open economy model is considered that is not directly derived from the assumption of optimizing agents. Using lower case letters to denote percentage deviations around the steady state, the equilibrium conditions can be expressed as

$$p_t = np_t(h) + (1 - n)[s_t + p_t^*(f)] \quad (11)$$

$$p_t^* = n[p_t(h) - s_t] + (1 - n)p_t^*(f) \quad (12)$$

$$y_t = \frac{1}{1 - q}[p_t - p_t(h)] + c_t^w \quad (13)$$

$$y_t^* = \frac{1}{1 - q}[p_t^* - p_t^*(f)] + c_t^w \quad (14)$$

$$nc_t + (1-n)c_t^* = c_t^w \quad (15)$$

$$c_{t+1} = c_t + r_t \quad (16)$$

$$c_{t+1}^* = c_t^* + r_t \quad (17)$$

$$(2-q)y_t = (1-q)c_t^w - c_t \quad (18)$$

$$(2-q)y_t^* = (1-q)c_t^w - c_t^* \quad (19)$$

$$m_t - p_t = c_t - \delta(r_t + \pi_{t+1}) \quad (20)$$

$$m_t^* - p_t^* = c_t^* - \delta(r_t + \pi_{t+1}^*), \quad (21)$$

where  $\delta = \beta/(\bar{\Pi} - \beta)$  and  $\bar{\Pi}$  is 1 plus the steady-state rate of inflation (assumed to be equal in both economies). Equations (11) and (12) express the domestic and foreign price levels as weighted averages of the prices of home- and foreign-produced goods expressed in a common currency. The weights depend on the relative sizes of the two countries as measured by  $n$ . Equations (13) and (14) are derived from (64) of the appendix and give the demand for each country's output as a function of world consumption and relative price. Increases in world consumption ( $c^w$ ) increase the demand for the output of both countries, while demand also depends on a relative price variable. Home country demand, for example, falls as the price of home production  $p(h)$  rises relative to the home price level. Equation (15) defines world consumption as the weighted average of consumption in the two countries.

Equations (16)-(21) are from the individual agent's first-order conditions (5), (6), and (7). The first two of these equations are simply the Euler condition for the optimal intertemporal allocation of consumption; the change in consumption is equal to the real rate of return. Equations (18) and (19) are implied by optimal production decisions. Finally, (20) and (21) give the real demand for home and foreign money as functions of consumption and nominal interest rates. While both countries face the same real interest rate  $r_t$ , nominal interest rates may differ if inflation rates differ between the two countries.

The equilibrium path of home and foreign production ( $y_t, y_t^*$ ), home, foreign, and world consumption ( $c_t, c_t^*, c_t^w$ ), prices and the nominal exchange rate ( $p_t(h), p_t, p_t^*(f), p_t^*, s_t$ ), and the real interest rate ( $r_t$ ) must be consistent with these equilibrium conditions.<sup>3</sup> Note that subtracting (12) from (11) implies

$$s_t = p_t - p_t^*, \quad (22)$$

while the addition of  $n$  times (13) and  $(1-n)$  times (14) yields the goods market-clearing relationship equating world production to world consumption:  $ny_t + (1-n)y_t^* = c_t^w$ .

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<sup>3</sup>Equations (20)-(21) differ somewhat from Obstfeld and Rogoff's specification due to differences in the methods used to obtain linear approximations. See chapter 10 of [Obstfeld and Rogoff \(1996\)](#).

## 2.2 Equilibrium with Flexible Prices

The linear version of the two-country model serves to highlight the channels that link open economies. With this framework, the role of money when prices are perfectly flexible is discussed first. As in the closed-economy case, the real equilibrium is independent of monetary phenomena when prices can move flexibly to offset changes in the nominal supply of money.<sup>4</sup> Prices and the nominal exchange rate will depend on the behavior of the money supplies in the two countries, and the adjustment of the nominal exchange rate becomes part of the equilibrating mechanism that insulates real output and consumption from monetary effects.

The assumption of a common capital market, implying that consumers in both countries face the same real interest rate, means from the Euler conditions (16) and (17) that  $c_{t+1} - c_{t+1}^* = c_t - c_t^*$ ; any difference in relative consumption is permanent. And world consumption  $c^w$  is the relevant scale variable for demand facing both home and foreign producers.

### 2.2.1 Monetary Dichotomy

With prices and the nominal exchange rate free to adjust immediately in the face of changes in either the home or foreign money supply, the model displays the classical dichotomy discussed in section 5.3.1 under which the equilibrium values of all real variables can be determined independently of the money supply and money demand factors. To see this, define the two relative price variables  $\chi_t \equiv p_t(h) - p_t$  and  $\chi_t^* \equiv p_t^*(f) - p_t^*$ . Equations (11) and (12) imply that

$$n\chi_t + (1 - n)\chi_t^* = 0,$$

while (13) and (14) can be rewritten as

$$y_t = -\frac{\chi_t}{1 - q} + c_t^w$$

$$y_t^* = -\frac{\chi_t^*}{1 - q} + c_t^w.$$

These equations, together with (15)-(19), suffice to determine the real equilibrium. The money demand equations (20) and (21) determine the price paths, while (22) determines the equilibrium nominal exchange rate given these price paths. Thus, an important implication of this model is that monetary policy (defined as changes in nominal money supplies) has no short-run effects on the real interest rate, output, or consumption in either country. Rather, only nominal interest rates, prices, and the nominal exchange rate are affected by variations in the nominal money stock. One-time

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<sup>4</sup>Recall from the discussion in chapter 2 that the dynamic adjustment outside the steady state is independent of money when utility is log separable, as assumed in (1). This result would also characterize this open economy model if it were modified to incorporate stochastic uncertainty due to productivity and money growth rate disturbances.



changes in  $m$  produce proportional changes in  $p$ ,  $p(h)$ , and  $s$ .

Changes in the growth rate of  $m$  affect inflation and nominal interest rates. Equation (20) shows that inflation affects the real demand for money, so different rates of inflation are associated with different levels of real money balances. Changes in nominal money growth rates produce changes in the inflation rate and nominal interest rates, thereby affecting the opportunity cost of holding money and, in equilibrium, the real stock of money. The price level and the nominal exchange rate jump to ensure that the real supply of money is equal to the new real demand for money.

Equations (21) can be subtracted from (20), yielding

$$m_t - m_t^* - (p_t - p_t^*) = (c_t - c_t^*) - \delta(\pi_{t+1} - \pi_{t+1}^*),$$

which, using (22), implies<sup>5</sup>

$$m_t - m_t^* - s_t = (c_t - c_t^*) - \delta(s_{t+1} - s_t). \quad (23)$$

Solving this forward for the nominal exchange rate, the no-bubbles solution is

$$s_t = \frac{1}{1 + \delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^i [(m_{t+i} - m_{t+i}^*) - (c_{t+i} - c_{t+i}^*)]. \quad (24)$$

Since (16) and (17) imply that  $c_{t+i} - c_{t+i}^* = c_t - c_t^*$ , the expression for the nominal exchange rate can be rewritten as

$$s_t = -(c_t - c_t^*) + \frac{1}{1 + \delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^i (m_{t+i} - m_{t+i}^*).$$

The current nominal exchange rate depends on the current and future path of the nominal money supplies in the two countries and on consumption differentials. The exchange rate measures the price of one money in terms of the other, and, as (24) shows, this depends on the relative supplies of the two monies. An increase in one country's money supply relative to the other's depreciates that country's exchange rate. From the standard steady-state condition that  $\beta R^{ss} = 1$  and the definition of  $\delta$  as  $\beta/(\bar{\Pi} - \beta)$ , the discount factor in (24),  $\delta/(1 + \delta)$ , is equal to  $\beta/\bar{\Pi} = 1/R^{ss}\bar{\Pi} = 1/(1 + i^{ss})$ . Future nominal money supply differentials are discounted by the steady-state nominal rate of interest. Because agents are forward-looking in their decision making, it is only the present discounted value of the relative money supplies that matters. In other words, the nominal exchange rate depends on a measure of the permanent money supply differential. Letting  $x_{t+i} \equiv (m_{t+i} -$

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<sup>5</sup>This uses the fact that  $\pi_{t+1} - \pi_{t+1}^* = (p_{t+1} - p_{t+1}^*) - (p_t - p_t^*) = s_{t+1} - s_t$ .

$m_{t+i}^* - (c_{t+i} - c_{t+i}^*)$ , the equilibrium condition for the nominal exchange rate can be written as

$$\begin{aligned} s_t &= \frac{1}{1+\delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1+\delta} \right)^i x_{t+i} = \frac{1}{1+\delta} x_t + \frac{\delta}{1+\delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1+\delta} \right)^i x_{t+1+i} \\ &= \frac{1}{1+\delta} x_t + \frac{\delta}{1+\delta} s_{t+1}. \end{aligned}$$

Rearranging and using (24) yields

$$\begin{aligned} s_{t+1} - s_t &= -\frac{1}{\delta} (x_t - s_t) \\ &= -\frac{1}{\delta} \left[ (m_t - m_t^*) - \frac{1}{1+\delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1+\delta} \right)^i (m_{t+1+i} - m_{t+1+i}^*) \right]. \end{aligned}$$

Analogously to Friedman's permanent income concept, the term

$$\frac{1}{1+\delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1+\delta} \right)^i (m_{t+1+i} - m_{t+1+i}^*)$$

can be interpreted as the permanent money supply differential. If the current value of  $m - m^*$  is high relative to the permanent value of this differential, the nominal exchange rate will fall (the home currency will appreciate). If  $s_t$  reflects the permanent money supply differential at time  $t$ , and  $m_t$  is temporarily high relative to  $m_t^*$ , then the permanent differential will be lower beginning in period  $t + 1$ . As a result, the home currency appreciates.

An explicit solution for the nominal exchange rate can be obtained if specific processes for the nominal money supplies are assumed. To take a very simple case, suppose  $m$  and  $m^*$  each follow constant, deterministic growth paths given by

$$m_t = m_0 + \mu t$$

and

$$m_t^* = m_0^* + \mu^* t.$$

Strictly speaking, (24) applies only to deviations around the steady state and not to money-supply processes that include deterministic trends. However, it is very common to specify (20) and (21), which were used to derive (24), in terms of the log levels of the variables, perhaps adding a constant to represent steady-state levels. The advantage of interpreting (24) as holding for the log levels of the variables is that one can then use it to analyze shifts in the trend growth paths of the nominal money supplies, rather than just deviations around the trend. It is important to keep in mind, however, that the underlying representative-agent model implies that the interest

rate coefficients in the money-demand equations are functions of the steady-state rate of inflation. Assume this is the same in both countries, implying that the  $\delta$  parameter was the same as well. The assumption of common coefficients in two country models is common, and it will maintain it in the following examples. The limitations of doing so should be kept in mind.

Then (24) implies

$$\begin{aligned} s_t &= -(c_t - c_t^*) + \frac{1}{1 + \delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^i [m_0 - m_0^* + (\mu - \mu^*)(t + i)] \\ &= s_0 + (\mu - \mu^*)t - (c_t - c_t^*), \end{aligned}$$

where  $s_0 = m_0 - m_0^* + \delta(\mu - \mu^*)$ .<sup>6</sup> In this case, the nominal exchange rate has a deterministic trend equal to the difference in the trend of money growth rates in the two economies (also equal to the inflation rate differentials since  $\pi = \mu$  and  $\pi^* = \mu^*$ ). If domestic money growth exceeds foreign money growth ( $\mu > \mu^*$ ),  $s$  will rise over time to reflect the falling value of the home currency relative to the foreign currency.

### 2.2.2 Uncovered Interest Parity

Real rates of return in the two countries have been assumed to be equal, so the Euler conditions for the optimal consumption paths (16 and 17) imply the same expected consumption growth in each economy. It follows from the equality of real returns that nominal interest rates must satisfy  $i_t - \pi_{t+1} = r_t = i_t^* - \pi_{t+1}^*$ , and this means, using (22), that

$$i_t - i_t^* = \pi_{t+1} - \pi_{t+1}^* = s_{t+1} - s_t.$$

The nominal interest rate differential is equal to the actual change in the exchange rate in a perfect-foresight equilibrium. This equality would not hold in the presence of uncertainty, since then variables dated  $t + 1$  would need to be replaced with their expected values, conditional on the information available at time  $t$ . In this case,

$$E_t s_{t+1} - s_t = i_t - i_t^*, \tag{25}$$

and nominal interest rate differentials would reflect expected exchange-rate changes. If the home country has a higher nominal interest rate in equilibrium, its currency must be expected to depreciate ( $s$  must be expected to rise) to equalize real returns across the two countries.

This condition, known as *uncovered nominal interest parity*, links interest rates and exchange rate expectations in different economies if their financial markets are integrated. Under rational expectations, the actual exchange rate at  $t + 1$  can be written as equal to the expectation of

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<sup>6</sup>This uses the fact that  $\sum_{i=0}^{\infty} ib^i = b/(1 - b)^2$  for  $|b| < 1$ .

the future exchange rate plus a forecast error  $\varphi_t$  uncorrelated with  $E_t s_{t+1}$ :  $s_{t+1} = E_t s_{t+1} + \varphi_t$ . Uncovered interest parity then implies

$$s_{t+1} - s_t = i_t - i_t^* + \varphi_{t+1}.$$

The ex post observed change in the exchange rate between times  $t$  and  $t+1$  is equal to the interest-rate differential at time  $t$  plus a random, mean zero forecast error. Since this forecast error will, under rational expectations, be uncorrelated with information, such as  $i_t$  and  $i_t^*$ , that is known at time  $t$ , one can recast uncovered interest parity in the form of a regression equation:

$$s_{t+1} - s_t = a + b(i_t - i_t^*) + \varphi_{t+1}, \quad (26)$$

with the null hypothesis of uncovered interest parity implying that  $a = 0$  and  $b = 1$ . Unfortunately, the evidence rejects this hypothesis.<sup>7</sup> In fact, estimated values of  $b$  are often negative.

One interpretation of these rejections is that the error term in an equation such as (26) is not simply due to forecast errors. Suppose, more realistically, that (25) does not hold exactly:

$$E_t s_{t+1} - s_t = i_t - i_t^* + v_t,$$

where  $v_t$  captures factors such as risk premia that would lead to divergences between real returns in the two countries. In this case, the error term in the regression of  $s_{t+1} - s_t$  on  $i_t - i_t^*$  becomes  $v_t + \varphi_{t+1}$ . If  $v_t$  and  $i_t - i_t^*$  are correlated, ordinary least squares estimates of the parameter  $b$  in (26) will be biased and inconsistent.

Correlation between  $v$  and  $i - i^*$  might arise if monetary policies are implemented in a manner that leads the nominal interest-rate differential to respond to the current exchange rate. For example, suppose that the monetary authority in each country tends to tighten policy whenever its currency depreciates. This could occur if the monetary authorities are concerned with inflation; depreciation raises the domestic currency price of foreign goods and raises the domestic price level. To keep the example simple for illustrative purposes, suppose that as a result of such a policy, the nominal interest rate differential is given by

$$i_t - i_t^* = \mu s_t + u_t, \quad \mu > 0,$$

where  $u_t$  captures any other factors affecting the interest rate differential.<sup>8</sup> Assume  $u$  is an exoge-

<sup>7</sup>For a summary of the evidence, see [Froot and Thaler \(1990\)](#). See also [McCallum \(1994\)](#), [Eichenbaum and Evans \(1995\)](#), and [Schlagenhauf and Wrase \(1995\)](#).

<sup>8</sup>The rationale for such a policy is clearly not motivated within the context of a model with perfectly flexible prices in which monetary policy has no real effects. The general point is to illustrate how empirical relationships such as (26) can depend on the conduct of policy.

nous, white noise process. Substituting this into the uncovered interest parity condition yields

$$E_t s_{t+1} = (1 + \mu)s_t + u_t + v_t, \quad (27)$$

the solution to which is<sup>9</sup>

$$s_t = - \left( \frac{1}{1 + \mu} \right) (u_t + v_t).$$

Since this solution implies that  $E_t s_{t+1} = -E_t(u_{t+1} + v_{t+1})/(1 + \mu) = 0$ , the interest parity condition is then given by

$$E_t s_{t+1} - s_t = - \left( \frac{1}{1 + \mu} \right) (u_t + v_t) = i_t - i_t^* + v_t$$

or  $i_t - i_t^* = (u_t - \mu v_t)/(1 + \mu)$ .

What does this imply for tests of uncovered interest parity? From the solution for  $s_t$ ,  $s_{t+1} - s_t = -(u_{t+1} - u_t + v_{t+1} - v_t)/(1 + \mu)$ . The probability limit of the interest-rate coefficient in the regression of  $s_{t+1} - s_t$  on  $i_t - i_t^*$  is equal to

$$\frac{\text{cov}(s_{t+1} - s_t, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} = \frac{\frac{1}{(1+\mu)^2} (\sigma_u^2 - \mu\sigma_v^2)}{\frac{1}{(1+\mu)^2} (\sigma_u^2 + \mu^2\sigma_v^2)} = \frac{\sigma_u^2 - \mu\sigma_v^2}{\sigma_u^2 + \mu^2\sigma_v^2},$$

which will not generally equal 1, the standard null in tests of interest parity. If  $u \equiv 0$ , the probability limit of the regression coefficient is  $-1/\mu$ . That is, the regression estimate uncovers the policy parameter  $\mu$ . Not only would a regression of the change in the exchange rate on the interest differential not yield the value of 1 predicted by the uncovered interest parity condition, but the estimate would be negative.

McCallum (1994) develops more fully the argument that rejections of uncovered interest parity may arise because standard tests compound the parity condition with the manner in which monetary policy is conducted. While uncovered interest parity is implied by the model independently of the manner in which policy is conducted, the outcomes of statistical tests may in fact be dependent on the behavior of monetary policy, since policy may influence the time-series properties of the nominal-interest-rate differential.

As noted earlier, tests of interest parity often report negative regression coefficients on the interest rate differential in (26). This finding is also consistent with the empirical evidence reported by Eichenbaum and Evans (1995). They estimate the impact of monetary shocks on nominal and real exchange rates and interest-rate differentials between the United States and France, Germany, Italy, Japan, and the United Kingdom. A contractionary U.S. monetary-policy shock leads to a

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<sup>9</sup>From (27), the equilibrium exchange-rate process must satisfy  $E_t s_{t+1} = (1 + \mu)s_t + u_t + v_t$  so that the state variables are  $u_t$  and  $v_t$ . Following McCallum (1983), the minimal state solution takes the form  $s_t = b_0 + b_1(u_t + v_t)$ . This implies that  $E_t s_{t+1} = b_0$ . So the interest parity condition becomes  $b_0 = (1 + \mu)(b_0 + b_1(u_t + v_t)) + u_t + v_t$ , which will hold for all realizations of  $u$  and  $v$  if  $b_0 = 0$  and  $b_1 = -1/(1 + \mu)$ .

persistent nominal and real appreciation of the dollar and a fall in  $i_t^* - i_t + s_{t+1} - s_t$ , where  $i$  is the U.S. interest rate and  $i^*$  is the foreign rate. Uncovered interest parity implies that this expression should have expected value equal to zero, yet it remains predictably low for several months. Rather than leading to an expected depreciation that offsets the rise in  $i$ , excess returns on U.S. dollar securities remain high for several months following a contractionary U.S. monetary-policy shock.<sup>10</sup>

## 2.3 Sticky Prices

Just as was the case with the closed economy, flexible-price models of the open economy appear unable to replicate the size and persistence of monetary shocks on real variables. And just as with closed-economy models, this can be remedied by the introduction of nominal rigidities. Obstfeld and Rogoff (1996) provided an analysis of their basic two-country model under the assumption that prices are set one period in advance.<sup>11</sup> The presence of nominal rigidities leads to real effects of monetary disturbances through the channels discussed in chapter 5, but new channels through which monetary disturbances have real effects are now also present.

Suppose  $p(h)$ , the domestic currency price of domestically produced goods, is set one period in advance and fixed for one period. A similar assumption is made for the foreign currency price of foreign-produced goods,  $p^*(f)$ . Although  $p(h)$  and  $p^*(f)$  are preset, the aggregate price indices in each country will fluctuate with the nominal exchange rate according to (11) and (12). Nominal depreciation, for example, raises the domestic price index  $p$  by increasing the domestic currency price of foreign-produced goods. This introduces a new channel, one absent in a closed economy, through which monetary disturbances can have an immediate impact on the price level. Recall that in the closed economy, there is no distinction between the price of domestic output and the general price level. Nominal price rigidities implied the price level cannot adjust immediately to monetary disturbances. Exchange-rate movements alter the domestic currency price of foreign goods, allowing the consumer price index to move in response to such disturbances even in the presence of nominal rigidities.

Now suppose that in period  $t$  the home country's money supply rises unexpectedly relative to that of the foreign country.<sup>12</sup> Under Obstfeld and Rogoff's simplifying assumption that prices adjust completely after one period, both economies return to their steady state one period after the change in  $m$ . But during the one period in which product prices are preset, real output and

<sup>10</sup>Eichenbaum and Evans measured monetary policy shocks in a variety of ways (VAR innovations to nonborrowed reserves relative to total reserves, VAR innovations to the federal funds rate, and Romer and Romer's 1989 measures of policy shifts). However, the identification scheme used in their VARs assumes that policy does not respond contemporaneously to the real exchange rate. This means that the specific illustrative policy response to the exchange rate that led to (27) is ruled out by their framework.

<sup>11</sup>See their Chapter 10. They also consider the case in which nominal wages are preset.

<sup>12</sup>An unexpected change is inconsistent with the assumption of perfect foresight implicit in the nonstochastic version of the model derived earlier. However, the linear approximation will continue to hold under uncertainty if future variables are replaced with their mathematical expectation.

consumptions levels will be affected.<sup>13</sup> And these real effects mean that the home country may run a current account surplus or deficit in response to the change in  $m$ . This effect on the current account alters the net asset positions of the two economies and can affect the new steady-state equilibrium.

Interpreting the model consisting of (11)-(21) as applying to deviations around the initial steady state, the Euler conditions (16) and (17), imply that  $c_{t+1} - c_{t+1}^* = c_t - c_t^*$ . Since the economies are in the new steady state after one period (i.e., in  $t + 1$ ),  $c_{t+1} - c_{t+1}^* \equiv \mathcal{C}$  is the steady-state consumption differential between the two countries. But since  $c_t - c_t^* = c_{t+1} - c_{t+1}^* = \mathcal{C}$ , this relationship implies that relative consumption in the two economies immediately jumps in period  $t$  to the new steady-state value. Equation (23), which expresses relative money demands in the two economies, can then be written  $m_t - m_t^* - s_t = \mathcal{C} - \delta(s_{t+1} - s_t)$ . Solving this equation forward for the nominal exchange rate (assuming no bubbles),

$$s_t = \frac{1}{1 + \delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^i (m_{t+i} - m_{t+i}^* - \mathcal{C}).$$

If the change in  $m_t - m_t^*$  is a permanent one-time change, one can let  $\Omega \equiv m - m^*$  without time subscripts denote this permanent change. The equilibrium exchange rate is then equal to

$$s_t = \frac{1}{1 + \delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^i (\Omega - \mathcal{C}) = \Omega - \mathcal{C}. \quad (28)$$

Since  $\Omega - \mathcal{C}$  is a constant, (28) implies that the exchange rate jumps immediately to its new steady state following a permanent change in relative nominal money supplies. If relative consumption levels do not adjust (i.e., if  $\mathcal{C} = 0$ ), then the permanent change in  $s$  is just equal to the relative change in nominal money supplies  $\Omega$ . An increase in  $m$  relative to  $m^*$  (i.e.,  $\Omega > 0$ ) produces a depreciation of the home country currency. If  $\mathcal{C} \neq 0$ , then changes in relative consumption affect the relative demand for money from (20) and (21). For example, if  $\mathcal{C} > 0$ , consumption, as well as money demand, in the home country is higher than initially. Equilibrium between home money supply and home money demand can be restored with a smaller increase in the home price level. Since  $p(h)$  and  $p^*(f)$  are fixed for one period, the increase in  $p$  necessary to maintain real money demand and real money supply equal is generated by a depreciation (a rise in  $s$ ). The larger the rise in home consumption, the larger the rise in real money demand and the smaller the necessary rise in  $s$ . This is just what (28) says.

Although the impact of a change in  $m - m^*$  on the exchange rate, given  $\mathcal{C}$ , has been determined, the real consumption differential is itself endogenous. To determine  $\mathcal{C}$  requires several steps. First,

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<sup>13</sup>If we consider a situation in which the economies are initially in a steady state, the preset values for  $p(h)$  and  $p^*(f)$  will equal zero.

the linear approximation to the current account relates the home country's accumulation of net assets to the excess of its real income over its consumption:  $b = y_t + [p_t(h) - p_t] - c_t = y_t - (1 - n)s_t - c_t$ , where  $p_t(h) - p_t = -(1 - n)s_t$  follows from (11) and the fact that  $p_t(h)$  is fixed (and equal to zero) during period  $t$ . Similarly, for the foreign economy,  $-nb/(1 - n) = y_t^* + ns_t - c_t^*$ . Together, these imply

$$\frac{b}{1 - n} = (y_t - y_t^*) - (c_t - c_t^*) - s_t. \quad (29)$$

From (13) and (14),  $y_t - y_t^* = s_t/(1 - q)$ , so (29) becomes

$$\frac{b}{1 - n} = \left(\frac{q}{1 - q}\right) s_t - (c_t - c_t^*) = \left(\frac{q}{1 - q}\right) s_t - \mathcal{C}, \quad (30)$$

where the definition of  $\mathcal{C}$  as the consumption differential has been used.

The last step is to use the steady-state relationship between consumption, income, and asset holdings given by (9) and (10) to eliminate  $b$  in (30) by expressing it in terms of the exchange-rate and consumption differences. In the steady state,  $b$  is constant and current accounts are zero, so consumption equals real income inclusive of asset income. In terms of the linear approximation, (9) and (10) become

$$c = rb + y + [p(h) - p] = rb + y - (1 - n)[s + p^*(f) - p(h)] \quad (31)$$

and

$$c^* = -\left(\frac{n}{1 - n}\right) rb + y^* + n[s + p^*(f) - p(h)]. \quad (32)$$

From the steady-state labor-leisure choice linking output and consumption given in (18) and (19),  $(2 - q)(y - y^*) = -(c - c^*)$ , and from the link between relative prices and demand from (13) and (14),  $y - y^* = [s + p^*(f) - p(h)]/(1 - q)$ . Using these relationships, one can now subtract (32) from (31), yielding

$$\begin{aligned} \mathcal{C} &= \left(\frac{1}{1 - n}\right) rb + (y - y^*) - [s + p^*(f) - p(h)] \\ &= \left(\frac{1}{1 - n}\right) rb + q(y - y^*) \\ &= \left(\frac{1}{1 - n}\right) rb + \left(\frac{q}{q - 2}\right) \mathcal{C}. \end{aligned}$$

Finally, this yields

$$b = \left(\frac{1 - n}{r}\right) \left(\frac{2}{2 - q}\right) \mathcal{C}. \quad (33)$$



Substituting (33) into (30),  $[2/(2-q)]C/r = qs_t/(1-q) - C$ . Solving for  $s$  in terms of  $C$ ,

$$s_t = \psi C, \tag{34}$$

where  $\psi = (1-q)[1+2/r(2-q)]/q > 0$ . But from (28),  $s_t = \Omega - C$ , so  $\psi C = \Omega - C$ . It follows that the consumption differential is  $C = \Omega/(1+\psi)$ . The equilibrium nominal exchange-rate adjustment to a permanent change in the home country's nominal money supply is then given by

$$s_t = \left( \frac{\psi}{1+\psi} \right) \Omega < \Omega.$$

With  $\psi > 0$ , the domestic monetary expansion leads to a depreciation that is less than proportional to the increase in  $m$ . This induces an expansion in domestic production and consumption. Consumption rises by less than income, so the home country runs a current account surplus and accumulates assets that represent claims against the future income of the foreign country. This allows the home country to maintain higher consumption forever. As noted, consumption levels jump immediately to their new steady state with  $C = \Omega/(1+\psi) > 0$ .

The two-country model employed in this section has the advantage of being based on the clearly specified decision problems faced by agents in the model. As a consequence, the responses of consumption, output, interest rates, and the exchange rate are consistent with optimizing behavior. Unanticipated monetary disturbances can have a permanent impact on real consumption levels and welfare when prices are preset. These effects arise because the output effects of a monetary surprise alter each country's current account, thereby altering their relative asset positions. A monetary expansion in the home country, for instance, produces a currency depreciation and a rise in the domestic price level  $p$ . This, in turn, induces a temporary expansion in output in the home country (see 13). With consumption determined on the basis of permanent income, consumption rises less than output, leading the home country to run a trade surplus as the excess of output over domestic consumption is exported. As payment for these exports, the home country receives claims against the future output of the foreign country. Home consumption does rise, even though the increase in output lasts only one period, as the home country's permanent income has risen by the annuity value of its claim on future foreign output.

A domestic monetary expansion leads to permanently higher real consumption for domestic residents; welfare is increased. This observation suggests that each country has an incentive to engage in a monetary expansion. However, a joint proportionate expansion of each country's money supply leaves  $m - m^*$  unchanged. There are then no exchange-rate effects, and relative consumption levels do not change. After one period, when prices fully adjust, a proportional change in  $p(h)$  and  $p^*(f)$  returns both economies to the initial equilibrium. But since output is inefficiently low because monopolistic competition is present, the one-period rise in output does increase welfare in both

countries. Both countries have an incentive to expand their money supplies, either individually or in a coordinated fashion.

This analysis involved changes in money supplies that were unexpected. If they had been anticipated, the level at which price setters would set individual goods prices would have incorporated expectations of money-supply changes. As noted in chapter 5, fully anticipated changes in the nominal money supply will not have the real effects that unexpected changes do when there is some degree of nominal wage or price rigidity. And as noted in chapter 6, the incentive to create surprise expansions can, in equilibrium, lead to steady inflation without the welfare gains an unanticipated expansion would bring.

Unanticipated, permanent changes in the money supply can have permanent effects on the international distribution of wealth in the Obstfeld-Rogoff model when there are nominal rigidities. Corsetti and Presenti (2001) have developed a two-country model with similar micro foundations as the Obstfeld-Rogoff model but in which preferences are specified so that changes in national money supplies do not cause wealth redistributions. Corsetti and Presenti assume the elasticity of output supply with respect to relative prices and the elasticity of substitution between the home-produced and foreign-produced goods are both equal to 1. Thus, an increase in the relative price of the foreign good (a decline in the terms of trade) lowers the purchasing power of domestic agents, but it also leads to a rise in the demand for domestic goods that increases nominal incomes. These two effects cancel, leaving the current account and international lending and borrowing unaffected. By eliminating current account effects, the Corsetti-Presenti model allows for a tractable closed-form solution with 1-period nominal wage rigidity, and it allows them to determine the impact of policy changes on welfare.

A large and growing literature has studied open-economy models that are explicitly based on optimizing behavior by firms and households but which also incorporate nominal rigidities. Besides the work of Obstfeld and Rogoff (1995), Obstfeld and Rogoff (1996) and Corsetti and Presenti (2001), examples include Betts and Devereux (2000), Obstfeld and Rogoff (2000), Benigno and Benigno (2008), Corsetti and Presenti (2002), and Kollman (2001). Lane (2001), Engel (2002), and Corsetti et al. (2010) provide surveys of the “new open economy macroeconomics.”

### 3 Policy Coordination

An important issue facing economies linked by trade and capital flows is the role to be played by policy coordination. Monetary policy actions by one country will affect other countries, leading to spillover effects that open the possibility of gains from policy coordination. As demonstrated in the previous section, the real effects of an unanticipated change in the nominal money supply in the two-country model depend on how  $m - m^*$  is affected. A rise in  $m$ , holding  $m^*$  unchanged, will produce a home country depreciation, shifting world demand toward the home country’s output. With

preset prices and output demand determined, the exchange-rate movement represents an important channel through which a monetary expansion affects domestic output. If both monetary authorities attempt to generate output expansions by increasing their money supplies, this exchange-rate channel will not operate, since the exchange rate depends on the relative money supplies. Thus, the impact of an unanticipated change in  $m$  depends critically on the behavior of  $m^*$ .

This dependence raises the issue of whether there are gains from coordinating monetary policy. [Hamada \(1976\)](#) is closely identified with the basic approach that has been used to analyze policy coordination, and in this section, develops a version of his framework. [Canzoneri and Henderson \(1989\)](#) provided an extensive discussion of monetary policy coordination issues; a survey is provided by [Currie and Levine \(1991\)](#).

Consider a model with two economies. Assume each economy's policy authority can choose its inflation rate and, because of nominal rigidities, monetary policy can have real effects in the short run. In this context, a complete specification of policy behavior is more complicated than in a closed-economy setting; one must specify how each national policy authority interacts strategically with the other policy authority. Two possibilities are considered. Coordinated policy is considered first, meaning that inflation rates in the two economies are chosen jointly to maximize a weighted sum of the objective functions of the two policy authorities. Noncoordinated policy is considered second, with the policy authorities interacting in a Nash equilibrium. In this setting, each policy authority sets its own inflation rate to maximize its objective function, taking as given the inflation rate in the other economy. These clearly are not the only possibilities. One economy may act as a Stackelberg leader, recognizing the impact its choice has on the inflation rate set by the other economy. Reputational considerations along the lines studied in chapter 6 can also be incorporated into the analysis (see [Canzoneri and Henderson \(1989\)](#)).

### 3.1 The Basic Model

The two-country model is specified as a linear system in log deviations around a steady state and represents an extension to the open-economy environment of the sticky-wage, AS-IS model of chapter 6. The LM relationship is dispensed with by assuming that the monetary policy authorities in the two countries set the inflation rate directly. An asterisk will denote the foreign economy, and  $\rho$  will be the real exchange rate, defined as the relative price of home and foreign output, expressed in terms of the home currency; a rise in  $\rho$  represents a real depreciation for the home economy. If  $s$  is the nominal exchange rate and  $p(h)$  and  $p^*(f)$  are the prices of home and foreign output, then  $\rho = s + p^*(f) - p(h)$ . The model should be viewed as an approximation that is appropriate when nominal wages are set in advance so that unanticipated movements in inflation affect real output. In addition to aggregate supply and demand relationships for each economy, an interest parity condition links the real interest-rate differential to anticipated changes in the real exchange

rate:

$$y_t = -b_1\rho_t + b_2(\pi_t - E_{t-1}\pi_t) + e_t \quad (35)$$

$$y_t^* = b_1\rho_t + b_2(\pi_t^* - E_{t-1}\pi_t^*) + e_t^* \quad (36)$$

$$y_t = a_1\rho_t - a_2r_t + a_3y_t^* + u_t \quad (37)$$

$$y_t^* = -a_1\rho_t - a_2r_t^* + a_3y_t + u_t^* \quad (38)$$

$$\rho_t = r_t^* - r_t + E_t\rho_{t+1}. \quad (39)$$

Equations (35) and (36) relate output to inflation surprises and the real exchange rate. A real exchange-rate depreciation reduces home aggregate supply by raising the price of imported materials and by raising consumer prices relative to producer prices. This latter effect increases the real wage in terms of producer prices. Equations (37) and (38) make demand in each country an increasing function of output in the other to reflect spillover effects that arise as an increase in output in one country raises demand for the goods produced by the other. A rise in  $\rho_t$  (a real domestic depreciation) makes domestically produced goods less expensive relative to foreign goods and shifts demand away from foreign output and toward home output.

To simplify the analysis, the inflation rate is treated as the choice variable of the policymaker. An alternative approach to treating inflation as the policy variable would be to specify money demand relationships for each country and then take the nominal money supply as the policy instrument. This would complicate the analysis without offering any new insights.

A third approach would be to replace  $r_t$  with  $i_t - E_t\pi_{t+1}$ , where  $i_t$  is the nominal interest rate, and treat  $i_t$  as the policy instrument. An advantage of this approach is that it more closely reflects the way most central banks actually implement policy. Because a number of new issues arise under nominal interest rate policies (see section ?? and chapter 8), policy is interpreted as choosing the rate of inflation in order to focus, in this section, on the role of policy coordination. Finally, a further simplification is reflected in the assumption that the parameters (the  $a_i$ 's and  $b_i$ 's) are the same in the two countries.

Demand ( $u_t, u_t^*$ ) and supply ( $e_t, e_t^*$ ) shocks are included to introduce a role for stabilization policy. These disturbances are assumed to be mean zero, serially uncorrelated processes, but they are allowed to be correlated to distinguish between common shocks that affect both economies and asymmetric shocks that originate in a single economy.

Equation (39) is an uncovered interest rate parity condition. Rewritten in the form  $r_t = r_t^* + E_t\rho_{t+1} - \rho_t$ , it implies that the home country real interest rate will exceed the foreign real rate if the home country is expected to experience a real depreciation.

Evaluating outcomes under coordinated and noncoordinated policies requires some assumption about the objective functions of the policymakers. In models built more explicitly on the behavior

of optimizing agents, alternative policies could be ranked according to their implications for the utility of the agents in the economies. Here a common approach is followed in which polices are evaluated on the basis of loss functions that depend on output variability and inflation variability:

$$V_t = \sum_{i=0}^{\infty} \beta^i (\lambda y_{t+i}^2 + \pi_{t+i}^2) \quad (40)$$

$$V_t^* = \sum_{i=0}^{\infty} \beta^i [\lambda (y_{t+i}^*)^2 + (\pi_{t+i}^*)^2]. \quad (41)$$

The parameter  $\beta$  is a discount factor between 0 and 1. The weight attached to output fluctuations relative to inflation fluctuations is  $\lambda$ . While these objective functions are ad hoc, they capture the idea that policymakers prefer to minimize output fluctuations around the steady state and fluctuations of inflation.<sup>14</sup> Objective functions of this basic form have played a major role in the analysis of policy. They reflect the assumption that steady-state output will be independent of monetary policy, so policy should focus on minimizing fluctuations around the steady state, not on the level of output.

The model can be solved to yield expressions for equilibrium output in each economy and for the real exchange rate. To obtain the real exchange rate, first subtract foreign aggregate demand (38) from domestic aggregate demand (37), using the interest parity condition (39) to eliminate  $r_t - r_t^*$ . This process yields an expression for  $y_t - y_t^*$ . Next, subtract foreign aggregate supply (36) from domestic aggregate supply (35) to yield a second expression for  $y_t - y_t^*$ . Equating these two expressions and solving for the equilibrium real exchange rate leads to the following:

$$\begin{aligned} \rho_t = & \frac{1}{B} \{b_2(1 + a_3) [(\pi_t - E_{t-1}\pi_t) - (\pi_t^* - E_{t-1}\pi_t^*)] \\ & + (1 + a_3)(e_t - e_t^*) - (u_t - u_t^*) + a_2 E_t \rho_{t+1}\}, \end{aligned} \quad (42)$$

where  $B \equiv 2a_1 + a_2 + 2b_1(1 + a_3) > 0$ . An unanticipated rise in domestic inflation relative to unanticipated foreign inflation or in  $e_t$  relative to  $e_t^*$  will increase domestic output supply relative to foreign output. Equilibrium requires a decline in the relative price of domestic output; the real exchange rate rises (depreciates), shifting demand toward domestic output. If the domestic aggregate demand shock exceeds the foreign shock,  $u_t - u_t^* > 0$ , the relative price of domestic output must rise ( $\rho$  must fall) to shift demand toward foreign output. A rise in the expected future exchange rate also leads to a rise in the current equilibrium  $\rho$ . If  $\rho$  were to increase by the same amount as the rise in  $E_t \rho_{t+1}$ , the interest differential  $r_t - r_t^*$  would be left unchanged, but the higher

<sup>14</sup>The steady-state values of  $y$  and  $y^*$  are zero by definition. The assumption that the policy loss functions depend on the variance of output around its steady-state level, and not on some higher output target, is critical for the determination of average inflation. Chapter 6 deals extensively with the time-inconsistency issues that arise when policy makers target a level of output that exceeds the economy's equilibrium level.

$\rho$  would, from (35) and (36), lower domestic supply relative to foreign supply. So  $\rho$  rises by less than the increase in  $E_t \rho_{t+1}$  to maintain goods market equilibrium.<sup>15</sup>

Notice that (42) can be written as  $\rho_t = A E_t \rho_{t+1} + v_t$ , where  $0 < A < 1$  and  $v_t$  is white noise, since the disturbances are assumed to be serially uncorrelated and the same will be true of the inflation forecast errors under rational expectations. It follows that  $E_t \rho_{t+1} = 0$  in any no-bubbles solution. The expected future real exchange rate would be nonzero if either the aggregate demand or aggregate supply shocks were serially correlated.

Now the expression for the equilibrium real exchange rate can be substituted into the aggregate supply relationships (35) and (36) to yield

$$\begin{aligned} y_t &= b_2 A_1 (\pi_t - E_{t-1} \pi_t) + b_2 A_2 (\pi_t^* - E_{t-1} \pi_t^*) \\ &\quad - a_2 A_3 E_t \rho_{t+1} + A_1 e_t + A_2 e_t^* + A_3 (u_t - u_t^*) \end{aligned} \quad (43)$$

$$\begin{aligned} y_t^* &= b_2 A_2 (\pi_t - E_{t-1} \pi_t) + b_2 A_1 (\pi_t^* - E_{t-1} \pi_t^*) \\ &\quad + a_2 A_3 E_t \rho_{t+1} + A_2 e_t + A_1 e_t^* - A_3 (u_t - u_t^*). \end{aligned} \quad (44)$$

The  $A_i$  parameters are given by

$$\begin{aligned} A_1 &= \frac{2a_1 + a_2 + b_1(1 + a_3)}{B} > 0 \\ A_2 &= \frac{b_1(1 + a_3)}{B} > 0 \\ A_3 &= \frac{b_1}{B} > 0. \end{aligned}$$

Equations (43) and (44) reveal the spillover effects through which the inflation choice of one economy affects the other economy when  $b_2 A_2 \neq 0$ . An increase in inflation in the home economy (assuming it is unanticipated) leads to a real depreciation. This occurs since unanticipated inflation leads to a home output expansion (see 35). Equilibrium requires a rise in demand for home country production. In the closed economy, this occurs through a fall in the real interest rate. In the open economy, an additional channel of adjustment arises from the role of the real exchange rate. Given that  $E_t \rho_{t+1} = 0$ , the interest parity condition (39) becomes  $\rho_t = r_t^* - r_t$ , so, for given  $r_t^*$ , the fall in  $r_t$  requires a rise in  $\rho_t$  (a real depreciation), which also serves to raise home demand.

The rise in  $\rho_t$  represents a real appreciation for the foreign economy, and this raises consumer-price wages relative to producer-price wages and increases aggregate output in the foreign economy (see 36). As a result, an expansion in the home country produces an economic expansion in the

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<sup>15</sup>The coefficient on  $E_t \rho_{t+1}$ ,  $a_2/B$  is less than 1 in absolute value.

foreign country. But as (42) shows, a surprise inflation by both countries leaves the real exchange rate unaffected. It is this link that opens the possibility that outcomes will depend on the extent to which the two countries coordinate their policies.

### 3.2 Equilibrium with Coordination

To focus on the implications of policy coordination, attention is restricted to the case of a common aggregate supply shock, common in the sense that it affects both countries. That is, suppose  $e_t = e_t^* \equiv \varepsilon_t$ , where  $\varepsilon_t$  is the common disturbance. For the rest of this section, assume  $u \equiv u^* \equiv 0$ , so that  $\varepsilon$  represents the only disturbance.

In solving for equilibrium outcomes under alternative policy interactions, the objective functions (40) and (41) simplify to a sequence of one-period problems (the problem is a static one with no link between periods). Assuming that the policy authority is able to set the inflation rate after observing the supply shock  $\varepsilon_t$ , the decision problem under a coordinated policy is

$$\min_{\pi, \pi^*} \left\{ \frac{1}{2} [\lambda y_t^2 + \pi_t^2] + \frac{1}{2} [\lambda (y_t^*)^2 + (\pi_t^*)^2] \right\}$$

subject to (43) and (44).<sup>16</sup> The first-order conditions are

$$\begin{aligned} 0 &= \lambda b_2 A_1 y_t + \pi_t + \lambda b_2 A_2 y_t^* \\ &= (1 + \lambda b_2^2 A_1^2 + \lambda b_2^2 A_2^2) \pi_t + 2\lambda b_2^2 A_1 A_2 \pi_t^* + \lambda b_2 \varepsilon_t \end{aligned}$$

$$\begin{aligned} 0 &= \lambda b_2 A_2 y_t + \lambda b_2 A_1 y_t^* + \pi_t^* \\ &= (1 + \lambda b_2^2 A_1^2 + \lambda b_2^2 A_2^2) \pi_t^* + 2\lambda b_2^2 A_1 A_2 \pi_t + \lambda b_2 \varepsilon_t, \end{aligned}$$

which uses the fact that  $A_1 + A_2 = 1$  and the result that the first-order conditions imply  $E_{t-1} \pi_t = E_{t-1} \pi_t^* = 0$ .<sup>17</sup> Solving these two equations yields the equilibrium inflation rates under coordination:

$$\pi_{c,t} = \pi_{c,t}^* = - \left( \frac{\lambda b_2}{1 + \lambda b_2^2} \right) \varepsilon_t \equiv -\theta_c \varepsilon_t. \quad (45)$$

Both countries maintain equal inflation rates. In response to an adverse supply shock ( $\varepsilon < 0$ ), inflation in both countries rises to offset partially the decline in output. Substituting (45) into the

<sup>16</sup>In defining the objective function under coordinated policy, we have assumed that each country's utility receives equal weight.

<sup>17</sup>Writing out the first order condition for  $\pi_t$  in full,  $0 = \pi_t + \lambda b_2^2 (A_1^2 + A_2^2) (\pi_t - E_{t-1} \pi_t) + 2\lambda b_2^2 A_1 A_2 (\pi_t^* - E_{t-1} \pi_t^*) + 2\lambda b_2 \varepsilon_t$ . Taking expectations conditional on time  $t-1$  information (i.e., prior to the realization of  $\varepsilon_t$ ), we obtain  $E_{t-1} \pi_t = 0$ .

expressions for output and the equilibrium real exchange rate,

$$y_{c,t} = y_{c,t}^* = \left( \frac{1}{1 + \lambda b_2^2} \right) \varepsilon_t < \varepsilon_t$$

and

$$\rho_t = 0.$$

The policy response acts to offset partially the output effects of the supply shock. The larger the weight placed on output in the loss function ( $\lambda$ ), the larger the inflation response and the more output is stabilized. Because both economies respond symmetrically, the real exchange rate is left unaffected.<sup>18</sup>

### 3.3 Equilibrium without Coordination

When policy is not coordinated, some assumption must be made about the nature of the strategic interaction between the two separate policy authorities. One natural case to consider corresponds to a Nash equilibrium; the policy authorities choose inflation to minimize loss, taking as given the inflation rate in the other economy. An alternative case arises when one country behaves as a Stackelberg leader, taking into account how the other policy authority will respond to the leader's choice of inflation. The Nash case is analyzed here, leaving the Stackelberg case to be studied as a problem at the end of the chapter.

The home policy authority picks inflation to minimize  $\lambda y_t^2 + \pi_t^2$ , taking  $\pi_t^*$  as given. The first-order condition is

$$\begin{aligned} 0 &= \lambda b_2 A_1 y_t + \pi_t \\ &= (1 + \lambda b_2^2 A_1^2) \pi_t + \lambda b_2^2 A_1 A_2 \pi_t^* + \lambda b_2 A_1 \varepsilon_t \end{aligned}$$

so that the home country's reaction function is

$$\pi_t = - \left( \frac{\lambda b_2^2 A_1 A_2}{1 + \lambda b_2^2 A_1^2} \right) \pi_t^* - \left( \frac{\lambda b_2 A_1}{1 + b_2^2 A_1^2} \right) \varepsilon_t. \quad (46)$$

A rise in the foreign country's inflation rate is expansionary for the domestic economy (see 43). The domestic policy authority lowers domestic inflation to partially stabilize domestic output. A parallel treatment of the foreign country policy authority's decision problem leads to the reaction function

$$\pi_t^* = - \left( \frac{\lambda b_2^2 A_1 A_2}{1 + \lambda b_2^2 A_1^2} \right) \pi_t - \left( \frac{\lambda b_2 A_1}{1 + \lambda b_2^2 A_1^2} \right) \varepsilon_t. \quad (47)$$

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<sup>18</sup>This would not be the case in response to an asymmetric supply shock. See problem 4.



Jointly solving these two reaction functions for the Nash equilibrium inflation rates yields

$$\pi_{N,t} = \pi_{N,t}^* = - \left( \frac{\lambda b_2 A_1}{1 + \lambda b_2^2 A_1} \right) \varepsilon_t \equiv -\theta_N \varepsilon_t. \quad (48)$$

How does stabilization policy with noncoordinated policies compare with the coordinated policy response given in (45)? Since  $A_1 < 1$ ,

$$|\theta_N| < |\theta_c|.$$

Policy responds less to the aggregate supply shock in the absence of coordination, and as a result, output fluctuates more:

$$y_{N,t} = y_{N,t}^* = \left( \frac{1}{1 + \lambda b_2^2 A_1} \right) \varepsilon_t > \left( \frac{1}{1 + \lambda b_2^2} \right) \varepsilon_t.$$

Because output and inflation responses are symmetric in the Nash equilibrium, the real exchange rate does not respond to  $\varepsilon_t$ .

Why does policy respond less in the absence of coordination? For each individual policymaker, the perceived marginal output gain from more inflation when there is an adverse realization of  $\varepsilon$  reflects the two channels through which inflation affects output. First, surprise inflation directly increases real output because of the assumption of nominal rigidities. This direct effect is given by the term  $b_2 (\pi_t - E_{t-1} \pi_t)$  in (35). Second, for given foreign inflation, a rise in home inflation leads to a real depreciation (see 42) and, again from (35), the rise in  $\rho_t$  acts to lower output, reducing the net impact of inflation on output. With  $\pi^*$  treated as given, the exchange-rate channel implies that a larger inflation increase is necessary to offset the output effects of an adverse supply shock. Since inflation is costly, the optimal policy response involves a smaller inflation response and less output stabilization. With a coordinated policy, the decision problem faced by the policy authority recognizes that a symmetric increase in inflation in both countries leaves the real exchange rate unaffected. With inflation perceived to have a larger marginal impact on output, the optimal response is to stabilize more.

The loss functions of the two countries can be evaluated under the alternative policy regimes (coordination and noncoordination). Because the two countries have been specified symmetrically, the value of the loss function will be the same for each. For the domestic economy, the expected loss when policies are coordinated is equal to

$$L^c = \frac{1}{2} \left( \frac{1}{1 + \lambda b_2^2} \right) \lambda \sigma_\varepsilon^2.$$

When policies are determined in a Nash noncooperative equilibrium,

$$L^N = \frac{1}{2} \left[ \frac{1 + \lambda b_2^2 A_1^2}{(1 + \lambda b_2^2 A_1)^2} \right] \lambda \sigma_\varepsilon^2.$$

Because  $0 < A_1 < 1$ , it follows that  $L^c < L^N$ ; coordination achieves a better outcome than occurs in the Nash equilibrium.

This example appears to imply that coordination will always dominate noncoordination. It is important to recall that the only source of disturbance was a common aggregate supply shock. The case of asymmetric shocks is addressed in problem 2. But even when there are only common shocks, coordination need not always be superior. Rogoff (1985) provided a counterexample. His argument is based on a model in which optimal policy is time inconsistent (see chapter 6), but one can briefly describe the intuition behind Rogoff's results. A coordinated monetary expansion leads to a larger short-run real-output expansion because it avoids changes in the real exchange rate. But this fact increases the incentive to engineer a surprise monetary expansion if the policymakers believe the natural rate of output is too low. Wage and price setters will anticipate this tactic, together with the associated higher inflation. Equilibrium involves higher inflation, but because it has been anticipated, output (which depends on inflation surprises) does not increase. Consequently, coordination leads to better stabilization but higher average inflation. If the costs of the latter are high enough, noncoordination can dominate coordination.

The discussion of policy coordination serves to illustrate several important aspects of open-economy monetary economics. First, the real exchange rate is the relative price of output in the two countries, so it plays an important role in equilibrating relative demand and supply in the two countries. Second, foreign shocks matter for the domestic economy; both aggregate supply and aggregate demand shocks originating in the foreign economy affect output in the domestic economy. As (43) and (44) show, however, the model implies that common demand shocks that leave  $u - u^*$  unaffected have no effect on output levels or the real exchange rate. Since these shocks do affect demand in each country, a common demand shock raises real interest rates in each country. Third, policy coordination can matter.

While the two-country model of this section is useful, it has several omissions that may limit the insights that can be gained from its use. First, the aggregate demand and aggregate supply relationships are not derived explicitly within an optimizing framework. As shown in chapter 5 and in the Obstfeld-Rogoff model, expectations of future income will play a role when consumption is determined by forward-looking, rational economic agents. Second, there is no role for current-account imbalances to affect equilibrium through their effects on foreign asset holdings. Third, no distinction has been drawn between the price of domestic output and the price index relevant for domestic residents. The loss function for the policymaker may depend on consumer price inflation. Fourth, the inflation rate was treated as the instrument of policy, directly controllable by the central

bank. This is an obvious simplification, one that abstracts from the linkages (and slippages) between the actual instruments of policy and the realized rate of inflation. Although such simplifications are useful for addressing many policy issues, chapters 11 and 12 will examine these linkages in more detail. Finally, the model, like the Obstfeld-Rogoff example, assumed one-period nominal contracts. Such a formulation fails to capture the persistence that generally characterizes actual inflation and the lags between changes in policy and the resulting changes in output and inflation.

## 4 The Small Open Economy

A two-country model provides a useful framework for examining policy interactions in an environment in which developments in one economy affect the other. For many economies, however, domestic developments have little or no impact on other economies. Decisions about policy can, in this case, treat foreign interest rates, output levels, and inflation as exogenous because the domestic economy is small relative to the rest of the world. The small open economy is a useful construct for analyzing issues when developments in the country of interest are unlikely to influence other economies.

In the small-open-economy case, the model of the previous section simplifies to become

$$y_t = -b_1\rho_t + b_2(p_t - E_{t-1}p_t) + e_t \quad (49)$$

$$y_t = a_1\rho_t - a_2r_t + u_t \quad (50)$$

$$\rho_t = r_t^* - r_t + E_t\rho_{t+1}. \quad (51)$$

The real exchange rate  $\rho$  is equal to  $s + p^* - p$ , where  $s$  is the nominal exchange rate and  $p^*$  and  $p$  are the prices of foreign and domestic output, all expressed in log terms. The aggregate supply relationship has been written in terms of the unanticipated price level rather than unanticipated inflation.<sup>19</sup> The dependence of output on price surprises arises from the presence of nominal wage and price rigidities. With foreign income and consumption exogenous, the impact of world consumption on the domestic economy can be viewed as one of the factors giving rise to the disturbance term  $u_t$ .

Consumer prices in the domestic economy are defined as

$$q_t = hp_t + (1 - h)(s_t + p_t^*), \quad (52)$$

where  $h$  is the share of domestic output in the consumer price index, while the Fisher relationship

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<sup>19</sup>Since  $p_t - E_{t-1}p_t = p_t - p_{t-1} - (E_{t-1}p_t - p_{t-1}) = \pi_t - E_{t-1}\pi_t$ , the two formulations are equivalent.

links the real rate of interest appearing in (50) and (51) with the nominal interest rate,

$$r_t = i_t - E_t p_{t+1} + p_t. \quad (53)$$

Uncovered interest parity links nominal interest rates. Since  $i^*$  will be exogenous from the perspective of the small open economy, one can write (51) as

$$i_t = E_t s_{t+1} - s_t + i_t^*, \quad (54)$$

where  $i^* = r^* + E_t p_{t+1}^* - p_t^*$ . Finally, real money demand is assumed to be given by

$$m_t - q_t = y_t - c i_t + v_t. \quad (55)$$

Notice that the basic structure of the model, like the closed-economy models of chapter 6 based on wage and/or price rigidity, displays the classical dichotomy between the real and monetary sectors if wages are flexible. That is, if wages adjust completely to equate labor demand and labor supply, the price-surprise term in (49) disappears.<sup>20</sup> In this case, (49)-(51) constitute a three-equation system for real output, the real interest rate, and the real exchange rate. Using the interest parity condition to eliminate  $r_t$  from the aggregate demand relationship, and setting the resulting expression for output equal to aggregate supply, yields the following equation for the equilibrium real exchange rate in the absence of nominal rigidities:

$$(a_1 + a_2 + b_1)\rho_t = a_2 (r_t^* + E_t \rho_{t+1}) + e_t - u_t.$$

This can be solved forward for  $\rho_t$ :

$$\begin{aligned} \rho_t &= \sum_{i=0}^{\infty} d^i E_t \left( \frac{a_2 r_{t+i}^* + e_{t+i} - u_{t+i}}{a_1 + a_2 + b_1} \right) \\ &= d \sum_{i=0}^{\infty} d^i E_t r_{t+i}^* + \frac{e_t - u_t}{a_1 + a_2 + b_1} \end{aligned}$$

where  $d \equiv a_2 / (a_1 + a_2 + b_1) < 1$  and the second equals sign follows from the assumption that  $e$  and  $u$  are serially uncorrelated processes. The real exchange rate responds to excess supply for domestic output; if  $e_t - u_t > 0$ , a real depreciation increases aggregate demand and lowers aggregate supply to restore goods market equilibrium.

The monetary sector consists of (52)-(55), plus the definition of the nominal exchange rate as

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<sup>20</sup>Recall from chapter 6 that the assumption behind an aggregate-supply function such as (49) is that nominal wages are set in advance on the basis of expectations of the price level, while actual employment is determined by firms on the basis of realized real wages (and therefore on the actual price level).

$s_t = \rho_t - p_t^* + p_t$ . When wages and prices are flexible, these determine the two price levels  $p$  (the price of domestic output) and  $q$  (the consumer price index). From the Fisher equation, the money demand equation, and the definition of  $q_t$ ,

$$m_t - p_t = y_t + (1 - h)\rho_t - c(r_t + E_t p_{t+1} - p_t) + v_t.$$

Because the real values are exogenous with respect to the monetary sector when there are no nominal rigidities, this equation can be solved for the equilibrium value of  $p_t$ :

$$p_t = \left( \frac{1}{1+c} \right) \sum_{i=0}^{\infty} \left( \frac{c}{1+c} \right)^i E_t (m_{t+i} - z_{t+i} - v_{t+i}),$$

where  $z_{t+i} \equiv y_{t+i} + (1 - h)\rho_{t+i} - cr_{t+i}$ . The equilibrium  $p_t$  depends not just on the current money supply, but also on the expected future path of  $m$ . Since (52) implies  $p_t = q_t - (1 - h)\rho_t$ , the equilibrium behavior of the domestic consumer price index  $q_t$  follows from the solutions for  $p_t$  and  $\rho_t$ .

When nominal wages are set in advance, the classical dichotomy no longer holds. With  $p_t - E_{t-1}p_t$  affecting the real wage, employment, and output, any disturbance in the monetary sector that was unanticipated will affect output, the real interest rate, and the real exchange rate. Since the model does not incorporate any mechanism to generate real persistence, these effects last only for one period.

With nominal wage rigidity, monetary policy affects real aggregate demand through both interest-rate and exchange-rate channels. As can be seen from (50), these two variables appear in the combination  $a_1\rho_t - a_2r_t$ . For this reason, the interest rate and exchange rate are often combined to create a *monetary conditions index*; in the context of the present model, this index would be equal to  $r_t - a_1\rho_t/a_2$ . Variations in the real interest rate and real exchange rate that leave this linear combination unchanged would be neutral in their impact on aggregate demand, since the reduction in domestic aggregate demand caused by a higher real interest rate would be offset by a depreciation in the real exchange rate.

## 4.1 Flexible Exchange Rates

Suppose that nominal wages are set in advance, but the nominal exchange rate is free to adjust flexibly in the face of economic disturbances. In addition, assume that monetary policy is implemented through control of the nominal money supply. In this case, the model consisting of (49)-(54) can be reduced to two equations involving the price level, the nominal exchange rate, and the nominal money supply (see the appendix for details). Equilibrium will depend on expectations of the period  $t + 1$  exchange rate, and the response of the economy to current policy actions may depend on how

these expectations are affected.

To determine how the exchange rate and the price level respond to monetary shocks, assume a specific process for the nominal money supply. To allow for a distinction between transitory and permanent monetary shocks, assume

$$m_t = \mu + m_{t-1} + \varphi_t - \gamma\varphi_{t-1}, \quad 0 \leq \gamma \leq 1, \quad (56)$$

where  $\varphi$  is a serially uncorrelated white noise process. If  $\gamma = 0$ ,  $m_t$  follows a random walk with drift  $\mu$ ; innovations  $\varphi$  have a permanent impact on the level of  $m$ . If  $\gamma = 1$ , the money supply is white noise around a deterministic trend. If  $0 < \gamma < 1$ , a fraction  $(1 - \gamma)$  of the innovation has a permanent effect on the level of the money supply.

To analyze the impact of foreign price shocks on the home country, let

$$p_t^* = \pi^* + p_{t-1}^* + \phi_t, \quad (57)$$

where  $\phi$  is a random white noise disturbance. This allows for a constant average foreign inflation rate of  $\pi^*$  with permanent shifts in the price path due to the realizations of  $\phi$ .

Using the method of undetermined coefficients, the appendix shows that the following solutions for  $p_t$  and  $s_t$  are consistent with (49)-(54), and with rational expectations:

$$p_t = k_0 + m_{t-1} + \frac{B_2[1 + c(1 - \gamma)]}{K} \varphi_t - \gamma\varphi_{t-1} + \frac{[(A_2 - B_2)u_t - A_2e_t - B_2v_t]}{K} \quad (58)$$

$$s_t = d_0 + m_{t-1} - p_{t-1}^* - \phi_t - \frac{B_1[1 + c(1 - \gamma)]}{K} \varphi_t - \gamma\varphi_{t-1} + \frac{[(B_1 - A_1)u_t + A_1e_t + B_1v_t]}{K}, \quad (59)$$

where  $A_1 = h - a_1 - a_2$ ,  $A_2 = 1 + c - A_1 > 0$ ,  $B_1 = -(a_1 + a_2 + b_1 + b_2) < 0$ ,  $B_2 = a_1 + a_2 + b_1 > 0$ , and  $K = -[(1 + c)B_1 + b_2A_1]$ . The constant  $k_0$  is given by

$$k_0 = (1 + c)\mu + \left[ c - \frac{a_2(1 - h - b_1)}{a_1 + b_1} \right] r^*,$$

and  $d_0 = k_0 - \pi^*$ .

Of particular note is the way a flexible exchange rate insulates the domestic economy from the foreign price shock  $\phi$ . Neither  $p_{t-1}^*$  nor  $\phi_t$  affects the domestic price level under a flexible exchange-rate system (see 58). Instead, (59) shows how they move the nominal exchange rate to maintain

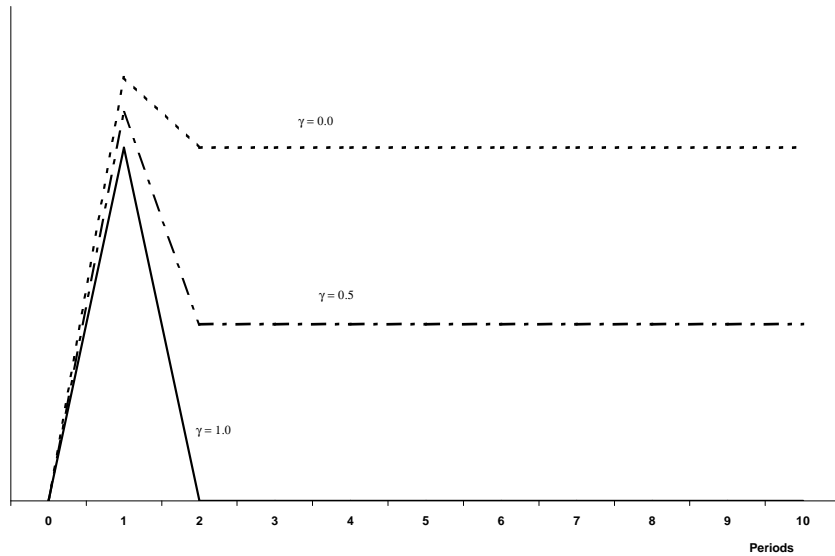


Figure 1: Nominal Exchange Rate Response to a Monetary Shock

the domestic currency price of foreign goods,  $s + p^*$ , unchanged. This insulates the real exchange rate and domestic output from fluctuations in the foreign price level.

With  $B_2[1+c(1-\gamma)]/K > 0$  and  $-B_1[1+c(1-\gamma)]/K > 0$ , a positive monetary shock increases the equilibrium price level and the nominal exchange rate. That is, the domestic currency depreciates in response to a positive money shock. The effect is offset partially the following period if  $\gamma > 0$ . The shape of the exchange rate response to a monetary shock is shown in figure 6.1 for different values of  $\gamma$ .

The  $\gamma < 1$  cases in figure 6.1 illustrate Dornbusch's overshooting result (Dornbusch (1976)). To the extent that the rise in  $m$  is permanent (i.e.,  $\gamma < 1$ ), the price level and the nominal exchange rate eventually rise proportionately. With one-period nominal rigidities, this occurs in period 2. A rise in the nominal money supply that increases the real supply of money reduces the nominal interest rate to restore money market equilibrium. From the interest parity condition, the domestic nominal rate can fall only if the exchange rate is expected to fall. Yet the exchange rate will be higher than its initial value in period 2, so to generate an expectation of a fall,  $s$  must rise more than proportionately to the permanent rise in  $m$ . It is then expected to fall from period 1 to period 2; the nominal rate overshoots its new long-run value.

The Dornbusch overshooting result stands in contrast to Obstfeld and Rogoff's conclusion, derived in section 2.3, that a permanent change in the nominal money supply does not lead to

overshooting. Instead, the nominal exchange rate jumps immediately to its new long-run level. This difference results from the ad hoc nature of aggregate demand in the model of this section. In the Obstfeld-Rogoff model, consumption is derived from the decision problem of the representative agent, with the Euler condition for consumption linking consumption choices over time. The desire to smooth consumption implies that consumption immediately jumps to its new equilibrium level. As a result, exchange-rate overshooting is eliminated in the basic Obstfeld-Rogoff model.

One implication of the overshooting hypothesis is that exchange-rate movements should follow a predictable or forecastable pattern in response to monetary shocks. A positive monetary shock leads to an immediate depreciation followed by an appreciation. The path of adjustment will depend on the extent of nominal rigidities in the economy, since these influence the speed with which the economy adjusts in response to shocks. Such a predictable pattern is not clearly evident in the data. In fact, nominal exchange rates display close to random-walk behavior over short time periods (Meese and Rogoff (1983)). In a VAR-based study of exchange-rate responses to U.S. monetary shocks, Eichenbaum and Evans (1995) do not find evidence of overshooting, but they do find sustained and predictable exchange-rate movements following monetary-policy shocks. A monetary contraction produces a small initial appreciation, with the effect growing so that the dollar appreciates for some time. However, in a study based on more direct measurement of policy changes, Bonser-Neal C. and Sellon Jr. (1998) find general support for the overshooting hypothesis. They measure policy changes by using data on changes in the Federal Reserve's target for the federal funds rate, rather than the actual funds rate, and restrict attention to time periods during which the funds rate was the Fed's policy instrument.

## 4.2 Fixed Exchange Rates

Under a system of fixed exchange rates, the monetary authority is committed to using its policy instrument to maintain a constant nominal exchange rate. This commitment requires that the monetary authority stand ready to buy or sell domestic currency for foreign exchange to maintain the fixed exchange rate. When it is necessary to sell foreign exchange, such a policy will be unsustainable if the domestic central bank's reserves of foreign exchange are expected to go to zero. Such expectations can produce speculative attacks on the currency.<sup>21</sup> The analysis here considers only the case of a sustainable fixed rate. And to draw the sharpest contrast with the flexible-exchange-rate regime, assume that the exchange rate is pegged. In practice, most fixed-exchange-rate regimes allow rates to fluctuate within narrow bands.<sup>22</sup>

Normalizing the fixed rate at  $s_t = 0$  for all  $t$ , the real exchange rate equals  $p_t^* - p_t$ . Assuming

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<sup>21</sup>See Krugman (1979) and Garber and Svensson (1995).

<sup>22</sup>Exchange-rate behavior under a target zone system was first analyzed by Krugman (1991).



the foreign price level follows (57), the basic model becomes

$$y_t = -b_1(p_t^* - p_t) + b_2(p_t - E_{t-1}p_t) + e_t$$

$$y_t = a_1(p_t^* - p_t) - a_2r_t + u_t$$

$$r_t = r^* + \pi^* - (E_t p_{t+1} - p_t).$$

The nominal interest rate has been eliminated, since the interest-parity condition and the fixed-exchange-rate assumption imply that  $i = r^* + \pi^*$ . These three equations can be solved for the price level, output, and the domestic real interest rate. The money demand condition plays no role because  $m$  must endogenously adjust to maintain the fixed exchange rate.

Solving for  $p_t$ ,

$$p_t = \frac{(a_1 + b_1)p_t^* - a_2(r^* + \pi^*) + a_2E_t p_{t+1} + b_2E_{t-1}p_t + u_t - e_t}{a_1 + a_2 + b_1 + b_2}.$$

Using the method of undetermined coefficients, one obtains

$$p_t = p_t^* - \frac{a_2 r^*}{a_1 + b_1} + \frac{u_t - e_t - b_2 \phi_t}{a_1 + a_2 + b_1 + b_2}. \quad (60)$$

Comparing (60) to (58) reveals some of the major differences between the fixed and flexible exchange-rate systems. Under fixed exchange rates, the average domestic rate of inflation must equal the foreign inflation rate:  $E(p_{t+1} - p_t) = E(p_{t+1}^* - p_t^*) = \pi^*$ . The foreign price level and foreign price shocks ( $\phi$ ) affect domestic prices and output under the fixed-rate system. But domestic disturbances to money demand or supply ( $\varphi$  and  $v$ ) have no price level or output effects. This situation is in contrast to the case under flexible exchange rates and is one reason why high-inflation economies often attempt to fix their exchange rates with low inflation countries. But when world inflation is high, a country can maintain lower domestic inflation only by allowing its nominal exchange rate to adjust.

The effects on real output of aggregate demand and supply disturbances also depend on the nature of the exchange-rate system. Under flexible exchange rates, a positive aggregate demand shock increases prices and real output. Goods-market equilibrium requires a rise in the real interest rate and a real appreciation. By serving to equilibrate the goods market and partially offset the rise in aggregate demand following a positive realization of  $u$ , the exchange-rate movement helps stabilize aggregate output. As a result, the effect of  $u$  on  $y$  is smaller under flexible exchange rates than under fixed exchange rates.<sup>23</sup>

The choice of exchange-rate regime influences the manner in which economic disturbances affect

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<sup>23</sup>This is consistent with the estimates for Japan reported in [Hutchison and Walsh \(1992\)](#). [Obstfeld \(1985\)](#) discussed the insulation properties of exchange rate systems.

the small open economy. While the model examined here does not provide an internal welfare criterion (such as the utility of the representative agent in the economy), such models have often been supplemented with loss functions depending on output or inflation volatility (see section 3 to study policy coordination), which are then used to rank alternative exchange-rate regimes. Based on such measures, the choice of an exchange-rate regime should depend on the relative importance of various disturbances. If volatility of foreign prices is of major concern, a flexible exchange rate will serve to insulate the domestic economy from real exchange-rate fluctuations that would otherwise affect domestic output and prices. If domestic monetary instability is a source of economic fluctuations, a fixed-exchange-rate system provides an automatic monetary response to offset such disturbances.

The role of economic disturbances in the choice of a policy regime is an important topic of study in monetary policy analysis. It figures most prominently in discussion of the choice between using an interest rate or a monetary aggregate as the instrument of monetary policy. This topic forms the major focus of chapter 9.

## 5 Appendix

### 5.1 The Obstfeld-Rogoff Model

This appendix provides a derivation of some of the components of the Obstfeld and Rogoff model developed in [Obstfeld and Rogoff \(1995\)](#) and [Obstfeld and Rogoff \(1996\)](#).

#### 5.1.1 Individual Product Demand

The demand functions faced by individual producers are obtained as the solution to the following problem:

$$\max \left[ \int_0^1 c(z)^q dz \right]^{\frac{1}{q}} \quad \text{subject to} \quad \int_0^1 P(z)c(z)dz = Z$$

for a given total expenditure  $Z$ . Letting  $\theta$  denote the Lagrangian multiplier associated with the budget constraint, the first-order conditions imply, for all  $z$ ,

$$c(z)^{q-1} \left[ \int_0^1 c(z)^q dz \right]^{\frac{1}{q}-1} = \theta P(z).$$

For any two goods  $z$  and  $z'$ , therefore,  $[c(z)/c(z')]^{q-1} = P(z)/P(z')$ , or

$$c(z) = c(z') \left[ \frac{P(z')}{P(z)} \right]^{\frac{1}{1-q}}.$$

If this expression is substituted into the budget constraint, one obtains

$$\int_0^1 P(z)c(z') \left[ \frac{P(z')}{P(z)} \right]^{\frac{1}{1-q}} dz = c(z')P(z')^{\frac{1}{1-q}} \left[ \int_0^1 P(z)^{\frac{q}{q-1}} dz \right] = Z. \quad (61)$$

Using the definition of  $P$  given in (3), both sides of (61) can be divided by  $P$  to yield

$$\frac{c(z')P(z')^{\frac{1}{1-q}} \left[ \int_0^1 P(z)^{\frac{q}{q-1}} dz \right]}{\left[ \int_0^1 P(z)^{\frac{q}{q-1}} dz \right]^{\frac{q-1}{q}}} = \frac{Z}{P}.$$

This can be simplified to

$$c(z') \left[ \frac{P(z')}{P} \right]^{\frac{1}{1-q}} = \frac{Z}{P} \quad \text{or} \quad c(z') = \left[ \frac{P(z')}{P} \right]^{\frac{1}{q-1}} C, \quad (62)$$

where  $C = \frac{Z}{P}$  is total real consumption of the composite good. Equation (62) implies that the demand for good  $z$  by agent  $j$  is equal to  $c^j(z) = [P(z)/P]^{1/(q-1)} C^j$ , so the world demand for product  $z$  will be equal to

$$\begin{aligned} y_t^d(z) &\equiv n \left[ \frac{P_t(z)}{P_t} \right]^{\frac{1}{q-1}} C_t + (1-n) \left[ \frac{P_t^*(z)}{P_t^*} \right]^{\frac{1}{q-1}} C_t^* \\ &= \left[ \frac{P_t(z)}{P_t} \right]^{\frac{1}{q-1}} C_t^w, \end{aligned} \quad (63)$$

where  $C^w = nC + (1-n)C^*$  is world real consumption. Notice that the law of one price has been used here, since it implies that the relative price for good  $z$  is the same for home and foreign consumers:  $P(z)/P = SP^*(z)/SP^* = P^*(z)/P^*$ . Finally, note that (63) implies

$$P_t(z) = P_t \left[ \frac{y_t^d(z)}{C_t^w} \right]^{q-1}. \quad (64)$$

### 5.1.2 The Individual's Decision Problem

Each individual begins period  $t$  with existing asset holdings  $B_{t-1}^j$  and  $M_{t-1}^j$  and chooses how much of good  $j$  to produce (subject to the world demand function for good  $j$ ), how much to consume, and what levels of real bonds and money to hold. These choices are made to maximize utility given by (1) and subject to the following budget constraint:

$$C_t^j + B_t^j + \frac{M_t^j}{P_t} \leq \frac{P_t(j)y_t(j)}{P_t} + R_{t-1}B_{t-1}^j + \frac{M_{t-1}^j}{P_t} + \tau_t$$

where  $\tau_t$  is the real net transfer from the government and  $R_t$  is the real gross rate of return. From (64), agent's  $j$ 's real income from producing  $y(j)$  will be equal to  $y(j)^q (C_t^w)^{1-q}$ , so the budget constraint can be written as

$$C_t^j + B_t^j + \frac{M_t^j}{P_t} = y_t(j)^q (C_t^w)^{1-q} + R_{t-1} B_{t-1}^j + \frac{M_{t-1}^j}{P_t} + \tau_t. \quad (65)$$

The value function for the individual's decision problem is

$$V(B_{t-1}^j, M_{t-1}^j) = \max \left\{ \log C_t^j + b \log \frac{M_t^j}{P_t} - \frac{k}{2} y_t(j)^2 + \beta V(B_t^j, M_t^j) \right\},$$

where the maximization is subject to (65). Letting  $\lambda$  denote the Lagrangian multiplier associate with the budget constraint, first-order conditions are

$$\frac{1}{C_t^j} - \lambda_t = 0 \quad (66)$$

$$\frac{b}{M_t^j} + \beta V_2(B_t^j, M_t^j) - \frac{\lambda_t}{P_t} = 0 \quad (67)$$

$$-k y_t(j) + \lambda_t q y_t(j)^{q-1} (C_t^w)^{1-q} = 0 \quad (68)$$

$$\beta V_1(B_t^j, M_t^j) - \lambda_t = 0 \quad (69)$$

$$V_1(B_{t-1}^j, M_{t-1}^j) = \lambda_t R_{t-1} \quad (70)$$

$$V_2(B_{t-1}^j, M_{t-1}^j) = \frac{\lambda_t}{P_t}. \quad (71)$$

In addition, the transversality condition  $\lim_{i \rightarrow \infty} \prod_{k=0}^i R_{t+k-1} (B_{t+i}^j + M_{t+i}^j / P_{t+i}) = 0$  must be satisfied.

These first-order conditions lead to the standard Euler condition for consumption with log utility:

$$C_{t+1}^j = \beta R_t C_t^j,$$

which is obtained using (66), (69), and (70). Equations (68) and (66) imply that the optimal production level the individual chooses satisfies

$$y_t(j)^{2-q} = \frac{q}{k} \frac{(C_t^w)^{1-q}}{C_t^j}. \quad (72)$$

Equation (67) yields an expression for the real demand for money,

$$\frac{M_t^j}{P_t} = bC_t^j \left( \frac{1+i_t}{i_t} \right),$$

where  $(1+i_t) = R_t P_{t+1}/P_t$  is the gross nominal rate of interest from period  $t$  to  $t+1$ . This expression should look familiar from chapter 2.

## 5.2 The Small-Open-Economy Model

This appendix employs the method of undetermined coefficients to obtain the equilibrium exchange rate and price level processes consistent with (49)-(57). The equations of the model are repeated here, where the real exchange rate  $\rho_t$  has been replaced by  $s_t + p_t^* - p_t$ ,  $r_t$  by  $r^* - (s_t + p_t^* - p_t) + E_t(s_{t+1} + p_{t+1}^* - p_{t+1})$ ,  $i_t$  by  $i_t^* + E_t s_{t+1} - s_t$ , and  $q_t$  by  $p_t + (1-h)(s_t + p_t^* - p_t)$ :

$$y_t = -b_1(s_t + p_t^* - p_t) + b_2(p_t - E_{t-1}p_t) + e_t \quad (73)$$

$$y_t = a_1(s_t + p_t^* - p_t) - a_2[r^* - (s_t + p_t^* - p_t) + E_t(s_{t+1} + p_{t+1}^* - p_{t+1})] + u_t \quad (74)$$

$$m_t - [p_t + (1-h)(s_t + p_t^* - p_t)] = y_t - c(i_t^* + E_t s_{t+1} - s_t) + v_t \quad (75)$$

$$m_t = \mu + m_{t-1} + \varphi_t - \gamma\varphi_{t-1}, \quad 0 \leq \gamma \leq 1 \quad (76)$$

$$p_t^* = \pi^* + p_{t-1}^* + \phi_t. \quad (77)$$

Substituting the aggregate demand relationship (74), the money supply process (76), and the foreign price process (77) into the money demand equation (75) yields, after some rearrangement,

$$\begin{aligned} A_1 p_t + A_2 s_t &= C_0 + \mu + m_{t-1} + \varphi_t - \gamma\varphi_{t-1} \\ &\quad - (1-h+a_1)(p_{t-1}^* + \phi_t) - a_2 E_t p_{t+1} \\ &\quad + (a_2+c)E_t s_{t+1} - u_t - v_t, \end{aligned} \quad (78)$$

where  $A_1 = h - a_1 - a_2$ ,  $A_2 = 1 - h + a_1 + a_2 + c > 0$  and  $C_0 = (c+a_2)r^* - (1-h+a_1-a_2-c)\pi^*$ . In deriving (78), two additional results have been used: from (77),  $E_t p_{t+1}^* = 2\pi^* + p_{t-1}^* + \phi_t$ , and  $i_t^* = r^* + E_t p_{t+1}^* - p_t^* = r^* + \pi^*$ .

Using the aggregate supply and demand relationships (73) and (74),

$$\begin{aligned} B_1 p_t + B_2 s_t &= -b_2 E_{t-1} p_t + a_2 (E_t s_{t+1} - E_t p_{t+1}) - (a_1 + b_1)\pi^* \\ &\quad + a_2 (r^* + \pi^*) + e_t - u_t - (a_1 + b_1)(p_{t-1}^* + \phi_t), \end{aligned} \quad (79)$$

where  $B_1 = -(a_1 + a_2 + b_1 + b_2) < 0$ , and  $B_2 = a_1 + a_2 + b_1 > 0$ .

The state variables at time  $t$  are  $m_{t-1}$ ,  $p_t^*$  and the various random disturbances. To rule out possible bubble solutions, follow McCallum (1983) and hypothesize minimum state variable solutions of the form:<sup>24</sup>

$$p_t = k_0 + m_{t-1} + k_1 p_{t-1}^* + k_2 \varphi_t + k_3 \varphi_{t-1} + k_4 u_t + k_5 e_t + k_6 v_t + k_7 \phi_t$$

$$s_t = d_0 + m_{t-1} + d_1 p_{t-1}^* + d_2 \varphi_t + d_3 \varphi_{t-1} + d_4 u_t + d_5 e_t + d_6 v_t + d_7 \phi_t.$$

These imply

$$E_{t-1} p_t = k_0 + m_{t-1} + k_1 p_{t-1}^* + k_3 \varphi_{t-1}$$

$$\begin{aligned} E_t p_{t+1} &= k_0 + m_t + k_1 p_t^* + k_3 \varphi_t \\ &= k_0 + \mu + m_{t-1} + (1 + k_3) \varphi_t - \gamma \varphi_{t-1} + k_1 (\pi^* + p_{t-1}^* + \phi_t) \end{aligned}$$

and

$$E_t s_{t+1} = d_0 + \mu + m_{t-1} + (1 + d_3) \varphi_t - \gamma \varphi_{t-1} + d_1 (\pi^* + p_{t-1}^* + \phi_t).$$

These expressions for  $p_t$  and  $s_t$ , together with those for the various expectations of  $p$  and  $s$ , can be substituted into (78) and (79). These then yield a pair of equations that must be satisfied by each pair  $(k_i, d_i)$ . For example, the coefficients on  $p_{t-1}^*$  in (78) and (79) must satisfy

$$A_1 k_1 + A_2 d_1 = -(1 - h + a_1) - a_2 k_1 + (a_2 + c) d_1$$

and

$$B_1 k_1 + B_2 d_1 = a_2(d_1 - k_1) - b_2 k_1 - (a_1 + b_1).$$

Using the definitions of  $A_i$  and  $B_i$  to cancel terms, the second equation implies that  $d_1 = k_1 - 1$ . Substituting this back into the first equation yields  $k_1 = 0$ . Therefore, the solution pair is  $(k_1, d_1) = (0, -1)$ . Repeating this process yields the values for  $(k_i, d_i)$  reported in (58) and (59).

## 6 Problems

1. Suppose  $m_t = m_0 + \gamma m_{t-1}$  and  $m_t^* = m_0^* + \gamma^* m_{t-1}^*$ . Use (24) to show how the behavior of the nominal exchange rate under flexible prices depends on the degree of serial correlation exhibited by the home and foreign money supplies.

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<sup>24</sup>The coefficient on  $m_{t-1}$  has been set equal to 1 in these trial solutions. It is easy to verify that this assumption is in fact correct.

2. In the model of section 3 used to study policy coordination, aggregate demand shocks were set equal to zero in order to focus on a common aggregate supply shock. Suppose instead that the aggregate supply shocks are zero, and the demand shocks are given by  $u \equiv x + \phi$  and  $u^* \equiv x + \phi^*$ , so that  $x$  represents a common demand shock and  $\phi$  and  $\phi^*$  are uncorrelated country-specific demand shocks. Derive policy outcomes under coordinated and (Nash) noncoordinated policy setting. Is there a role for policy coordination in the face of demand shocks? Explain.
3. Continuing with the same model as in the previous question, how are real interest rates affected by a common aggregate demand shock?
4. Policy coordination with asymmetric supply shocks: Continuing with the same model as in the previous two questions, assume that there are no demand shocks but that the supply shocks  $e$  and  $e^*$  are uncorrelated. Derive policy outcomes under coordinated and uncoordinated policy setting. Does coordination or noncoordination lead to a greater inflation response to supply shocks? Explain.
5. Assume that the home country policymaker acts as a Stackelberg leader and recognizes that foreign inflation will be given by (47). How does this change in the nature of the strategic interaction affect the home country's response to disturbances?
6. In a small open economy with perfectly flexible nominal wages, the text showed that the real exchange rate and domestic price level were given by

$$\rho_t = \sum_{i=0}^{\infty} d^i \mathbf{E}_t \left( \frac{a_2 r_{t+i}^* + e_{t+i} - u_{t+i}}{a_1 + a_2 + b_1} \right)$$

and

$$p_t = \left( \frac{1}{1+c} \right) \sum_{i=0}^{\infty} \left( \frac{c}{1+c} \right)^i \mathbf{E}_t (m_{t+i} - z_{t+i} - v_{t+i}),$$

where  $z_{t+i} \equiv y_{t+i} + (1-h)\rho_{t+i} - cr_{t+i}$ . Assume that  $r^* = 0$  for all  $t$  and that  $e$ ,  $u$ , and  $z + v$  all follow first-order autoregressive processes (e.g.,  $e_t = \rho_e e_{t-1} + x_{e,t}$  for  $x_e$  white noise). Let the nominal money supply be given by

$$m_t = g_1 e_{t-1} + g_2 u_{t-1} + g_3 (z_{t-1} + v_{t-1}).$$

Find equilibrium expressions for the real exchange rate, the nominal exchange rate, and the consumer price index. What values of the parameters  $g_1$ ,  $g_2$ , and  $g_3$  minimize fluctuations in  $s_t$ ? In  $q_t$ ? In  $\rho_t$ ? Are there any conflicts between stabilizing the exchange rate (real or nominal) and stabilizing the consumer price index?

7. Equation (42) for the equilibrium real exchange rate in the two-country model of section 3.1 takes the form  $\rho_t = A\mathbb{E}_t\rho_{t+1} + v_t$ . Suppose  $v_t = \gamma v_{t-1} + \psi_t$ , where  $\psi_t$  is a mean-zero, white noise process. Suppose the solution for  $\rho_t$  is of the form  $\rho_t = bv_t$ . Find the value of  $b$ . How does it depend on  $\gamma$ ?
8. Section ?? demonstrated how a simple open-economy model with nominal price stickiness could be expressed in a form that paralleled the closed-economy new Keynesian model of chapter 6. Would this same conclusion result in a model of sticky wages with flexible prices? What if both wages and prices are sticky?
9. [McCallum and Nelson \(2000\)](#) have proposed a new Keynesian open economy model in which imported goods are only used as inputs into the production of the domestic good and households consume only the domestically produced good. If  $e_t$  is the nominal exchange rate, and  $s_t$  is the real exchange rate, The model can be summarize by the following equations:

$$c_t = \mathbb{E}_t c_{t+1} - b_1 [R_t - \mathbb{E}_t \pi_{t+1} - r_t]$$

$$im_t = y_t - \sigma s_t$$

$$ep_t = y_t^* + \sigma^* s_t$$

$$s_t = e_t - p_t + p_t^*$$

$$R_t = R_t^* + \mathbb{E}_t e_{t+1} - e_t,$$

$$y_t = (1 - \alpha)(n_t + \varepsilon_t) + \alpha im_t$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t$$

where  $im_t$  denotes imports,  $ep_t$  denotes exports, and all variables are expressed relative to their flexible-price equivalents. Foreign variables are denoted by \*. The linearized production function is

$$y_t = (1 - \alpha)(n_t + \varepsilon_t) + \alpha im_t$$

and the goods market equilibrium condition takes the form

$$y_t = \omega_1 c_t + \omega_2 g_t + \omega_3 ep_t.$$

Show that this open economy model can be reduced to two equations corresponding to the *IS* relationship and the Phillips curve that, when combined with a specification of monetary policy, could be solved for the equilibrium output gap and inflation rate. How does the interest elasticity of the output gap depend on the openness of the economy?



10. Assume the utility function of the representative household in a small open economy is

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(C_t) - \frac{N_t^{1+\eta}}{1+\eta} + \frac{a_m}{1-\gamma_m} \left( \frac{M_t}{P_t} \right)^{1-\gamma_m} \right\},$$

where  $C$  is total consumption,  $N$  is labor supply, and  $M/P$  is real money holdings.  $C_t$  is defined by (??), and utility is maximized subject to the sequence of constraints given by

$$P_t C_t + M_t + e_t \frac{1}{1+i_t^*} B_t^* + \frac{1}{1+i_t} B_t \leq W_t N_t + M_{t-1} + e_t B_{t-1}^* + B_{t-1} + \Pi_t - \tau_t.$$

Let  $P_t^h$  ( $P_t^f$ ) be the average price of domestically (foreign) produced consumption goods.

- (a) Derive the first-order conditions for the household's problem.
  - (b) Show that the choice of domestic-produced consumption goods relative to foreign-produced consumption good basket depends on the terms of trade. Why is it not a function of the real exchange rate?
  - (c) Derive an expression of the price index  $P_t$ .
11. Using the first-order conditions derived in question 10a, derive the Euler equations and obtain the uncovered interest rate parity condition. Can you provide economic intuition to explain this equation.
  12. For the economy of questions 10 and 11, show how to derive the Fisher equation by introducing an extra asset into this economy and using the appropriate Euler equations.

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