Policy in NK model

Fall 2015
The basic new Keynesian model

Three core elements:

1. Expectational IS curve, expressed in terms of the output gap:
\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t x_{t+1} - r^n_t) \]

2. An expectational Phillips Curve:
\[ (\pi_t - \pi^T) = \beta E_t (\pi_{t+1} - \pi^T) + \kappa x_t + e_t \]

3. In lecture, we set the target inflation rate \( \pi^T \) to equal zero.

4. A monetary policy rule:
\[ i_t = \phi_\pi (\pi_t - \pi^T) + \phi_x x_t \]

4. In lecture, we simplified and set \( \phi_x x_t \) so that monetary policy only responded directly to inflation.
The basic new Keynesian model: serially uncorrelated shocks

- If the aggregate demand shock $r^n$ and the inflation shock $e$ are serially uncorrelated with mean zero so that $E_t r_{t+1}^n = E_t e_{t+1} = 0$, then $E_t x_{t+1} = E_t \pi_{t+1} = 0$ and model becomes

$$x_t = -\sigma (i_t - r_t^n)$$

$$\pi_t - \pi^T = \kappa x_t + e_t$$

$$i_t = \phi_\pi \left( \pi_t - \pi^T \right) + \phi_x x_t$$
The basic new Keynesian model: serially uncorrelated shocks

- Substitute the policy rule into the IS equation to obtain an aggregate demand relationship between $x$ and $\pi$:

$$x_t = -\sigma(i_t - r^n_t)$$

$$= -\sigma \left[ \phi_\pi (\pi_t - \pi^T) + \phi_x x_t - r^n_t \right]$$

$$= -\left( \frac{\sigma}{1 + \sigma \phi_x} \right) \left[ \phi_\pi (\pi_t - \pi^T) - r^n_t \right]$$

- Rewrite to express $\pi$ as a function of $x$ and $r^n$:

$$\pi_t = \pi^T - \left( \frac{1 + \sigma \phi_x}{\sigma \phi_\pi} \right) x_t + \left( \frac{1}{\phi_\pi} \right) r^n_t$$
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- Two equations for $x$ and $\pi$:

$$\pi_t = \pi^T - \left( \frac{1 + \sigma \phi_x}{\sigma \phi_\pi} \right) x_t + \left( \frac{1}{\phi_\pi} \right) r^n_t$$

and

$$\pi_t = \pi^T + \kappa x_t + e_t$$

- The first defines a negatively sloped line in ($x$, $\pi$) space, the second a positively sloped line.
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[Image of a graph showing the relationship between inflation rate and output gap]
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The effects of a positive inflation (π) shock

\[ π \]

\[ π^T \]

\[ x \]

Output gap

Inflation rate

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![Diagram showing the effects of a positive demand (r_n) shock with output gap and inflation rate axes. The diagram includes two lines labeled $\phi_\pi = 1.5$ and $\phi_\pi = 3$. The image also contains notes indicating the effects of a positive demand shock.](image-url)
The basic new Keynesian policy trade off frontier

Policy tradeoff

policy rule: $i = r^f + \phi^*\pi$

small $\phi$

large $\phi$
The basic new Keynesian model

- Policy objectives – assume central bank wants to minimize

\[ \frac{1}{2} \left[ \left( \pi_t - \pi^T \right)^2 + \lambda x_t^2 \right] \]

- True output gap appears in loss – eliminates source of inflation bias in the earlier models we examined.

- In this case, is there any difference between policy under discretion and policy under commitment?
Central bank treats expectations as given.

Central bank wants to minimize

$$\frac{1}{2} \left[ \left( \pi_t - \pi^T \right)^2 + \lambda x_t^2 \right]$$

subject to

$$\left( \pi_t - \pi^T \right) = \beta E_t \left( \pi_{t+1} - \pi^T \right) + \kappa x_t + e_t$$

Expectations IS curve not a constraint – always move $i_t$ to offset $r^n_t$.

- Only a problem when a big negative $r^n_t$ shock pushes $i_t$ to zero.
- Will discuss this case later.
Optimal policy under discretion

- Use Phillips curve to eliminate output gap from objective:

\[
x_t = \frac{(\pi_t - \pi^T) - \beta E_t(\pi_{t+1} - \pi^T) - e_t}{\kappa}
\]

so minimize

\[
\frac{1}{2} \left\{ \left( \pi_t - \pi^T \right)^2 + \lambda \left[ \frac{(\pi_t - \pi^T) - \beta E_t(\pi_{t+1} - \pi^T) - e_t}{\kappa} \right]^2 \right\}
\]

treating expectations as given.
Optimal policy under discretion

\[ L \equiv \frac{1}{2} \left\{ \left( \pi_t - \pi^T \right)^2 + \lambda \left[ \frac{(\pi_t - \pi^T) - \beta E_t (\pi_{t+1} - \pi^T) - e_t}{\kappa} \right] \right\}^2 \]

- First order condition – differentiate with respect to \( \pi \) and set equal to zero:

\[ \frac{\partial L}{\partial \pi} = \left( \pi_t - \pi^T \right) + \lambda \left( \frac{1}{\kappa} \right) \left[ \frac{(\pi_t - \pi^T) - \beta E_t (\pi_{t+1} - \pi^T) - e_t}{\kappa} \right] = 0 \]

- Simplify by writing this as

\[ \frac{\partial L}{\partial \pi} = \left( \pi_t - \pi^T \right) + \left( \frac{\lambda}{\kappa} \right) \times = 0 \]
Central bank’s first-order condition takes the form

\[(\pi_t - \pi^T) + \left(\frac{\lambda}{\kappa}\right)x = 0\]

Called a targeting rule or targeting criterion. Keep this relationship between inflation (relative to target) and the output gap equal to zero.

- If inflation is above target, the output gap should be negative; if inflation is below target, the output gap should be positive.
- If \(\pi_t > \pi^T\) and \(x_t > 0\), policy is too loose; if \(\pi_t < \pi^T\) and \(x_t < 0\), policy is too tight.
The Qvigstad graph

Too loose
1988-1990

Too tight
2005-2007

2008-2014
What about optimal policy if central bank can commit?

- Under commitment, central bank can affect expectations by making promises about future inflation.
- First-order condition turns out to imply a targeting rule of the form

\[ \pi_t - \pi^T + \left( \frac{\lambda}{\kappa} \right) (x_t - x_{t-1}) = 0 \]
Illustrating commitment versus discretion in the simple NK model

![Graphs](Graphs_Compare.m)

MATLAB

dynare

NewKeynesianModels

NKM_basic

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\[
\left( \pi_t - \pi^T \right) = \beta E_t \left( \pi_{t+1} - \pi^T \right) + \kappa x_t + e_t
\]

- Can stabilize better under commitment by using both \( x_t \) and future expectations as instruments.
- Under discretion, \( x_t \) is the central bank's only instrument.
- Responding to past conditions (via \( x_{t-1} \)) reflects the fact policy under commitment can't ignore promises made in the past.
- Under discretion, bygones are bygones – promises made in the past are ignored.
Friedman’s Presidential Address

- Emphasized limits to monetary policy.
- Emphasized role of misperceptions in explaining short-run real effects of monetary policy while monetary policy has no long-run real effects.
- Brought expectations and the way they are formed to the center of macroeconomics.

Lucas and rational expectations

- If policy makers cannot systematically fool people, can’t use monetary and fiscal policies to systematically stabilize the economy.
If natural rate holds, and inflation brings no benefit, and everyone understands this, why do countries find themselves with high inflation?

Kydland and Prescott and the time-inconsistency of optimal policy.

- Policy announcements are designed to affect behavior.
- When it comes time to implement policy, either behavior has been affected or it hasn’t. Now it is too late for the policy to affect behavior so one of the benefits is no longer available – only the costs of implementing the policy may remain.

If central bank announces low inflation, and if everyone believes this and makes decisions based on expecting low inflation, when it comes time to implement policy, the marginal benefits of a bit of inflation will exceed the marginal costs, so the central bank will choose to inflate.

- Expecting this, no one believes the low-inflation announcement.
Time inconsistency: Example 1

- Suppose

\[ y = y^n + a(\pi - \pi^e) \]

- Central bank acts to maximize

\[ W \equiv \lambda y - \frac{1}{2} (\pi - \pi^T)^2 \]

- Policy makers prefer more output to less.
Eliminate $y$ so problem becomes pick $\pi$, taking $\pi^e$ as given, to minimize

$$L \equiv \lambda [y^n + a (\pi - \pi^e)] - \frac{1}{2} (\pi - \pi^T)^2.$$ 

First-order condition is

$$a \lambda - (\pi - \pi^T) = 0 \Rightarrow \pi = \pi^T + a \lambda > \pi^T$$

Expected inflation is

$$\pi^e = \pi^T + a \lambda$$

Output is

$$y = y^n$$
Time inconsistency: Example 2

- Suppose
  \[ y = y^n + a(\pi - \pi^e) + e \]

- Central bank acts to minimize
  \[ L \equiv \frac{1}{2} \left[ (\pi - \pi^T)^2 + \lambda (y - y^*)^2 \right] \]

where \( y^* = y^n + k > y^n \).
Eliminate $y$ so problem is to minimize

$$L \equiv \frac{1}{2} \left[ \left( \pi - \pi^T \right)^2 + \lambda \left( y^n + a (\pi - \pi^e) + e - y^* \right)^2 \right]$$

by picking $\pi$, given expectations.

First-order condition is

$$\left( \pi - \pi^T \right) + a\lambda \left( y^n + a (\pi - \pi^e) + e - y^* \right) = 0$$

or

$$\pi = \frac{\pi^T + a\lambda \left( y^* - y^n \right)}{1 + a^2\lambda} + \frac{a^2\lambda}{1 + a^2\lambda} \pi^e - \frac{a\lambda}{1 + a^2\lambda} e_t$$
Private agents understand the decision problem of the central bank. Assuming their best guess of $e$ is that it will be zero, they will expect inflation to be

$$\pi^e = \pi^T + a\lambda (y^* - y^n) + \frac{a^2 \lambda}{1 + a^2 \lambda} \pi^e$$

$$= \pi^T + a\lambda (y^* - y^n) = \pi^T + a\lambda k > \pi^T$$

Actual inflation is

$$\pi = \frac{\pi^T + a\lambda k}{1 + a^2 \lambda} + \frac{a^2 \lambda}{1 + a^2 \lambda} (\pi^T + a\lambda k) - \frac{a\lambda}{1 + a^2 \lambda} e_t$$

$$= \pi^T + a\lambda k - \frac{a\lambda}{1 + a^2 \lambda} e_t$$

and

$$\pi - \pi^e = -\frac{a\lambda}{1 + a^2 \lambda} e_t$$
Output is then given by

\[ y = y^n + a(\pi - \pi^e) + e \]

\[ = y^n - \frac{a^2 \lambda}{1 + a^2 \lambda} e_t + e \]

\[ = y^n + \left( \frac{1}{1 + a^2 \lambda} \right) e_t \]
Appoint a conservative central banker (i.e., one with a $\lambda$ less than society’s).

Structure the incentives correctly

- Assign a target and hold central bank accountable for meeting it (goal dependence but instrument independent).
- Make the goals achievable with monetary policy – no unrealistic output or unemployment goals. Recall what Friedman said monetary policy could and could not do.
Credibility is critical.

Being willing to be tough can be necessary to convince people inflation will be brought under control and then maintained at a low and stable rate.

Credibility allows flexibility when it is needed to respond to economic shocks.

Anchoring longer run expectations can aid stabilization policies, allowing expectations to move in the short-run in ways that help make policy more effective.