- Discussion of Friedman’s 1968 model.
- Role of expectations.
Fisher equation and the central bank’s policy rule

- Implications of the Fisher relationship and a policy rule
- Fisher equation
  \[ i = r + \pi \]
- Policy rule
  \[ i = r + \pi^T + \phi (\pi - \pi^T) \]
- Taylor Principle: \( \phi > 1 \).
Fisher equation and the central bank’s policy rule

\[ r = 0.02, \ \pi^T = 0.02, \ \phi = 1.5 \]

blue line: \[ i = r + \pi \]

red line: \[ i = r + \pi^T + \phi^*(\pi - \pi^T) \]
Fisher equation and the central bank’s policy rule

\[ r = 0.02, \quad \pi^T = 0.02, \quad \phi = 1.5 \]

- **Blue line:** \( i = r + \pi \)
- **Red line:** \( i = r + \pi^T + \phi(\pi - \pi^T) \)
- **Red dotted line:** \( i = r + 0.05 + \phi(\pi - 0.05) \)
U.S. output gaps, unemployment and PCE inflation, 1960-2015

U.S. output gaps, unemployment and PCE inflation

percent


-10 -5 0 5 10 15

output gap (cbo)
output gap (hp)
unemployment rate
PCE inflation
The labor market – real wage adjusts to balance labor supply and labor demand.

Full information case – long-run:
- Firms – hire labor based on the real wage.
  - The real wage is the nominal wage divided by the price index: $W_t/P_t$.
- Workers – supply labor based on the real wage.
- Supply-demand diagram.
Less than full information case – short-run

- Firms – hire labor based on the real wage.
  - Firms have good information on the wages they have to pay and the prices at which they sell their output.

- Workers – supply labor based on their perceived real wage.
  - Perceived real wage is nominal wage divided by perceived or expected prices: $W_t / P_t^e$. 
Suppose prices rise. Suppose wages rise in proportion so the real wage is unchanged.

Workers take time to fully perceive all prices have risen.
- Initially, $P > P^e$. So $W / P^e > W / P$.
- Workers misperceive rise in nominal wages as a rise in real wages.
- Labor supply curve shifts to right.

Equilibrium employment (and therefore output) rises.
- Actual real wage falls.
As workers spend their higher wage income, they discover prices have risen.

- Looks like inflation has robbed them of their wage gains, but wages only rose because of inflation.

As $P^e$ rises, labor supply curve shifts back to left.

Equilibrium employment eventually returns to its original level.
A slightly more formal model

- Labor demand is a negative function of the real wage:
  \[ L^d_t = X_t \left( \frac{W_t}{P_t} \right)^{-b} \]
  where \( X_t \) represents other factors affecting labor demand.

- Labor supply is a positive function of the perceived real wage:
  \[ L^s_t = Z_t \left( \frac{W_t}{P^e_t} \right)^a \]
  where \( X_t \) represents other factors affecting labor demand.
A slightly more formal model

- Convenient to take logs so that we have a linear system:
  - Labor demand:
    \[ l^d_t \equiv \log(L^d_t) = x_t - b(w_t - p_t) \]
    where \( x_t = \log(X_t) \), \( w_t = \log(W_t) \) and \( p_t = \log(P_t) \).
  - Labor supply:
    \[ l^s_t \equiv \log(L^s_t) = z_t + a(w_t - p^e_t) \]
    where \( z_t = \log(Z_t) \).
- Rewrite labor supply as
  \[
  l^s_t = z_t + a(w_t - p^e_t - p_t + p_t) \\
  = z_t + a(w_t - p_t) + a(p_t - p^e_t).
  \]
A slightly more formal model

- Model is

\[ l^d_t = x_t - b (w_t - p_t) \]
\[ l^s_t = z_t + a (w_t - p_t) + a (p_t - p^e_t). \]

and

\[ l_t = l^d_t = l^s_t. \]

- Solving,

\[ x_t - b (w_t - p_t) = z_t + a (w_t - p_t) + a (p_t - p^e_t) \Rightarrow \]

\[ w_t - p_t = \left( \frac{1}{a + b} \right) [x_t - z_t - a (p_t - p^e_t)] \]
Equilibrium

Equilibrium real wage and employment are given by

\[ w_t - p_t = \left( \frac{1}{a + b} \right) [x_t - z_t - a(p_t - p_t^e)] \]

\[ l_t = x_t - b(w_t - p_t) = \left( \frac{ax_t + bz_t}{a + b} \right) + \left( \frac{ab}{a + b} \right)(p_t - p_t^e). \]

A price rise that isn’t perceived as a general increase in prices leads to a fall in actual real wages and a rise in employment (and therefore output).
Adjustment of expectations

- Suppose perceptions about prices adjust in the following manner:

\[ p_t^e = q p_t + (1 - q) p_{t-1}^e. \]

- Complete model is now

\[
\begin{align*}
\omega_t - p_t &= \left( \frac{1}{a + b} \right) [x_t - z_t - a(p_t - p_t^e)] \\
l_t &= \left( \frac{a x_t + b z_t}{a + b} \right) + \left( \frac{a b}{a + b} \right) (p_t - p_t^e) \\
p_t^e &= q p_t + (1 - q) p_{t-1}^e. 
\end{align*}
\]

- See LaborMarket_FriedmanModel.xlsx
Adding aggregate demand

- Assume a simple quantity theory model for aggregate demand:

\[ M_t V_t = P_t Y_t \]

where \( M \) is the nominal money supply, \( V \) is velocity, and \( PY \) is nominal GDP.

- Assume \( M \) is exogenous and controled by the central bank.
- Assume \( V \) is exogenous and follows a stochastic process.
- Take logs,

\[ m_t + v_t = p_t + y_t \]

or

\[ y_t = (m_t - p_t) + v_t. \]
Adding aggregate supply

- Assume an aggregate production function:
  \[ y_t = s l_t + e_t \]
  where \( e \) is an aggregate supply shock.

- From our earlier expression for equilibrium employment,
  \[
  l_t = \left( \frac{ax_t + bz_t}{a + b} \right) + \left( \frac{ab}{a + b} \right) (p_t - p_t^e) \\
  = \delta (p_t - p_t^e) + u_t
  \]
  where \( u \) will be treated as an exogenous shock.

- Aggregate supply equation becomes
  \[ y_t = s \delta (p_t - p_t^e) + su_t + e_t \]
Adding aggregate supply

- An aggregate supply equation of form

\[ y_t = s \delta (p_t - p_t^e) + su_t + e_t \]

often called a Lucas supply equation.

- Lucas’s island model.

- Often convenient to rewrite it in terms of inflation surprises:

\[
\begin{align*}
  y_t &= s \delta (p_t - p_{t-1} - p_t^e + p_{t-1}) + su_t + e_t \\
  &= s \delta (\pi_t - \pi_t^e) + su_t + e_t
\end{align*}
\]
Putting supply and demand together

- Consider the basic model of Friedman:
  \[ y_t = (m_t - p_t) + v_t \]
  \[ y_t = \phi (p_t - p_t^e) + e_t \]

- Solve these two equations:
  \[ (m_t - p_t) + v_t = \phi (p_t - p_t^e) + e_t \]

  which implies
  \[ p_t = \left( \frac{1}{1 + \phi} \right) (m_t + \phi p_t^e + v_t - e_t) \]

  and
  \[ y_t = \left( \frac{\phi}{1 + \phi} \right) (m_t + \phi p_t^e + v_t - e_t) - \phi p_t^e + e_t \]
  \[ = \left( \frac{\phi}{1 + \phi} \right) (m_t - p_t^e + v_t) - \left( \frac{1}{1 + \phi} \right) e_t. \]
What is the effect of a change in the money supply on output?

\[ y_t = \left( \frac{\phi}{1 + \phi} \right) (m_t - p_t^e + \nu_t) - \left( \frac{1}{1 + \phi} \right) e_t \]

- Is it \( \left( \frac{\phi}{1 + \phi} \right) \)?

- No. It is \( \left( \frac{\phi}{1 + \phi} \right) \left[ 1 - (\text{effect of change in } m_t \text{ on } p_t^e) \right] \).

- Can’t answer this question without knowing how expectations are affected.
policy as systematic

...[equilibrium methods] will focus attention on the need to think of policy as the choice of stable rules of the game, well understood by economic agents. Only in such a setting will economic theory help predict the actions agents will choose to take. (Lucas and Sargent 1978)
What is the effect of a change in the money supply on output?

Ignore the exogenous shocks to demand and supply

- Suppose $p_t^e = \rho p_{t-1}$ and $m_t = \gamma m_{t-1} + z_t$. Then

\[
\begin{align*}
    p_t &= \left( \frac{1}{1 + \phi} \right) (m_t + \phi p_t^e + \nu_t - e_t) \\
    &= \left( \frac{1}{1 + \phi} \right) (\gamma m_{t-1} + z_t + \phi \rho p_{t-1} + \nu_t - e_t)
\end{align*}
\]

and

\[
\begin{align*}
    y_t &= \left( \frac{\phi}{1 + \phi} \right) (m_t - p_t^e + \nu_t) - \left( \frac{1}{1 + \phi} \right) e_t. \\
    &= \left( \frac{\phi}{1 + \phi} \right) (\gamma m_{t-1} + z_t - \rho p_{t-1} + \nu_t) - \left( \frac{1}{1 + \phi} \right) e_t.
\end{align*}
\]
What is the effect of a change in the money supply on output?

- Assume $\phi = 2$. Assume $\gamma = 1.0$ and $\rho = 0.9$.
  - Run ratexp.m
- Now assume $\gamma = 0.9$ and $\rho = 0.9$.
  - Run ratexp.m again.
Friedman and Lucas put expectations at center stage

- Role of expectations could explain shifting Phillips Curve.
- Go back to inflation version of Lucas supply curve:
  \[ y_t = s\delta (\pi_t - \pi_t^e) + su_t + e_t. \]
- Rewrite this to look like a Phillips Curve:
  \[ \pi_t = \pi_t^e + \left( \frac{1}{s\delta} \right) (y_t - su_t - e_t) \]
- Coefficient on expected inflation is equal to 1.
Friedman and Lucas put expectations at center stage

- Huge literature in the late 1960s and into the 1970s estimating Phillips Curves of the form

\[ \pi_t = a\pi_t^e + \left( \frac{1}{s\delta} \right) (y_t - su_t - e_t). \]

- Hypothesis of interest was \( H_0 : a = 1 \).
\[ \pi_t = a\pi_t^e + \left(\frac{1}{s\delta}\right) (y_t - su_t - e_t). \]

- If \( a \neq 1 \), then once inflation came to be expected so that \( \pi = \pi^e \), and assume on average the shocks were zero, then
  \[ y = s\delta (1 - a) \pi. \]

- There is a long-run tradeoff between average inflation and average output levels.
  - If \( a = 1 \), no long-run tradeoff.

- Empirical estimates of \( a \) kept rising until the hypothesis that \( a = 1 \) could not be rejected – Friedman was victorious!
Rational expectations and its implications for policy

- policy as systematic

  ...[equilibrium methods] will focus attention on the need to think of policy as the choice of stable rules of the game, well understood by economic agents. Only in such a setting will economic theory help predict the actions agents will choose to take. (Lucas and Sargent 1978)

- but unpredictable

  ...the government countercyclical policy must itself be unforeseeable by private agents...while at the same time be systematically related to the state of the economy. Effectiveness, then, rests on the inability of private agents to recognize systematic patterns in monetary and fiscal policy. (Lucas and Sargent 1978).
Suppose private agents understand the model

If they know

\[ p_t = \left( \frac{1}{1 + \phi} \right) (m_t + \phi p_t^e + v_t - e_t), \]

they should use this knowledge in forming expectations.
Suppose private agents understand the model

- Suppose $p_t^e$ is agents’ best guess of $p_t$, given the information they have.
- Suppose they know model, and all past history of the variables.
- Then

$$p_t^e = \left( \frac{1}{1 + \phi} \right) E_{t-1} (m_t + \phi p_t^e + \nu_t - e_t)$$

$$= \left( \frac{1}{1 + \phi} \right) (\gamma m_{t-1} + \phi p_t^e)$$

- So

$$(1 + \phi) p_t^e = (\gamma m_{t-1} + \phi p_t^e) \Rightarrow p_t^e = \gamma m_{t-1}.$$
Suppose private agents understand the model: What is the equilibrium?

Equilibrium conditions were

\[ p_t = \left( \frac{1}{1+\phi} \right) \left( m_t + \phi p_t^e + \nu_t - e_t \right) \]

and

\[ y_t = \left( \frac{\phi}{1+\phi} \right) (m_t - p_t^e + \nu_t) - \left( \frac{1}{1+\phi} \right) e_t. \]
What is the rational expectations equilibrium?

- Use rational expectations:

\[ p_t^e = \gamma m_{t-1}. \]

- Price level is

\[
\begin{align*}
p_t & = \left( \frac{1}{1+\phi} \right) (\gamma m_{t-1} + z_t + \phi p_t^e + v_t - e_t) \\
    & = \left( \frac{1}{1+\phi} \right) (\gamma m_{t-1} + z_t + \phi \gamma m_{t-1} + v_t - e_t) \\
    & = \gamma m_{t-1} + \left( \frac{1}{1+\phi} \right) (z_t + v_t - e_t)
\end{align*}
\]
What is the ration expectations equilibrium?

Output is

\[ y_t = \left( \frac{\phi}{1 + \phi} \right) (\gamma m_{t-1} + z_t - \gamma m_{t-1} + \nu_t) - \left( \frac{1}{1 + \phi} \right) e_t \]

\[ = \left( \frac{\phi}{1 + \phi} \right) (z_t + \nu_t) - \left( \frac{1}{1 + \phi} \right) e_t \]
Real equilibrium, i.e., output, is independent of the systematic part of the money supply (the $\gamma m_{t-1}$) part.

Exercise:

- Suppose the central bank wants to stabilize output in the face of $e$ shocks.
- Suppose central bank and the public have a forecast of $e$, call it $e^f_t$, so policy sets $m_t = \kappa e^f_t$.
- Show that under rational expectations, output is independent of $\kappa$. 
Suppose output is given by

\[ y_t = \gamma \delta (\pi_t - \pi_t^e) + \gamma u_t + e_t \]
Suppose output is given by

\[ y_t = \gamma \delta (\pi_t - \pi^e_t) + \gamma u_t + e_t \]

And suppose inflation is determined by monetary policy.
Suppose output is given by

\[ y_t = \gamma \delta (\pi_t - \pi_t^e) + \gamma \mu_t + e_t \]

And suppose inflation is determined by monetary policy.

Then any inflation rate that comes to be expected yields the same output behavior.
Suppose output is given by

\[ y_t = \gamma \delta (\pi_t - \pi^e_t) + \gamma \mu_t + e_t \]

And suppose inflation is determined by monetary policy.

Then any inflation rate that comes to be expected yields the same output behavior.

And if inflation is ultimately controlled by the central bank and if low inflation is preferred to high inflation, why did so many countries find themselves with high inflation rates during the 1970s?