Review questions for the midterm: Partial answers

1. Briefly summarize what you feel were the major points Milton Friedman made in his 1968 American Economic Association Presidential Address?

2. If workers know their nominal wage but are slow to perceive changes in general prices, explain how a surprise inflation would create an economic expansion. What happens to employment and the real wage? What happens as workers come to perceive that prices have risen? Use a simple labor demand, labor supply graph to illustrate your answers. A rise in prices and nominal wages causes workers to misinterpret the rise in wages as a rise in real wages if they are slow to perceive the corresponding rise in prices. Given what they perceive as a rise in real wages, workers increase their labor supply (for a given actual real wage, the labor supply curve shifts to the right). The new equilibrium involves a lower actual real wage and a higher level of employment (and therefore output). Hence, the surprise rise in prices increases output. Eventually, workers come to perceive that inflation had increases wages and prices and that their real wages had actually fallen. When perceptions correct, the labor supply curve shifts back to its original location and employment returns to its natural rate.

3. Kydland and Prescott demonstrated the importance of credibility and commitment in setting in which optimal policies are time inconsistent. Explain what is meant by saying an optimal policy is time inconsistent. Provide an example in which the best policy is time inconsistent. A policy is time inconsistent when the policy that it is optimal to announce

4. Consider the following simple model. At the start of each period, the central bank announces its target for the inflation rate. The private sector then sets nominal wages based on their expectations of inflation. Finally, the central bank implements actual policy which determines actual inflation. Unemployment in this economy is given by

\[ u = u^u - a (\pi - \pi^e) + v, \]

where \( u \) is the unemployment rate, \( u^u \) is the natural rate of unemployment, \( \pi \) is inflation, \( \pi^e \) is expected inflation, and \( v \) is a random error term that captures any other factors affecting unemployment. Assume \( v \) has an expected value of zero. When it implements policy (i.e., picks \( \pi \), the central bank tries to maximize an objective function given by

\[ L = \lambda u + \frac{1}{2} (\pi - \pi^*)^2, \]

where \( \lambda \) is a positive constant and \( \pi^* \) is the policy maker’s desired inflation rate.
(a) Given \( \pi^e \), what value of \( \pi \) minimizes the central bank’s loss function \( L \)? Substitute the equation for \( u \) into the objective function \( L \) to obtain

\[
L = \lambda [u^n - a(\pi - \pi^e) + v] + \frac{1}{2} (\pi - \pi^e)^2
\]

and take the first-order condition with respect to \( \pi \) and set it equal to zero:

\[
\frac{\partial L}{\partial \pi} = -a \lambda + (\pi - \pi^e) = 0
\]

\[\Rightarrow \pi = \pi^e + a \lambda\]

(b) If private agents understand the decisions of the central bank, what inflation rate will they expect? \( \pi^e = \pi^* + a \lambda \).

(c) What is the equilibrium rate of inflation? What is the equilibrium rate of unemployment? What is the value of \( L \) in this equilibrium? Equilibrium inflation is equal to \( \pi^* + a \lambda \) and unemployment is \( u = u^n - a[\pi^* + a \lambda - (\pi^* + a \lambda)] + v = u^n + v \). Finally, \( L = \lambda (u^n + v) + \frac{1}{2} (a \lambda)^2 \).

(d) If the inflation rate you found in part (c) is not equal to \( \pi^* \), explain why? What factors cause \( \pi \) to differ from \( \pi^* \)? If the public were to expect \( \pi^e \), then the marginal cost of delivering inflation of \( \pi^* \) is \( (\pi - \pi^*) = \pi^* - \pi^e = 0 \) while the marginal benefit is a reduction of \( a \) in unemployment which is worth \( a \lambda \) to the policy maker. So the policy maker has an incentive to aim for a bit more inflation. Expecting this, the public will expect higher inflation. Inflation rises until the marginal cost \( \pi - \pi^* \) is just equal to the marginal benefit of \( a \lambda \), or \( \pi = \pi^* + a \lambda \). The inflation bias \( a \lambda \) is larger the more a bit of inflation reduces unemployment (i.e., the larger is \( a \)) and the more valuable the fall in unemployment is (i.e., the larger is \( \lambda \)).

5. Consider the same setup as in the previous question, but now suppose the central bank will also deliver the inflation rate it announced at the start of the period. What inflation rate should it announce? Is there a gain (measured by achieving a lower value of \( L \)) if the central bank can commit? (I.e., compare the value for \( L \) obtained in part (d) of the previous question to the value of \( L \) in the equilibrium when the central bank commits to its announced inflation rate.) If the central bank announces \( \pi^* \) and delivers it, the value of \( L \) will be \( \lambda (u^n + v) - \frac{1}{2} (\pi^* - \pi^e)^2 = \lambda (u^n + v) \), which is lower than the value of \( L \) under the discretionary policy found in question 4.

6. Consider the same model as question 3 so

\[
u = u^n - a (\pi - \pi^e) + v,
\]
but now the central bank wants to minimize

\[ L = \frac{1}{2} \lambda (u - u^n + k)^2 + \frac{1}{2} (\pi - \pi^*)^2, \]

where \( k > 0 \) and so \( u^n - k \) is the central bank’s goal for unemployment.

Assume the central bank sets inflation after observing the shock \( v \) but private agents form expectations before observing \( v \) (assume their best guess is that it will be zero).

(a) Given \( \pi^e \), what value of \( \pi \) minimizes the central bank’s loss function \( L \)? Repeat the process used in question 4 to write \( L \) as

\[ L = \frac{1}{2} \lambda [u^n - a (\pi - \pi^e) + v - u^n + k]^2 + \frac{1}{2} (\pi - \pi^*)^2 \]

\[ = \frac{1}{2} \lambda [-a (\pi - \pi^e) + v + k]^2 + \frac{1}{2} (\pi - \pi^*)^2. \]

The first-order condition for the optimal \( \pi \), taking expectations as given, is

\[ \frac{\partial L}{\partial \pi} = -a \lambda [-a (\pi - \pi^e) + v + k] + (\pi - \pi^*). \]

Solve for \( \pi \):

\[ \pi = \frac{\pi^* + a \lambda k + a \lambda v}{1 + a^2 \lambda} + \frac{a^2 \lambda}{1 + a^2 \lambda} \pi^e \]

\[ = \frac{\pi^* + a \lambda k + a \lambda v}{1 + a^2 \lambda} + \frac{a^2 \lambda}{1 + a^2 \lambda} \pi^e, \]

(b) If private agents understand the decisions of the central bank, what inflation rate will they expect? They will use equation (1) and expect

\[ \pi^e = \frac{\pi^* + a \lambda k}{1 + a^2 \lambda} + \frac{a^2 \lambda}{1 + a^2 \lambda} \pi^e \]

where it is assumed their best guess for \( v \) is 0. Solving for \( \pi^e \),

\[ \pi^e = \pi^* + a \lambda k. \]

(c) What is the equilibrium rate of inflation? What is the equilibrium rate of unemployment? What is the value of \( L \) in this equilibrium? Substituting (2) into (1),

\[ \pi = \frac{\pi^* + a \lambda k + a \lambda v}{1 + a^2 \lambda} + \frac{a^2 \lambda}{1 + a^2 \lambda} \pi^e \]

\[ = \frac{\pi^* + a \lambda k + a \lambda v}{1 + a^2 \lambda} + \frac{a^2 \lambda}{1 + a^2 \lambda} (\pi^* + a \lambda k) \]

\[ = \pi^* + a \lambda k + \left( \frac{a \lambda}{1 + a^2 \lambda} \right) v \]
Equilibrium unemployment is

\[ u = u^n - a (\pi - \pi^e) + v \]  

\[ = u^n - a \left( \frac{a\lambda}{1 + a^2\lambda} \right) v + v \]  

\[ = u^n + \left( \frac{1}{1 + a^2\lambda} \right) v \]  

(d) If the inflation rate you found in part (c) is not equal to \( \pi^* \), explain why? What factors cause \( \pi \) to differ from \( \pi^* \)? See the answer to 4(d). The new factor now is \( k \), it shows up because it is the difference between the economy’s natural rate of unemployment \( u^n \) and the policy maker’s target unemployment rate of \( u^n - k \). The larger \( k \) is, the lower is the policy maker’s goal and the greater the incentive to try to engineer surprise inflation.

7. How did Ken Rogoff propose solving the inflation bias that can arise under discretionary monetary policy? What trade oﬀ between costs and beneﬁts would determine how conservative the central banker should be? Use your results from question 6 to answer this question. From (3), the lower is \( \lambda \) (the more conservative – in Rogoff’s terms – the central banker is), the smaller is the average inﬂation bias \( (a\lambda k) \), but lowering \( \lambda \) below society’s value means unemployment becomes more volatility (the coeﬃcient on the shock \( v \) in equation 4 is decreasing in \( \lambda \) so a smaller \( \lambda \) means the coeﬃcient increases and a shock causes a larger fluctuation in unemployment).

8. Discuss three possible solutions to the inflation bias you found in question 6. An independent Rogoff conservative central banker, optimal incentives via assigned objectives and accountability, pure independence so that the central bank focuses on stabilizing around the natural rate of unemployment.

9. Do Blinder and Rudd in their 2012 paper agree that the high inﬂation of the 1970s was the result of the times of time inconsistency and credibility issues highlighted by Bernanke in his 2005 paper? If they disagree, what do Blinder and Rudd point to as the source(s) of the high inﬂation? No. Supply shocks.

10. Consider the following simple new Keynesian model:

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1} - r^n) \]  

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \]  

\[ i_t = \phi \pi_t \]  

where \( x \) is the output gap, \( \pi \) is inﬂation, \( i \) is the nominal interest rate, \( r^n \) and \( e \) are stochastic shocks, and \( \sigma, \beta, \kappa, \) and \( \phi \) are constants. Assume \( r^n \) and \( e \) are mean zero, serially uncorrelated random shocks so that there future expected values are zero. In this case, \( E_t x_{t+1} = E_t \pi_{t+1} = 0 \) as well.
(a) Use equations (5) and (7) (together with $E_t x_{t+1} = E_t \pi_{t+1} = 0$) to obtain an aggregate demand relationship between the output gap and inflation. Draw a graph with $x$ on the horizontal axis and $\pi$ in the vertical axis and illustrate this aggregate demand relationship. What is its slope? Explain how the slope depends on $\phi$. Draw the case of a large $\phi$ and a small $\phi$. The two equation are a negatively sloped aggregate demand relationship obtained from (5) and (7):

$$x_t = -\sigma \phi \pi_t + \sigma r^n_t$$

Rewrite this as a line is $x, \pi$ space:

$$\pi_t = - \left( \frac{1}{\sigma \phi} \right) x_t + \left( \frac{1}{\phi} \right) r^n_t$$

This has a slope equal to $- (1/\sigma \phi \pi)$ which decreases as $\phi$ increases (i.e., it gets flatter as $\phi$ increases.

(b) Use equation (6) together with $E_t x_{t+1} = E_t \pi_{t+1} = 0$ to obtain a Phillips curve relating $x$ and $\pi$. Add this relationship to your graph. What is its slope? Suppose $\kappa$ is larger in an economy with more flexible prices. How is the slope of the Phillips Curve affected? The Phillips curve is positively sloped and given by

$$\pi_t = \kappa x_t + \epsilon_t$$

that is equation (6).

(c) Using your graph, illustrate (i) the effects of a positive demand shock ($r^n > 0$) and (ii) the effects of a negative inflation shock ($\epsilon < 0$). For each shock, explain how the effects on $x$ and $\pi$ differ if $\phi$ is large or small and if $\kappa$ is large or small. (So for each shock, discuss four cases: $\phi$ large, $\kappa$ large; $\phi$ large, $\kappa$ small; $\phi$ small $\kappa$ large; $\phi$ small $\kappa$ small.) See lecture slides Slides_Econ105_Fall2015_OptimalPolicy.pdf.