Workers, Capitalists, Wages, and Employment*

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Abstract

Standard new Keynesian models assume the functional distribution of income is irrelevant. This stands in contrast to much of the public debate which presumes that a shift in the distribution of income towards workers will boost aggregate demand and employment. Under this latter view, falling wages during a recession can worsen unemployment. In contrast, neoclassical economics often views unemployment as a direct reflection of the failure of wages to fall. In this paper, I construct a sticky-price, sticky-wage model of workers and capitalists in which the functional distribution of income matters. Wage income directly affects aggregate demand. I derive a second order approximation to social welfare and show the welfare consequences of greater wage flexibility depends critically on monetary policy and the relative importance of different sources of economic fluctuations.

1 Introduction

Are labor costs too high in recessions? Are slow employment recoveries caused by the failure of real wages to fall? Would greater wage flexibility lead to more stable employment? These are old questions in macroeconomics and, as Galí (2013) has recently emphasized, they were central to Keynes’s General Theory and to the immediate reaction to the General Theory. They have remained central to macro debates during periods of high unemployment ever since.

In public discussions, a direct link between wages and employment is often drawn, but these discussions often hypothesize a positive link between wage increases and employment increases.

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An increase in wages is seen as translating into an increase in aggregate spending which, in turn, boosts output and employment. Put in "old Keynesian" terms, the marginal propensity to consume out of wage income is higher than the marginal propensity to consume out of nonwage income. As a consequence, policies that boost labor income relative to profits will increase aggregate demand, and, when prices and wages are sticky, will increase total output and employment. This view stands in contrast to the neoclassical view that emphasizes a negative link between wage increases and employment decreases consistent with a downward-sloping labor demand schedule. In this latter view, high unemployment during a recession is a sign that real wages are too high, and that the failure of wages to fall prevents the return to full employment.

While new Keynesian models emphasize the effects of aggregate demand on markups, employment and wages, these models also focus on the effects of wages on marginal costs and inflation but they exclude any direct effect of wages on aggregate demand.¹ For example, as Galí (2013) shows, if monetary policy were to maintain a constant real interest rate, employment would be a function of exogenous shocks only and the dynamic evolution of employment and output would, in Galí’s words, be "decoupled from wages. In other words, there is no direct impact of wage adjustment on labor demand and employment." (p. 991, emphasis in original).

The standard macro model employed for policy analysis is populated with households who optimally allocate consumption over time while receiving income in the from of wages, interest and profits. In both neoclassical and new Keynesian models, aggregate demand is independent of the functional distribution of income between wages and profits. A rise in labor income that, ceterius peribus, lowers profit income does not affect consumption, as all households pool income regardless of source. These models are convenient in sweeping away any effects of the functional distribution of income on spending. As such, however, they may be poorly specified from the perspective of understanding how wage income may feed directly into aggregate demand.

In this paper, I investigate the properties of a model in which the functional distribution of income matters. To do so, I distinguish between capitalists and workers. Capitalists own the economy’s production technology and receive income as a return to capital and as profits resulting from imperfect competition in goods markets. Workers receive wage income. Taken

¹Galí (2013) provides an excellent discussion of wages and employment through the lens of new Keynesian models. These models place price and wage rigidities at the heart of the analysis of business cycle fluctuations, or at least at the heart of the analysis of aggregate demand policies designed to promote macroeconomic stability.
by itself, this modification would have little effect if both capitalists and workers had access to a common financial market. So I also assume workers do not have access to financial markets; workers are therefore similar to the rule-of-thumb households analyzed by, for example, Mankiw (2000), Galí et al. (2004), Galí et al. (2007), Amato and Laubach (2003), Colciago (2011), Furlanetto and Seneca (2012), and others. These models have typically been used to investigate the effects of fiscal policy, as the presence of rule-of-thumb households leads to deviations from Ricardian equivalence.

The basic model is a simple modification of the standard sticky prices, sticky wages model of Erceg et al. (2000). The modification is designed to explore the implications of heterogeneity by income source and financial market access. Heterogeneity can have important implications for the types of general equilibrium models commonly employed for policy analysis. For example, Bilbiie (2008) shows how limited asset market participation can reverse the sign of the impact of interest rates on aggregate demand. Ravenna and Walsh (2012) show how heterogeneity across worker productivity can generate unemployment composition effects that have a significant impact on dynamics in the face of large shocks. Models of fiscal policy such as Galí et al. (2007) rely on rule-of-thumb households to generate deviations from Ricardian equivalence. While these examples focus on the implications for the economy’s response to shocks and to policy, there is also interest in the distributional consequences of macroeconomic fluctuations, see for example, Gorodnichenko et al. (2012).

In models with rule-of-thumb households, a fixed fraction of households behave as standard utility maximizing agents with full access to financial markets. The marginal utility of consumption of these households satisfies a standard intertemporal Euler condition. The remaining fraction of households simply consume their current (labor) income. In the present paper, I take this dichotomy to the extreme in assuming two types of households differentiated by their source of income. Capitalists own the economy’s productive technology and its (fixed) stock of capital. Capitalists also have access to a market for bonds which, in equilibrium, are in zero net supply. Workers, as their name suggest, earn income from supplying labor. They do not have access to financial markets and therefore, in each period, consume their labor income. In equilibrium, capitalists also consume their current income, but this is an equilibrium outcome and not a constraint on capitalists’ choices.

Many rule-of-thumb models assume both rule-of-thumb households and optimizing households are, when it comes to the labor market, identical. Or, as in Colciago (2011), there are differentiated labor types, but each household supplies all labor types and so labor income is identical across households, and, importantly, is independent of whether the household is a
rule-of-thumb household or an optimizing household. By distinguishing clearly between recipients of labor income and of non-labor income, and by assuming each worker is the monopoly supplier of a single labor type, the worker-capitalist model developed here generates a cross-sectional distribution of labor income and consumption among workers, as well as consumption differences between workers and capitalists.

For a given real interest rate and employment, a temporary rise in real wages has two effects in the present model: first, it raises the relative price of labor, and second, it shifts income to workers and away from capitalists. This second effect increases aggregate demand as workers’ consumption rises fully with the wage increase and capitalists consumption falls by less than the decline in profits as they desire to intertemporally smooth the fall in their income. These are partial equilibrium effects, and the general equilibrium effects will clearly depend on the response of the monetary authority to the wage increase.

A model of workers and capitalists is admittedly extreme in the restrictions it imposes, but no more so than the standard model of a representative household. Exploring the consequences of the model developed here is useful, as it is not clear it is a less realistic abstraction than the assumption of a representative rational intertemporal optimizer. The model is set out in section 2. It is compared to the benchmark new Keynesian model with sticky prices and wages due to Erceg et al. (2000), and some of the implications of the model are discussed. Section 3 discusses the calibration of the model, implications for determinacy, and presents impulse responses to aggregate demand, productivity, and markup subsidy shocks, in each case comparing them to the results in the EHL model. The effects of a wage subsidy and a payroll tax are also discussed. The consequences of increased wage flexibility, holding monetary policy constant, are considered in section 4. The consequences of wage flexibility under optimal monetary policy is considered in section 5, while conclusions are summarized in a final section.

\section{A model of workers and capitalists}

Two types of households populate the economy: capitalists and workers. There is a continuum of measure one of capitalists and workers. Capitalists own the economy’s productive technology, have access to financial markets to share risk, and receive income solely from profits. Workers receive income solely from wages and do not have access to financial markets.\footnote{Workers are able to share risk intertemporally by adjusting their labor supply. See Heathcote et al. (2014) for evidence this channel plays a role in insuring against permanent wage fluctuations.}

Consumption by workers is equal to wage income, so workers in the model are similar to the...
rule-of-thumb consumers in Campbell and Mankiw (1991), Mankiw (2000), Galí et al. (2004), and Galí et al. (2007). Unlike the optimizing consumers in these papers, capitalists do not supply labor. As owners of the productive technology, capitalists operate firms and hire labor and rent capital to produce differentiated consumption goods. For simplicity, I treat the aggregate capital stock as fixed. In addition, there is zero-profit employment agency that hires the differentiated labor serves of workers and rents a labor aggregate to firms who use it to produce output.

Both the markets for consumption goods and for labor are characterized by monopolistic competition, and both prices and wages are sticky. With the exception of the distinction among households based on their income source and access to financial markets, the model is similar to the standard new-new Keynesian model of Erceg et al. (2000) in its treatment of sticky prices and wages. The specification of the labor market, though, differs from those common in the RoT literature. In standard RoT models such as Galí et al. (2007), both RoT households and optimizing households face the same wage and supply the same amount of labor. Furlanetto (2011) discusses alternative labor market specifications in a model with RoT households. However, in these alternative specifications, as in Colciago (2011), all RoT households end up with the same level of consumption. Thus, despite lacking access to financial markets, these households are able to pool any idiosyncratic consumption risk. In the model developed here, capitalists, by assumption, optimize with respect to their consumption choice but do not supply labor. Thus, only the preferences of workers will be relevant for the determination of labor supply decisions. In addition, workers are assumed to supply differentiated labor services that are imperfect substitutes in production, but rather than assume each worker/household supplies all types of labor services, I assume each worker supplies a unique labor type, implying that, with sticky wages, labor income and therefore consumption will differ across worker/households.

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3 See also Colciago (2011), Natvik (2012).

4 The income from capital ownership that capitalists receive could be interpreted as income from human capital and therefore as a form or wage income. The key distinction then is that capitalists supply their human capital inelastically and the return to human capital is not sticky.

5 See Natvik (2012) for a discussion of the implicit assumptions about wealth redistributions in RoT models.
2.1 Capitalists

2.1.1 As consumers

As a consumer, capitalist $h$ maximizes

$$E_t \sum_{i=0}^{\infty} \beta^i U(C^c_{t+i}(h)) = E_t \sum_{i=0}^{\infty} \beta^i \left[ C^c_{t+i}(h) \right]^{1-\sigma_c} \frac{1}{1-\sigma_c},$$

where

$$C^c_t(h) = \int c^c_{j,t}(h) \frac{\theta_j^{\rho_p-1}}{\theta_t^{\rho_p}} dj$$

is a Dixit-Stiglitz aggregate of the consumption of the individual goods by capitalist $h$. The maximization of (1) is subject to a budget constraint given by

$$(1 - \tau_t^\pi) \Pi_t(h) + \left( \frac{1 + i_{t-1}}{1 + \pi_t} \right) b_{t-1}(h) - T^c_t - C^c_t(h) - b_t(h) \geq 0,$$

where $\Pi_t$ is the real profit income of capitalist $h$, $b$ are real bond holdings, $\pi_t = (P_t/P_{t-1}) - 1$ is the inflation rate, where $P_t$ is the aggregate price level, given by

$$P_t \equiv \left[ \int P_{j,t}^{1-\theta_j^\rho_p} dj \right]^{\frac{1}{1-\theta_j^\rho_p}}.$$

$\tau^\pi$ is a profits tax, and $T^c$ is a lump-sum tax on capitalists. Assuming a complete contingent claims market for consumption among capitalists, $C^c_t(h) = C^c_t$ for all $h$ and

$$(C^c_t)^{-\sigma_c} = \beta E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) (C^c_{t+1})^{-\sigma_c}.$$

Hence, in terms of deviations around the steady state,

$$\hat{c}^c_t = E_t \hat{c}^c_{t+1} - \left( \frac{1}{\sigma_c} \right) (i_t - E_t \pi_{t+1})$$

Bonds are in zero net supply, so the aggregate (per capita) budget constraint for capitalists is given by

$$C^c_t = (1 - \tau_t^\pi) \Pi_t - T^c_t.$$
2.1.2 As firm owners

Capitalists own the (fixed) aggregate stock of capital and the economy’s production technology. Capitalist \( j \) operates firm \( j \), producing a consumption good \( c_{j,t} \) using the technology

\[
c_{j,t} = Z_t N_j^{1-a} K_j^a, \quad 0 < a \leq 1,
\]

where \( Z \) is a common stochastic productivity factor, \( N_{j,t} \) is employment at firm \( j \), and \( K_{j,t} \) is the firm’s capital. All firms hire labor services at a common real wage \( \omega_t \) and capital at a common rental rate \( r_t \). This implies \( K_{j,t}/N_{j,t} \) is independent of \( j \).

The firm faces a demand curve (whose derivation is discussed below) given by

\[
c_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta^p_t} C_t,
\]

where \( P_{j,t} \) is the price of \( c_{j,t} \), and \( C_t \) is aggregate consumption. The price elasticity of demand, \( \theta^p_t \), is common across all firms and may vary stochastically. Its mean value is \( \theta^p > 1 \).

Defining aggregate output as

\[
Y_t = \left[ \int c_{j,t}(h) \left( \frac{\theta^p_{j,t}}{\theta^p_t} \right)^{\theta^p_{j,t}} \right]^{\theta^p_t} - 1
\]

goods clearing implies,

\[
Y_t \equiv C_t.
\]

Given \( c_{j,t} \), the solution to the firm’s problem of producing \( c_{j,t} \) at minimum cost yields

\[
(1 - a) \left( \frac{c_{j,t}}{N_{j,t}} \right) = \mu^P_t \left( 1 + \tau^P_t \right) \omega_t
\]

and

\[
a \left( \frac{c_{j,t}}{K_{j,t}} \right) = \mu^P_t r_t,
\]

where \( \mu^P_t = \theta^P_t / (\theta^P_t - 1) \) is the firm’s markup and \( \tau^P_t \) is a payroll tax. Given common factor prices faced by all firms, \( N_{j,t}/K_{j,t} = N_t/K_t = ar_t / \left[ (1 - a) \left( 1 + \tau^P_t \right) \omega_t \right] \) is independent of \( j \).

In the steady state,

\[
(1 + \tau^P) \omega = (1 - a) \left( \frac{Y}{\mu^P N} \right),
\]
where $\mu^p = \theta^p / (\theta^p - 1)$ is the steady-state markup. The steady-state share of profits in total income is then equal to

$$\frac{\Pi}{\bar{Y}} = 1 - \frac{1}{\mu^p} = \frac{\mu^p - 1}{\mu^p},$$

and the aggregate share of steady-state income received by capitalists is $(\Pi + \tau K)/\bar{Y} = (\mu^p + a - 1)/\mu^p$.

Firms are subject to sticky prices, modeled by the standard Calvo adjustment process, where $1 - \varphi^p$ of the firms are randomly chosen to adjust each period. In addition, firms face a payroll tax $\tau^p$. Marginal cost for firm $j$ is

$$mc_{j,t} = \omega_t \left(1 + \tau^p + \sum_{j=1}^{N_t} (1 - a)^t \left( c_{j,t} - N_{j,t} \right) \right). \quad (7)$$

The Calvo assumption leads to the well-known solution for the optimal price chosen by those firms that can adjust their price at time $t$.

Let $\hat{x}$ denote the log deviation of $X_t$ around its steady-state value. Then the log-linearized version of (7) is

$$\hat{y}_t = (1 - a) \hat{n}_t + z_t \quad (8)$$

and the linearized inflation equation is:6

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_p (\hat{mc}_t + \hat{\mu}_t), \quad (9)$$

where from (7),

$$\hat{mc}_t = \hat{\omega}_t + \tau^p + a \hat{n}_t - \hat{z}_t \quad (10)$$

is real marginal cost and

$$\kappa_p \equiv \frac{(1 - \varphi^p) (1 - \beta \varphi^p)}{\varphi^p} \left( \frac{1 - a}{1 - a + a \theta^p} \right).$$

### 2.2 Workers

Each worker-household supplies a differentiated labor type and purchases market consumption goods. When wages are sticky and adjust according to a standard Calvo mechanism, wages,

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6See, for example, Galí (2008).
hours, and therefore labor income and consumption will differ across workers. These differences will reflect any dispersion of relative wages generated by staged wage adjustment. I also differ from the segmented labor market model of Furlanetto (2011) who treats the wage choice of rule-of-thumb households as a static choice. Instead, I assume a Calvo process for wage setting and that workers are forward-looking in their wage setting behavior. Thus, while workers do not have access to financial markets and so simply consume their current labor income, they recognize that with sticky wages, their choice when they do adjust their wage will affect current and expected future labor income and consumption until another opportunity to adjust the wage arrives.

Worker \( s \) supplies labor hours \( H_t(s) \) to a labor employment agency at wage \( \omega_t(s) \). Preferences of household \( s \) are given by

\[
E_t \sum_{i=0}^{\infty} \beta^i U^w \left( C_{t+i}^w(s), H_{t+i}(s) \right) = E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{(C_{t+i}^w(s))^{1-\sigma^w}}{1-\sigma^w} - \frac{\lambda H_{t+i}(s)^{1+\eta}}{1+\eta} \right]
\]

and are maximized subject to

\[
C_t^w(s) = (1 + \tau^w_t) \omega_t(s) H_t(s) - T^w_t
\]

where \( \omega_t(s) \) is the wage of worker \( s \), \( \tau^w_t \) is a wage subsidy, and \( T^w_t \) is a lump-sum tax.\(^7\) As with capitalists,

\[
C_t^w(s) = \left[ \int c_{j,t}(s) \frac{\theta^w_{t-1}}{\theta^j_t} \, dj \right]^{\frac{\theta^j_t}{\theta^w_{t-1}}}
\]

The employment agency sells labor services \( N_t \) to firms, where \( N_t \) is an aggregate of differentiated labor services supplied by households defined by

\[
N_t = \int \left[ H_t(s) \frac{\theta^w_{t-1}}{\theta^t} \, ds \right]^{\frac{\theta^w_t}{\theta^w_{t-1}}}
\]

The employment agency takes wages as given. Thus, the demand for type \( s \) labor by the employment agency is a function of relative wages:

\[
H_t(s) = \left( \frac{\omega_t(s)}{\omega_t} \right)^{-\theta^w_t} N_t.
\]

\(^7\)I assume the wage subsidy is common to all workers and not a function of the worker’s individual type.
The linearized budget constraint of household $s$ is
\[ \omega H \left( \hat{\omega}_t(s) + \hat{h}_t(s) + \tau_t^w \right) - T^w t^w_t = C^w_t(s). \]

Since the (linearized) demand for labor type $s$ is
\[ \hat{h}_t(s) = -\theta_t^w (\hat{\omega}(s) - \hat{\omega}_t) + \hat{n}_t, \]
the budget constraint of worker $s$ can be written as
\[ \hat{c}_t^w(s) = \left( \frac{\omega H}{C^w} \right) [\hat{\omega}_t(s) - \theta_t^w (\hat{\omega}(s) - \hat{\omega}_t) + \hat{n}_t + \tau_t^w] - \left( \frac{T^w}{C^w} \right) t^w_t. \]

or, since
\[ \int (\hat{\omega}(s) - \hat{\omega}_t) ds = 0, \]
the aggregate (per capita) consumption by workers is
\[ \hat{c}_t^w = \left( \frac{\omega N}{C^w} \right) (\hat{\omega}_t + \hat{n}_t + \tau_t^w) - \left( \frac{T^w}{C^w} \right) t^w_t \quad (11) \]

If the worker could reset her wage each period, she would set $\omega_t(s)$ to maximize
\[ U^w (C^w_t(s), H_t(s)) = U^w ((1 + \tau_t^w) \omega_t(s) H_t(s) - T^w_t, H_t(s)), \]
subject to the demand for its labor type. Thus, with flexible wages, the worker’s problem could be written as
\[
\max_{\omega_t(s)} U^w \left( (1 + \tau_t^w) \omega_t \left( \frac{\omega_t(s)}{\omega_t} \right)^{1-\theta_t^w} H_t - T^w_t, \left( \frac{\omega_t(s)}{\omega_t} \right)^{-\theta_t^w} N_t \right) \]
and the first order condition for this problem is
\[ (1 + \tau_t^w) (1 - \theta_t^w) \left( \frac{\omega_t(s)}{\omega_t} \right)^{-\theta_t^w} N_t U^w_t - \theta_t^w \left( \frac{\omega_t(s)}{\omega_t} \right)^{-\theta_t^w} N_t \left( \frac{1}{\omega_t(s)} \right) U^w_H = 0 \]
or (recall $U_H < 0$),
\[ (1 + \tau_t^w) \omega_t(s) = -\frac{\theta_t^w}{\theta_t^w - 1} \frac{U^w_H(s)}{U^w_C(s)} = -\mu_t^w \frac{U^w_H(s)}{U^w_C(s)} \quad (12) \]
Log linearizing (12) around the steady state yields, when wages are flexible,

\[ \eta \hat{h}_t(s) + \sigma^w \hat{c}_t^w(s) = \tau_t^w + \hat{\omega}_t(s) - \hat{\mu}_t^w. \]

Aggregating over all workers,

\[ \left( \eta \hat{h}_t + \sigma^w \hat{c}_t^w \right) - \left( \tau_t^w + \hat{\omega}_t - \hat{\mu}_t^w \right), \tag{13} \]

where \( \hat{h}_t = \int \hat{h}_t(s) ds \) and \( \hat{c}_t^w = \int \hat{c}_t^w(s) ds \).

Rather than assume flexible wages, workers, like firms, are assumed to face a constant probability \( 1 - \varphi^w \) each period of changing their wage. Thus, any wedge, current or expected, between the marginal rate of substitution term \( -\mu^w U_H^w(s)/U_C^w(s) \) and the gross of subsidy real wage \( (1 + \tau^w) \omega_t(s) \) will induce workers who have an opportunity to change their wage to do so. The marginal rate of substitution between leisure and consumption some future date \( t + k \) for a worker setting her wage at time \( t \), \( mrs_{t+k/t} \), and the aggregate average marginal rate of substitution at \( t + k \), \( mrs_{t+k} \), can be expressed as

\[ mrs_{t+k/t} = mrs_{t+k} + \sigma^w \left( \hat{c}_{t+k/t} - \hat{c}_{t+k} \right) + \eta \left( \hat{h}_{t+k/t} - \hat{h}_{t+k} \right) \]
\[ = mrs_{t+k} + \sigma^w \left( \frac{\omega H}{C^w} \right) \left( \hat{\omega}_{t+k/t} - \omega_{t+k} \right) + \left[ \sigma^w \left( \frac{\omega H}{C^w} \right) + \eta \right] \left( \hat{h}_{t+k/t} - \hat{h}_{t+k} \right) \]
\[ = mrs_{t+k} - \gamma \left( w_{t+k/t} - w_{t+k} \right) \]

where \( w \) is nominal wage,

\[ \tilde{\sigma}^w \equiv \sigma^w \left( \frac{\omega H}{C^w} \right), \]

and

\[ \gamma \equiv \theta^w (\hat{\sigma}^w + \eta) - \tilde{\sigma}^w. \]

Using this result in the standard first order condition for the wage chosen by adjusting workers yields the following wage inflation equation:

\[ \tau_t^w = \beta E_t \tau_{t+1}^w + \kappa_w \left( mrs_t - \hat{\omega}_t + \hat{\mu}_t^w \right), \tag{14} \]

where

\[ mrs_t = \eta \hat{h}_t + \sigma^w \hat{c}_t^w - \tau_t^w \tag{15} \]
and
\[ \kappa_w \equiv \frac{(1 - \varphi^w)(1 - \beta \varphi^w)}{\varphi^w} \left( \frac{1}{1 - \sigma^w + (\sigma^w + \eta) \theta^w} \right) . \]

The aggregate real wage then evolves according to
\[ \omega_t = \omega_{t-1} + \pi_t^w - \pi_t. \]  

(16)

2.3 Market clearing

Goods market clearing for each firm implies

\[ c_{j,t} = \left[ \int \left( \frac{P_{j,t}}{P_t} \right) d_j \right]^{\theta^p} \left( C_t^c + C_t^w \right) . \]

From the definition of \( Y_t \),

\[ Y_t = \left[ \int c_{j,t}(h) \left( \frac{P_{j,t}}{P_t} \right) d_j \right]^{\theta^p} \left( C_t^c + C_t^w \right) \]
\[ = \left[ \int \left( \frac{P_{j,t}}{P_t} \right)^{1 - \theta^p} d_j \right]^{\theta^p} \left( C_t^c + C_t^w \right) \]
\[ = C_t^c + C_t^w = C_t. \]

From the production function of firm \( j \) and the demand curve faced by firm \( j \),

\[ N_{j,t} = \left( \frac{C_{j,t}^c}{Z_t K^a} \right)^{\frac{1}{1-a}} \left( \frac{Y_t}{Z_t K^a} \right)^{\frac{1}{1-a}} \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{\theta^p}{1-a}} . \]

Therefore

\[ N_t = \int N_{j,t} d_j = \left( \frac{Y_t}{Z_t K^a} \right)^{\frac{1}{1-a}} \Delta_{p,t} \]  

(17)

where

\[ \Delta_{p,t} \equiv \int \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{\theta^p}{1-a}} d_j \]
reflects relative price dispersion across firms. To a first order approximation,

\[ \hat{n}_t = \left( \frac{1}{1 - a} \right) \left( \hat{y}_t - z_t - \theta^y \Delta_{p,t} \right) = \left( \frac{1}{1 - a} \right) (\hat{y}_t - z_t) \]

as the measure of relative price dispersion is equal to zero to first order.

The labor supplied by household \( s \) is

\[ H_t(s) = \left( \frac{\omega_t(s)}{\omega_t} \right)^{-\theta^w} N_t \]

which when aggregated across workers yields

\[ H_t = \int H_t(s) ds = \left[ \int \left( \frac{\omega_t(s)}{\omega_t} \right)^{-\theta^w} ds \right] N_t = \Delta_{w,t} N_t, \]

where

\[ \Delta_{w,t} = \left[ \int \left( \frac{\omega_t(s)}{\omega_t} \right)^{-\theta^w} ds \right] \geq 1 \]

is a function of relative wage dispersion. Hence,

\[ N_t = \Delta_{w,t}^{-1} H_t < H_t. \]

Using (17),

\[ Y_t = \Delta_{p,t}^a Z_t K^a N_t^{1-a} = (\Delta_{p,t} \Delta_{w,t})^{a-1} Z_t K^a H_t^{1-a} \]

illustrating how both wage and price dispersion drive wedges between aggregate labor hours supplied \((H)\) and final output produced. However, to first order,

\[ \hat{n}_t = \hat{h}_t. \] (18)

### 2.4 Monetary and fiscal policy

Monetary policy is represented by a simple Taylor-type instrument rule which, when linearized, takes the form

\[ i_t = \phi_n \pi_t + \phi_{nw} \pi^w_t + \phi_y \hat{y}_t. \] (19)
Fiscal policy is Ricardian. The government’s budget linearized constraint is

\[
\left( \frac{T^c}{C} \right) \tilde{c}_t + \left( \frac{T^w}{C} \right) \tilde{w}_t + \left( \frac{\omega N}{C} \right) \tau^p_t + \left( \frac{\Pi}{C} \right) \tau^\pi_t = \left( \frac{\omega H}{C} \right) \tau^w_t.
\]

For simplicity, I set \( \tau^\pi = 0 \). Then

\[
\left( \frac{T^c}{C} \right) \tilde{c}_t + \left( \frac{T^w}{C} \right) \tilde{w}_t = \left( \frac{\omega H}{C} \right) (\tau^w_t - \tau^p_t). \tag{20}
\]

Because of the presence of workers who simply consume their after-tax wage income, variations in \( \tilde{w}_t \) have the potential to affect aggregate spending. This means the effects of the wage subsidy and the payroll tax will depend on how \( \tilde{c}_t \) and \( \tilde{w}_t \) are adjusted to maintain the government’s budget balance. Assume the government follows tax rules of the form

\[
\left( \frac{T^w}{C} \right) \tilde{w}_t = \delta \left( \frac{\omega H}{C} \right) (\tau^w_t - \tau^p_t) \tag{21}
\]

and

\[
\left( \frac{T^c}{C} \right) \tilde{c}_t = (1 - \delta) \left( \frac{\omega H}{C} \right) (\tau^w_t - \tau^p_t) \tag{22}
\]

so that \( \delta \in [0, 1] \) is the share of the costs of wage subsidies net of profit tax revenues financed by lump-sum taxes on workers.

2.5 The log-linear model

The log-linearized model consists of equations (4), (8)–(10), (11), (14)–(19), (21) and (22) that jointly solve for \( \tilde{c}_t, \tilde{w}_t, \tilde{n}_t, \tilde{h}_t, \tilde{i}_t, \tilde{m}_t, \tilde{m}_p, \tilde{\omega}_t, \tilde{\pi}_t, \tilde{\pi}^w_t, \tilde{\tau}_t \) and \( \tilde{\tau}_t \). The model equations are summarized in the appendix, together with the log-linear equations for the model of Erceg et al. (2000) which will serves as a benchmark for comparison.

2.5.1 With flexible wages and prices

When both wages and prices are flexible, the economy’s labor market equilibrium, in terms of log deviations around the steady state, is characterized by

\[
\eta \tilde{n}_t + \sigma^{cw} \tilde{c}_t - \tau^c_t + \tilde{\mu}^{cw}_t = \tilde{\omega}_t = z_t - a \tilde{n}_t - (\tau^p_t + \mu^p_t). \tag{23}
\]
Using (11) to eliminate $c^w$, the left side of (23) yields a labor supply relationship of the form
\[
\hat{n}_t^s = \left(1 - \frac{\bar{\sigma}^w}{\eta + \bar{\sigma}^w}\right)(\hat{\omega}_t + \tau_t^w) - \left(\frac{1}{\eta + \bar{\sigma}^w}\right)\left[\hat{\mu}_t^w - \bar{\sigma}^w \left(\frac{T^w}{\omega H}\right) \hat{t}_t^w\right].
\]

From the right side of (23), labor demand is given by
\[
\hat{n}_t^d = \left(\frac{1}{a}\right)(z_t - \hat{\omega}_t - \tau_t^P - \mu_t^P).
\]

Equating labor supply to labor demand and using the aggregate production function, the flex-price-wage equilibrium real wage, employment, and output are
\[
\hat{\omega}_t^f = \phi^w (\eta + \bar{\sigma}^w) (z_t - \tau_t^P - \mu_t^P)
+ a \phi^w \left[\hat{\mu}_t^w - \bar{\sigma}^w \left(\frac{T^w}{\omega H}\right) \hat{t}_t^w\right] - a \phi^w (1 - \bar{\sigma}^w) \tau_t^w;
\]
\[
\hat{n}_t^f = \phi^w (1 - \bar{\sigma}^w) [z_t - (\tau_t^P + \mu_t^P)]
- \phi^w \left[\hat{\mu}_t^w - \bar{\sigma}^w \left(\frac{T^w}{\omega H}\right) \hat{t}_t^w\right] + \phi^w (1 - \bar{\sigma}^w) \tau_t^w;
\]
\[
\hat{y}_t^f = \phi^w (1 + \eta) z_t + \phi^w (1 - a) (1 - \bar{\sigma}^w) (\tau_t^w - \tau_t^P - \mu_t^P)
- \phi^w (1 - a) \left[\hat{\mu}_t^w - \bar{\sigma}^w \left(\frac{T^w}{\omega H}\right) \hat{t}_t^w\right],
\]
where
\[
\phi^w \equiv \frac{1}{\eta + \bar{\sigma}^w + a (1 - \bar{\sigma}^w)}.
\]

An interesting feature of the case of flexible prices and wages is that, because aggregate demand affects the real interest rate but not employment or output, the inverse elasticity of intertemporal substitution of capitalists, $\sigma^c$, is irrelevant for the behavior of output and employment. It is only relevant for financial market variables. If the adjusted inverse elasticity of intertemporal substitution of workers, $\bar{\sigma}^w$, is equal to 1, the flex-price-wage output is independent of the payroll tax, the profit tax, and price markups unless the former two lead to changes in the lump-sum tax on workers $\hat{t}_t^w$. When $\bar{\sigma}^w = 1$, labor supply is completely

This result depends on the absence of investment.
inelastic with respect to the real wage; an increase in price markups lowers the real wage but leaves employment, and therefore output, unaffected. In contrast, a wage markup does affect \( \hat{y}_t^F \) by reducing labor supply, even when \( \hat{\sigma}^w = 1 \).

In a standard NK model, the linearized labor market equilibrium condition with flexible prices and wages takes the form

\[
\eta \hat{n}_t + \sigma \hat{y}_t - \tau_t^w + \hat{\mu}_t^W = \hat{\omega}_t = z_t - a \hat{n}_t - \left( \tau_t^P + \mu_t^P \right),
\]

since aggregate consumption is equal to aggregate income. The key difference with (23) is that when \( \hat{c}_t = \hat{y}_t \), the marginal rate of substitution between leisure and consumption no longer depends, as it does in the worker-capitalist model, on worker’s share of total income and therefore on the functional distribution of income. If \( \hat{n}_{sf}^t \) and \( \hat{y}_{sf}^t \) denote the flex-price and wage employment and output in the standard (EHL) model, it follows that

\[
\hat{n}_{sf}^t = \phi^s \left( 1 - \sigma \right) z_t + \phi^s \left( \tau_t^w - \tau_t^P - \mu_t^P - \mu_t^w \right)
\]

and

\[
\hat{y}_{sf}^t = \phi^s \left( 1 + \eta \right) z_t + \phi^s \left( 1 - a \right) \left( \tau_t^w - \tau_t^P - \mu_t^P - \mu_t^w \right).
\]

where

\[
\phi^s \equiv \frac{1}{\eta + \sigma + a \left( 1 - \sigma \right)},
\]

and \( \sigma \) reflects the preferences of the representative household.

Thus, in the standard model, markups in either product or labor markets, a payroll tax, and a wage subsidy all affect the flexible price and wage equilibrium in an equivalent (up to sign) fashion. If \( \sigma^w = \sigma \) and \( T^w = 0 \) so that \( \omega H/C^w = 1 \), then \( \phi^w = \phi^s = \phi \) and

\[
\hat{y}_t^F = \phi \left( 1 + \eta \right) z_t + \phi \left( 1 - a \right) \left( \tau_t^w - \tau_t^P - \mu_t^P \right) \\
-\phi \left( 1 - a \right) \left[ \hat{\mu}_t^w - \sigma \left( \frac{T^w}{\omega H} \right) \hat{\mu}_t^w \right].
\]

Comparing \( \hat{y}_t^F \) and \( \hat{y}_{sf}^t \) in this case shows that they are affected similarly by productivity and wage markup shocks. The former has standard effects while the latter reduces labor supply and generates a fall in output. However, price markup, payroll tax and wage subsidy shocks have different effects in the two models. With log utility, for example, so that \( \sigma = 1 \), none of these shocks affects output in the worker-capitalist model as individual workers internalize
the equal proportional effect of wages on consumption, while in the standard model the first two reduce output and the third increases output. When \( \sigma \neq 1 \),

\[
\hat{y}_t^f - \hat{y}_t^{sf} = \phi \sigma (1 - a) \left[ \mu_t^P + \tau_t^P - \tau_t^w + \left( \frac{T^w}{\omega H} \right) \hat{t}_t^w \right].
\]

In the case of constant returns to scale in labor \( (a = 1) \), \( \hat{y}_t^f = \hat{y}_t^{sf} \). In general though, with \( a < 1 \), a positive price markup shock, payroll tax shock, or a negative wage subsidy shock, all of which reduce both \( \hat{y}_t^f \) and \( \hat{y}_t^{sf} \), reduce the flex-equilibrium output less in the worker-capitalist economy than in the standard NK economy. Notice also that productivity shocks and wage-markup shocks affect flex-equilibrium output in exactly the same manner in the worker-capitalist model and the standard new Keynesian model.

Lump-sum taxes on workers, which play no role in determining flexible price and wage output in the standard model, do so in the worker-capitalist model. Under the fiscal rule \( (21) \),

\[
\hat{y}_t^f - \hat{y}_t^{sf} = \phi \sigma (1 - a) \left[ \mu_t^P + (1 - \delta) \left( \tau_t^P - \tau_t^w \right) \right].
\]

If variations in the government’s budget are financed through adjustments of lump-sum taxes on workers \( (\delta = 1) \), then payroll taxes and wage subsidies affect the flex-equilibrium output identically in both the EHL and the worker-capitalist model (WCM). This will not be the case if \( 0 \leq \delta < 1 \).

### 2.5.2 With sticky wages and prices

When prices and wages are sticky, both the worker-capitalist model and the standard EHL model imply the price markup and the payroll tax enter only in the form \( \hat{\mu}_t^P + \tau_t^P \), as can be seen by substituting the expression for \( \hat{m} \hat{c}_t \) into the price inflation equation. Positive innovations to either increase firms’ desired wedge between the wage paid to labor and the marginal product of labor. In the EHL model, the wage markup and the wage subsidy appear only in the form \( \hat{\mu}_t^w - \tau_t^w \), so positive innovations to either affect equilibrium in an equal but opposite manner. Thus, equal positive innovations to both have no effect on the equilibrium. This stands in contrast to the implications of the worker-capitalist model. Equal positive innovations to the wage markup and the wage subsidy do not affect workers’ desired wage and so do not directly affect wage inflation (see 13). However, \( \tau_t^w \) has a direct (positive) effect on the consumption of workers by increasing their income (as long as it is not offset by changes in \( \hat{t}_t^w \)), for a given real wage and employment level.
2.6 Wages and aggregate demand

One purpose of the present model is to allow for a direct channel from wages and employment to aggregate demand. To highlight this channel, the goods clearing condition can be used to eliminate capitalists’ consumption from their Euler condition to obtain

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \left( \frac{C^c}{C} \right) \left( \frac{1}{\sigma} \right) \left( i_t - E_t \pi_{t+1} - \rho \right) - \left( \frac{C^w}{C} \right) \left( E_t \hat{c}^w_{t+1} - \hat{c}^w_t \right). \]

Using the budget constraint of workers to eliminate \( \hat{c}^w_t \) and \( E_t \hat{c}^w_{t+1} \) yields

\[ \hat{y}_t = E_t \hat{y}_{t+1} - G \left( \frac{1}{\sigma^c} \right) \left( i_t - E_t \pi_{t+1} - \rho \right) + G \left( \frac{\omega N}{C} \right) \left( \frac{1}{1-a} \right) E_t \Delta \hat{z}_{t+1} \]

\[ -G \left( \frac{\omega N}{C} \right) E_t \left[ \Delta \hat{\omega}_{t+1} + \Delta \tau^w_{t+1} - \left( \frac{T^w}{\omega N} \right) E_t \Delta t^w_{t+1} \right], \]

where \( E_t \Delta x_{t+1} = E_t x_{t+1} - x_t \),

\[ \hat{\sigma}^c = \sigma^c \left( \frac{C}{C^c} \right), \]

and

\[ G \equiv \left[ 1 - \left( \frac{1}{1-a} \right) \left( \frac{\omega N}{C} \right) \right]^{-1} > 1. \]

The factor \( G \) reflects a Keynesian multiplier that arises from the impact of labor income on aggregate demand.\(^9\) A (non-productivity-driven) rise in output increases labor income. This directly increases the consumption of workers, re-enforcing the initial rise in aggregate demand. Substituting in the fiscal rule,

\[ \hat{y}_t = E_t \hat{y}_{t+1} - G \left( \frac{1}{\sigma^c} \right) \left( i_t - E_t \pi_{t+1} - \rho \right) + G \left( \frac{\omega N}{C} \right) \left( \frac{1}{1-a} \right) E_t \Delta \hat{z}_{t+1} \]

\[ -G \left( \frac{\omega N}{C} \right) E_t \left[ \Delta \hat{\omega}_{t+1} + (1 - \delta) E_t \Delta \tau^w_{t+1} + \delta E_t \Delta \tau^p_{t+1} \right], \]

showing how both monetary policy (via the nominal interest rate) and the fiscal tax policy affect aggregate demand for given expected future output, productivity and wage growth. If one assumes \( z_t, \tau^p, \) and \( \tau^w \) are AR(1) processes with autocorrelation coefficients \( \rho_z, \rho_p \) and

\(^9\)\( G \) is also equal to \( \mu^p / (\mu^p - 1) \) since the real wage is the marginal product of labor divided by the firm’s markup.
\( \rho_w \), the Euler condition can be solved forward to obtain\(^{10} \)

\[
\hat{y}_t = -G \left( \frac{1}{\sigma^c} \right) E_t \sum_{i=0}^{\infty} (i_{t+i} - E_t \pi_{t+1+i} - \rho) - G \left( \frac{\omega N}{C} \right) \left( \frac{1}{1 - a} \right) z_t \\
- G \left( \frac{\omega N}{C} \right) E_t \sum_{i=0}^{\infty} \Delta \hat{w}_{t+1+i} + G \left( \frac{\omega N}{C} \right) \left[ (1 - \delta) \tau^w_t + \delta \tau^p_t \right]. \tag{26}
\]

If all fiscal adjustment occurs through changes in lump-sum taxes on capitalists so that \( \delta = 1 \),

\[
\hat{y}_t = -G \left( \frac{1}{\sigma^c} \right) E_t \sum_{i=0}^{\infty} (i_{t+i} - E_t \pi_{t+1+i} - \rho) + G \left( \frac{\omega N}{C} \right) \left[ \tau^w_t - \left( \frac{1}{1 - a} \right) z_t \right] \\
- G \left( \frac{\omega N}{C} \right) E_t \sum_{i=0}^{\infty} \Delta \hat{w}_{t+1+i}.
\]

Using the aggregate production function, employment is

\[
\hat{n}_t = - \left( \frac{1}{1 - a} \right) G \left( \frac{1}{\sigma^c} \right) E_t \sum_{i=0}^{\infty} (i_{t+i} - E_t \pi_{t+1+i} - \rho) - G \left( \frac{1}{1 - a} \right) z_t \\
- G \left( \frac{1}{1 - a} \right) \left( \frac{\omega N}{C} \right) \tau^w_t - \left( \frac{1}{1 - a} \right) G \left( \frac{\omega N}{C} \right) E_t \sum_{i=0}^{\infty} \Delta \hat{w}_{t+1+i}. \tag{27}
\]

Galí (2013) argues that, if the central bank were to maintain a fixed real interest rate, employment would be independent of wages. As can be seen from (27), this is no longer true in the worker-capitalist model. Unlike the case in a standard EHL model, employment does depend on wages. So there is a direct impact of wage adjustment on employment. A temporary rise

\(^{10}\)This uses the fact that, for example,

\[
E_t \sum_{i=0}^{\infty} \Delta z_{t+i} = E_t \sum_{i=0}^{\infty} (\rho_z - 1) z_{t+i} \\
= - (1 - \rho_z) E_t \sum_{i=0}^{\infty} z_{t+i} \\
= - (1 - \rho_z) \sum_{i=0}^{\infty} \rho_z^i z_t = -z_t.
\]
in wages increases employment. To see this, suppose

\[ \hat{\omega}_t = \rho_{\omega} \hat{\omega}_{t-1} + \zeta \hat{\omega}_t, \quad 0 < \rho_{\omega} < 1 \]

is an exogenous process. Then (27) becomes,

\[
\hat{n}_t = -\left( \frac{1}{1 - a} \right) G \left( \frac{1}{\sigma} \right) \mathbb{E}_t \sum_{i=0}^{\infty} (\eta_{t+i} - \mathbb{E}_t \pi_{t+1+i} - \rho) + \left( \frac{1}{1 - a} \right) G \left( \frac{\omega N}{C} \right) \tau^w_t - z_t + \left( \frac{\omega N}{C} \right) \omega_t.
\]

Employment is increasing in the real wage. Furthermore, a wage subsidy increases employment (as long as \( \delta < 1 \)), i.e., as long as the wage subsidy is partially financed through lump-sum taxes on capitalists. If \( \delta < 1 \), an increase in the tax on profits also increases employment as the revenue from the profits tax is (partially) used to reduce the lump-sum tax on workers. This increases consumption by workers and the resulting rise in aggregate demand boosts employment.

### 3 Numerical exercises

In this section, I investigate the properties of a calibrated version of the model. The basic EHL model is used as a benchmark for comparison. As baseline values for the model’s parameters, I adopt the calibration employed by Galí (2013) and the basic policy coefficients from Taylor (1993). These values are reported in Table 1.
Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma^c$</td>
<td>1/EIS capitalists</td>
</tr>
<tr>
<td>$\sigma^w$</td>
<td>1/EIS workers</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1/wage elasticity</td>
</tr>
<tr>
<td>$\theta^p$</td>
<td>Goods demand elasticity</td>
</tr>
<tr>
<td>$\theta^w$</td>
<td>Labor demand elasticity</td>
</tr>
<tr>
<td>$\varphi^p$</td>
<td>Calvo price adjustment</td>
</tr>
<tr>
<td>$\varphi^w$</td>
<td>Calvo wage adjustment</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Policy response, inflation</td>
</tr>
<tr>
<td>$\phi_\delta$</td>
<td>Policy response, output</td>
</tr>
</tbody>
</table>

3.1 Determinacy

The presence of rule-of-thumb households similar to the workers in the present model has potential implications for the class of interest rate rules consistent with a locally unique stationary rational expectations equilibrium. For example, Galí et al. (2004) showed that a stronger response to inflation than implied by the standard Taylor Principle ($\phi_\pi > 1$) may be required to ensure determinacy.\(^{11}\) Colciago (2011) demonstrates that the presence of wage stickiness serves to restore the traditional results on the Taylor Principle by reducing the volatility of real wages (which feed through directly into consumption volatility with rule-of-thumb households), and Natvik (2009) and Natvik (2012) show how determinacy is affected by the size of government consumption and the role played by wealth inequality.

The results of Colciago (2011) on determinacy carry over to the worker-capitalist model, with the Blanchard-Kahn conditions for a locally unique rational expectations equilibrium holding for $\phi_\pi > 1$ when $\phi_y = 0$. As the response to output rises above zero, the critical value of $\phi_\pi$ that ensures determinacy falls below one, consistent with Bullard and Mitra (2002). Results are, however, sensitive to the degree of wage frigidity. Figure 1 shows the determinacy region when wages are much more flexible than under the baseline calibration ($\varphi^w = 0.25$ as opposed to $\varphi^w = 0.75$ in the baseline calibration). Now, even responding strongly to output is not consistent with determinacy unless $\phi_\pi > 1$. And weak responses to both inflation and

\(^{11}\)See also Natvik (2009) who shows how, in the presence of rule-of-thumb households, determinacy can be affected by the size of government consumption.
output no longer results in multiple equilibria. Instead, this case results in too many roots outside the unit circle; no solution exits.

3.2 Impulse responses

3.2.1 Demand and policy shocks

A useful way to begin an evaluation of the behavior of the worker-capitalist model is to compare the effects of a demand shock in the model with the results from the EHL model. The demand shock is modelled as a preference shock that increases the marginal utility of capitalists’ current consumption. This is equivalent, in the EHL model, to the demand shock considered by Galí (2013) which increases the marginal utility of current consumption while leaving the marginal rate of substitution between leisure and consumption unchanged. The shock is assumed to be persistent, with an AR(1) coefficient of 0.9. The effects of such a shock in the EHL model are shown by the dotted line in Figure 2 (which replicates Galí’s figure 11, p. 985). The solid line shows the results in the worker-capitalist model (WCM). The chief differences are in the weaker output response and stronger response of inflation in the WCM model, and the higher peak increase and greater persistence of the real wage relative to EHL. These differences arise primarily because, with the baseline parameter calibrations (specifically \( \sigma^{w} \) close to 1), the real wage has only a weak direct negative effect on wage inflation, unlike the case in the EHL model. This reflects offsetting wage effects on the gap between the marginal rate of substitution between leisure and consumption by workers and the real wage. With log preferences, the effect of a rise in \( \omega_{t} \) on workers’ consumption increases the MRS by the exact percentage amount by which the real wage rises, leaving the gap between the two unaffected. In the EHL model, the rise in \( \omega_{t} \) dampens wage inflation and therefore firms’ marginal costs rise less, reducing the inflationary effects of the shock.

Figure 3 shows the impulse responses to a persistent contractionary monetary policy shock in the two models.\(^{12}\) The negative real effects of the policy shock are larger in the worker-capitalist model. The fall in the real wage and employment (not shown, but employment moves in proportion to output) depresses consumption by workers, thus generating a multiplier effect on spending. As the lower panels of the figure show, consumption of workers and capitalists are affected differentially by the monetary policy shock; contractionary monetary policy increases consumption of capitalists and depresses consumption of workers. As reflected in the decline in inflation, real marginal costs for firms fall in the face of the interest rate rise, and this

\(^{12}\)The AR(1) coefficient for the policy shock is set at 0.75.
increases profit. Interestingly, the impulse responses for worker and capitalist consumption are (roughly) similar to the effects of contractionary monetary policy on low-net worth and high-net worth households reported by Gorodnichenko et al. (2012) (see their figure 9, p. 41).

3.2.2 Productivity shock

The effects of a positive and persistent productivity shock are shown in figure 4. This shock has the same effect on flex-price and wage output in each model. In the EHL model, output rises but by less than flex-output does, so the output gap falls. Hours decline as do price inflation and wage inflation. The former falls more than the latter, so the real wage rises. In the worker-capitalist model, in contrast, the drop in employment reduces aggregate demand through the fall in workers’ consumption. Because of sticky prices, this fall in demand amplifies the decline in employment and results in an actual decline in output and a large, negative effect on the output gap. Consumption of workers follows the path of hours closely, as the decline in real wages is quite small. In contrast, consumption of capitalists rises significantly in the face of the positive productivity shock.

3.2.3 Markups, payroll taxes and wage subsidies

In the EHL model, markup and distortionary fiscal shocks have similar effects. For example, price markups and the payroll tax appear only in the form \( \hat{\mu}_t^p + \tau_t^p \) and so have equivalent effects macroeconomic effects. Similarly, wage markups and wage subsidies enter the EHL model in the form \( \hat{\mu}_t^w - \tau_t^w \) so that a negative wage markup shock has the same effects as a positive wage subsidy. The effects of markup shocks, payroll tax shocks, and wage subsidy shocks may differ in the worker-capitalist model depending on how lump-sum taxes are adjusted to maintain a balanced fiscal budget. If taxes on workers are adjusted, for example, the effects of a wage subsidy on aggregate demand would be offset by the rise in taxes on workers to finance the wage subsidy.\(^{13}\) In the baseline calibration, \( \delta = 1 \) and all adjustment occurs through adjustments of the lump-sum tax on capitalists.

Wage and price markup shocks  Wage markup shocks have the same negative effect on output with flexible prices and wages in both models. When prices and wages are sticky, the fall in employment reduces total labor income and causes the consumption of workers to

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\(^{13}\)The importance of distributional effects for results on fiscal multipliers in models with rule-of-thumb households is discussed in Natvik (2012).
fall. This in turn generates a drop in aggregate demand, an affect absent in the EHL model. Consequently, output (and employment) fall more in the worker-capitalist model than in the EHL model.

**Price markup and payroll tax shocks**  A payroll tax shock is equivalent to a price markup shock in the EHL as only $\tau_t^p + \mu_t^p$ appears in the equilibrium conditions. When $\hat{p}_t$ adjusts to balance the fiscal budget, then only $\tau_t^p + \mu_t^p$ matters in both models, but even so, the effects of a positive innovation to $\tau_t^p + \mu_t^p$ are quite different in the two models, as shown in figure 6. In the baseline calibration, $\bar{\sigma}^w = \sigma (\omega N/C^w)$ is close to 1, so, from (24), employment and output are little affected by price markup shocks. In the EHL model, such shocks reduce output when prices and wages are flexible. When they are sticky, price markup shocks have the usual effect in EHL, but in WCM the rise in markups increases capitalists’ consumption (which is not fully offset by monetary policy under the assumed Taylor rule) and aggregate demand rises. This increases output.

Results differ when the payroll tax revenue is used to reduce the lump-sum taxes on workers, as shown in figure ?? . Now, consumption of workers rises and this increases aggregate demand, leading to larger increases in output and employment.

**Wage subsidies**  In EHL, a wage subsidy increases labor supply. With flexible wages and prices, this increases output and the real wage falls. With sticky wages and prices, the wage subsidy causes a negative output gap and reduces wage inflation. The resulting decline in the real wage lowers firms’ marginal costs and inflation falls. The overall effects in the EHL model appear small, however, compared to the response of the economy to an increase in the wage subsidy in the worker-capitalist model. With sticky prices and wages, actual output increases substantially as the wage subsidy acts directly to boost spending by workers. Consumption of capitalists falls as they bear the tax needed to finance the subsidy. In contrast, then the wage subsidy is financed by a tax on workers, their is no longer a direct effect on workers’ income and spending. The aggregate effects on output are close to zero.

4 Wage flexibility

If output is constrained by aggregate demand and wages have a direct effect on spending, a fall in real wages could reduce labor demand and exacerbate unemployment rather than reduce it. In the face of a negative demand shock, sticky wages might help stabilize the economy.
by limiting the decline in wage income and workers' spending. The feedback or multiplier effect that arises when current income directly affects current spending could, in principle, be neutralized by monetary policy to the extent that the central bank is not constrained by the zero lower bound on its policy interest rate. If the initial shock to the economy arose from an exogenous fluctuation in aggregate demand and the economy were away from the zero lower bound, the monetary policy could simply output; no adjustment of wages or prices would be required. If monetary policy failed to fully insulate spending in the face of such a shock, then the potential feedback from a fall in labor demand to wages, labor incomes and spending would come into play.

While wage flexibility might be irrelevant if the economy is subject only to demand shocks and these are neutralized by monetary policy, the degree of wage flexibility will be relevant in the face of other shocks, such as productivity and markup shocks. For all such shocks, however, the effects on output, employment, price and wage inflation will be dependent on the manner in which monetary policy reacts. To focus on the role of wage flexibility, therefore, I consider the responses to shocks holding the monetary rule fixed while varying the Calvo parameter $\varphi^w$.

Figure 10 shows the response to an aggregate demand shock for $\varphi^w = 0.4$, 0.6, and 0.75 (the baseline value). Recalling that an increase in $\varphi^w$ implies an increase in wage rigidity, output and employment are significantly more stable when wages are more flexible. Thus, despite the fact the model builds in a labor-income spending multiplier, and the real wage rises more in the face of the positive demand shock when wages are more flexible, wage flexibility reduces the overall responses of employment and output. The lower left plot in the figure shows that the response of consumption spending by workers is largest when wages are the most rigid (i.e., when $\varphi^w = 0.75$), and this accounts for the larger expansion of output and employment. Conversely, given the linearized model, a negative demand shock causes a larger decline in employment and workers' consumption when wages are relatively rigid. Of course, as wages becomes more rigid, the impact of the demand shock on firms' marginal costs and therefore on inflation is dampened.\footnote{\cite{Coibion et al. (2012)} show that downward nominal wage rigidity can be stabilizing at the zero lower bound on nominal interest rates by reducing the chances the economy experiences a destabilizing deflation. It does so by limiting the fall in real marginal costs that would otherwise occur.}

The effects of wage flexibility on the economy's response to a productivity shock are shown in figure 11. Here, both output and inflation are more stable when wages are more rigid. However, the output gap, hours, and worker consumption are less stable when wages are more
rigid.

In standard new Keynesian models, markup shocks in labor and product markets are inefficient shocks that pose trade-offs for monetary policy makers. The effects of wage rigidity on the responses to price market shocks are similar to those for productivity shocks. Greater wage flexibility dampens the effects of price-markup shocks on output, and employment. Not surprisingly, wage inflation becomes more volatile in response to a price markup shock as the Calvo parameter falls. The effects of wage rigidity on the response to wage markup shocks is shown in figure 12. Greater wage flexibility increases the response of the output gap, employment, inflation, and wage inflation to wage-markup shocks. Thus, except for the case of wage markup shocks, greater wage flexibility does not represent an unambiguous improvement from the perspective of a typical flexible-inflation-targeting central bank.

5 Welfare and wage rigidity

The previous sections have employed a simple Taylor rule to represent monetary policy. Because increases in wage flexibility increased the volatility of inflation and wage inflation in the face of productivity, price markup, and wage markup shocks and decreased the volatility of the output gap in the case of the first two of these shocks, any conclusions about the costs of wage rigidity will depend on the relative weights given to the volatility of output, inflation, and wage inflation. In this section, I employ a second order approximation to the welfare of workers and capitalists to assess the implications of wage rigidity. I do this first under the assumption that monetary policy continues to be characterized by a Taylor rule. I then reassess the costs of wage rigidity under an optimal time-consistent monetary policy.

I define social welfare as the sum of the welfare of workers and capitalists, divided by total population. Thus,

$$\begin{align*}
W_t &= \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \int \frac{(C_{t+i}^c(h))^{1-\sigma^c}}{1 - \sigma^c} dh + \int \left[ \frac{(C_{t+i}^w(s))^{1-\sigma^w}}{1 - \sigma^w} - \frac{\chi H_{t+i}(s)^{1+\eta}}{1 + \eta} \right] ds \right\}. \quad (28)
\end{align*}$$

The Appendix derives a second order approximation to $W_t$ around an efficient steady state and shows that optimal policy should minimize

$$\begin{align*}
-\frac{1}{2} \left( \frac{\beta^w}{\kappa_p} \right) E_t \sum_{i=0}^{\infty} \beta^i L_{t+i},
\end{align*}$$

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where
\[
L_t = \pi_{t+1}^2 + \lambda_x x_{t+1}^2 + (\lambda_{1,\pi^w} + \lambda_{2,\pi^w}) \left( \pi_{t+1}^w \right)^2 + \lambda_c \left( \hat{c}_t^e - c_t^w \right)^2
\]  
(29)
and
\[x_t \equiv \hat{y}_t - \hat{y}_t^t,\]
\[
\lambda_x \equiv \left( \frac{\kappa_p}{\theta^p} \right) \left( \frac{a + \eta + \sigma(1 - a)}{1 - a} \right)
\]
\[
\lambda_{1,\pi^w} \equiv (1 - a) \left( \frac{\theta^w}{\kappa_w} \right)
\]
\[
\lambda_{2,\pi^w} \equiv \left( \frac{(1 - \theta^w)^2}{[1 - \hat{d}^{\pi^w} + (\hat{d}^{\pi^w} + \eta) \theta^w]} \right) \left( \frac{\kappa_p}{\theta^p} \right) \left( \frac{\theta^w}{\kappa_w} \right)
\]
\[
\lambda_c \equiv \sigma \left( \frac{\kappa_p}{\theta^p} \right) \left( \frac{C^c}{Y^s} \right) \left( \frac{C^w}{Y^s} \right).
\]

The weights \(\lambda_x\) and \(\lambda_{1,\pi^w}\) are identical in form to those appearing on the output gap and wage inflation terms in the standard sticky-price, sticky wage new Keynesian model. In the worker-capitalist model, an additional weight, \(\lambda_{2,\pi^w}\), is given to wage inflation volatility.\(^\text{15}\) This additional cost arises because relative wage dispersion generates an inefficient dispersion of consumption across workers. Further, the final terms in the loss function reflects imperfect risk sharing across workers and capitalists. In an efficient equilibrium, consumption for the two groups would move together.\(^\text{16}\)

To evaluate \(L_t\), I consider values of \(\varphi^w\) from 0.25 to 0.95. By fixing the parameters in the policy rule while varying the degree of wage rigidity, the exercise serves to isolate the effects of wage rigidity without compounding the effects of any endogenous response of policy to changes in the economy’s structure. I then repeat this exercise under the optimal, time-consistent monetary policy.

Figure 13 shows the four components of \(L_t\) due to inflation, wage inflation, output gap volatility, and asymmetric consumption volatility in the presence of persistent productivity shocks. When wages are fairly flexible, the bulk of the loss arises from inflation variability. This falls significantly as wages become more rigid, as does the contribution of wage inflation volatility to loss. The other two components are same, but the cost of output gap volatility increases as wages become more rigid. However, the over all loss declines dramatically as

\(^{15}\)While \(\lambda_{1,\pi^w}\) is identical in form to the weight on wage inflation volatility in EHL, its value differs as \(\kappa_w\) has \(1 - \hat{d}^{\pi^w} (1 - \theta^w) + \eta \theta^w\) in the denominator rather than \(1 + \eta \theta^w\) as in EHL.

\(^{16}\)Amato and Laubach (2003) derive a quadratic approximation to welfare for a rule-of-thumb model, but in their version, rule-of-thumb households set current consumption equal to lagged aggregate consumption.
wages become more rigid. This, the greater stability of $\pi$ and $\pi^w$ in response to productivity shocks as $\varphi^w$ increases that was seen in figure 11 dominates in welfare terms over the greater output gap instability that occurs when wages are stickier.

A similar decomposition for the case of price markup shocks is shown in figure 14 and for wage markup shocks in 15) Loss is increasing in wage rigidity initially and peaks at $\varphi^w = 0.75$, the value used in the baseline calibration. As wage rigidity increases beyond that point, loss declines due to the fall in costs associated with price and wage inflation.

The results in figures 13 – 15 were conditional on a fixed rule describing monetary policy. Using the loss function $L_t$, one can evaluate loss as a function of wage rigidity under an optimal monetary policy that will itself depend on the degree of wage rigidity. In all cases, outcomes under an optimal, time-consistent monetary policy are very different than those under a fixed Taylor Rule as seen in figures 16 – 18. Loss is monotonically increasing in wage rigidity, a result consistent with the findings of Galí (2013) for the EHL model. While the costs associated with the volatility of wage inflation fall as wages become more rigid, this is more than offset by rising volatility in inflation and the costs of asymmetric consumption movements.

In contrast, loss is monotonically decreasing in wage rigidity under optimal policy when fluctuations are driven by price markup shocks, as shown in figure 17. For all values of $\varphi^w$, the costs of price inflation volatility dominate the total loss. For wage markup shocks, welfare is monotonically increasing in wage rigidity as was the case for productivity shocks. While the costs of wage inflation volatility essentially goes to zero as wages becomes more rigid, the costs of volatility associated with price inflation dominates.

6 Conclusions

In this paper, I have developed a simply variant of a rule-of-thumb model that distinguishes between workers earning labor income and unable to participate in financial markets and capitalists who own the economy’s productive technology and can engage in financial markets to share consumption risk. While most models that include rule-of-thumb households have focused on government spending shocks, I consider a broader range of shocks and show how the effects of wage subsidies and payroll taxes depend on the way lump-sum taxes on workers and/or capitalists are adjusted to balance the government’s budget, illustrating how the distributional assumptions in standard models and in rule-of-thumb models are not innocuous. This is a point Natvik (2012) has made with respect to fiscal spending shocks.

When worker supply differentiated labor types and wages are sticky, relative wage disper-
sion generates a cross-sectional variance of consumption among workers. This represents an additional cost of wage inflation absent in representative agent models. It is also absent in traditional rule-of-thumb models which usually assume consumption is equalized across rule-of-thumb households. Welfare costs also arise from imperfect risk sharing between workers and capitalists. When workers and capitalists have identical preferences over consumption, a quadratic approximation to welfare can be obtained that is directly comparable to quadratic loss functions in other new Keynesian models.

The present model is very stylized, as are standard representative agent models. The structure of the model serves to highlight the role the distribution of income between labor and capital may play for the way aggregate shocks affect output, employment, and inflation. The presence of workers who consume wage income introduces a direct aggregate demand channel for wages and employment that leads to a Keynesian multiplier effect. In the context of the model, greater wage flexibility does help to stabilize output and employment; whether it stabilizes the consumption of workers depends on the underlying source of the shock.

The consequences of wage rigidity depend critically on the conduct of monetary policy and on the source of economic fluctuations. When monetary policy is determined by a fixed Taylor rule, welfare is improved when wages become less flexible. However, this result is reversed for productivity and wage markup shocks when policy is instead set optimally. Under an optimal monetary policy, the demand channel of labor income would be offset by an appropriate adjustment of the interest rate. At the ZLB, policy would not be able to offset the multiplier effect of a fall in labor income. Away from the ZLB, the dichotomy between workers and capitalists is still relevant in affecting the behavior of wage inflation, real wages, and price inflation.

7 Appendix

7.1 The log-linearized model

The WCM log-linear model consists of 13 equations in $\hat{c}^c, \hat{c}^w, \hat{y}, \hat{n}, i, \pi, \lambda, \hat{m}, \hat{s}, \hat{\omega}, \pi^w, \hat{\pi}^w, \hat{\pi}^c, \hat{y}^f$:

$$\hat{y}_t = \left( \frac{C^c}{C^c_t} \right) \hat{c}^c_t + \left( \frac{C^w}{C^w_t} \right) \hat{c}^w_t$$

$$\hat{c}^c_t = E_t \hat{c}^c_{t+1} - \left( \frac{1}{\sigma^c} \right) (i_t - E_t \pi_{t+1} - \rho).$$
\[
\begin{align*}
\dot{c}_t^w &= \left( \frac{\omega H}{C^w} \right) (\dot{\omega}_t + \dot{n}_t + \tau_w^t) - \left( \frac{T^w}{C^w} \right) \dot{t}_t^w \\
i_t &= \phi_{\pi} \pi_t + \phi_{\pi^w} \pi_t^w + \phi_y \dot{y}_t \\
(1 - a) \dot{n}_t &= \dot{y}_t - \dot{z}_t \\
\dot{m_c}_t &= \dot{\omega}_t + \tau_p^t - \dot{z}_t + a \dot{n}_t \\
\dot{\pi}_t &= \beta E_t \pi_{t+1} + \kappa_p (\dot{m_c}_t + \dot{\mu}_t^p) , \\
\dot{m_r s}_t &= \eta \dot{n}_t + \sigma^w \dot{c}_t^w \\
\pi^w_t &= \beta E_t \pi_{t+1}^w + \kappa_w (\dot{m_r s}_t - \tau^w_t + \dot{\mu}_t^w - \dot{\omega}_t) \\
\omega_t &= \omega_{t-1} + \pi_t^w - \pi_t \\
\dot{y}_t^f &= \phi^w \{ (1 + \eta) z_t + (1 - a) \left[ (1 - \hat{\sigma}^w) \left( \tau^w_t - \tau^p_t - \mu_t^p \right) \right. \\
&\left. - \hat{\mu}_t^w + \sigma^w \dot{t}_t^w \right] \} \\
\left( \frac{T^w}{C} \right) \dot{t}_t^w &= \delta \left( \frac{\omega H}{C} \right) \left( \tau^w_t - \tau^p_t \right) \\
\left( \frac{T^c}{C} \right) \dot{c}_t &= (1 - \delta) \left( \frac{\omega H}{C} \right) \left( \tau^w_t - \tau^p_t \right)
\end{align*}
\]

where
\[
\hat{\sigma}^w = \sigma \left( \frac{\omega H}{C^w} \right)
\]
\[
\kappa_p = \frac{(1 - \varphi^p) \left( 1 - \beta \varphi^p \right)}{\varphi^p} \left( \frac{1 - a}{1 - a + a \theta^p} \right); \\
\kappa_w = \frac{(1 - \varphi^w) \left( 1 - \beta \varphi^w \right)}{\varphi^w} \left( \frac{1}{1 - \hat{\sigma}^w + (\hat{\sigma}^w + \eta) \theta^w} \right); \\
\phi^w = \frac{1}{\eta + \hat{\sigma}^w + a (1 - \hat{\sigma}^w)}.
\]

### 7.2 The standard EHL case

The standard linearized model of Erceg, Henderson and Levin takes the form
\[
\dot{y}_t = E_t \dot{y}_{t+1} - \left( \frac{1}{\sigma} \right) (\dot{i}_t - E_t \pi_{t+1} - \rho).
\]
\[ i_t = \phi_i \pi_t + \phi_{i\tau} \pi_t^{\tau} + \phi_{i\eta} \gamma_t \]

\[ (1 - a) \hat{n}_t = \hat{y}_t - \hat{z}_t \]

\[ \lambda_t = \hat{\omega}_t + \tau_t^p - \hat{z}_t + a \hat{n}_t \]

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa_p \left( \hat{\lambda}_t + \hat{\mu}_t^p \right), \]

\[ mrs_t = \eta \hat{n}_t + \sigma \hat{y}_t \]

\[ \pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w \left( mrs_t - \tau_t^w + \hat{\mu}_t^w - \hat{\omega}_t \right) \]

\[ \omega_t = \omega_{t-1} + \pi_t^w - \pi_t \]

\[ \gamma_t^{sf} = \phi \left[ (1 + \eta) \gamma_t + (1 - a) \left( \tau_t^w - \tau_t^p - \hat{\mu}_t^p - \hat{\mu}_t^w \right) \right] \]

where

\[ \kappa_w = \frac{(1 - \varphi^w)(1 - \beta \varphi^w)}{\varphi^w} \left( \frac{1}{1 + \eta \theta^w} \right); \]

\[ \phi \equiv \frac{1}{\eta + \sigma + a (1 - \sigma)}. \]

These nine equations yield the standard Erceg et al. (2000) (EHL) model that can be solved for \( \hat{y}_t, \pi_t, \pi_t^w, \omega_t, i, \lambda_t, \gamma_t^{sf}, \hat{n}_t \) and \( mrs_t \).

**References**


Coibion, O., Gorodnichenko, Y., Wieland, J. 2012. The Optimal Inflation Rate in New Key-


Gorodnichenko, Y., Kueng, L., Silvia, J., Coibion, O. 2012. Innocent Bystanders? Monetary Policy and Inequality in the US.


Figure 1: Determinacy regions for worker-capitalist model when $\phi^w = 0.25$, other parameters fixed at their baseline values. Key: Green = unique equilibrium, Blue = multiple equilibria, Brown = no equilibria.

Figure 2: Demand shock (Baseline parameters)
Figure 3: Policy shock (baseline parameters)

Figure 4: Productivity shock (baseline parameters)
Figure 5: Wage markup shock (baseline parameters)

Figure 6: Price markup or payroll tax shock (baseline parameters) when revenue is used to reduce tax on capitalists.
Figure 7: Wage subsidy financed by tax on capitalists (baseline parameters)

Figure 8: Payroll tax shock in WCM. Solid line: $\delta = 0$ and payroll tax revenues offset by adjusting lump-sum taxes on capitalists. Dashed line: $\delta = 1$ and payroll tax revenues offset by adjusting lump-sum taxes on workers.
Figure 9: Wage subsidy shock in WCM. Solid line: $\delta = 0$ and wage subsidy financed through lump-sum taxes on capitalists. Dashed line: $\delta = 1$ and wage subsidy financed through lump-sum taxes on workers.

Figure 10: The effects of wage rigidity on the response to a demand shock: $\varphi_1^w = 0.75$, $\varphi_2^w = 0.6$, $\varphi_3^w = 0.4$. 
Figure 11: The effects of wage rigidity on the response to a productivity shock: $\phi^w_1 = 0.75$, $\phi^w_2 = 0.6$, $\phi^w_3 = 0.4$.

Figure 12: The effects of wage rigidity on the response to a wage-markup shock: $\phi^w_1 = 0.75$, $\phi^w_2 = 0.6$, $\phi^w_3 = 0.4$. 
Figure 13: Components of loss to productivity shocks as function of wage rigidity under a Taylor Rule: inflation (blue), wage inflation (aqua), output gap (yellow) and asymmetric consumption (brown).

Figure 14: Components of loss under Taylor rule to price markup shocks as function of wage rigidity: inflation (blue), wage inflation (aqua), output gap (yellow) and asymmetric consumption (brown).
Figure 15: Components of loss under Taylor rule to wage markup shocks as function of wage rigidity: inflation (blue), wage inflation (aqua), output gap (yellow) and asymmetric consumption (brown).

Figure 16: Components of loss to productivity shocks as function of wage rigidity under optimal discretion: inflation (blue), wage inflation (aqua), output gap (yellow) and asymmetric consumption (brown).
Figure 17: Components of loss to price markup shocks as function of wage rigidity under optimal discretion: inflation (blue), wage inflation (aqua), output gap (yellow) and asymmetric consumption (brown).

Figure 18: Components of loss under optimal discretion to wage markup shocks as function of wage rigidity: inflation (blue), wage inflation (aqua), output gap (yellow) and asymmetric consumption (brown).