BHARAT TREHAN
CARL E. WALSH

Testing Intertemporal Budget Constraints: Theory and Applications to U.S. Federal Budget and Current Account Deficits

1. INTRODUCTION

RECENT RESEARCH HAS EMPHASIZED the role of intertemporal budget constraints in a variety of contexts. For example, Sargent and Wallace (1981) examined the implications of the government's budget constraint for the behavior of monetary and fiscal authorities, while Obstfeld (1986) examined the implications of intertemporal current account balance for the behavior of exchange rates. However, until just a few years ago, researchers were content to assume the existence of such constraints, with little effort being directed towards determining whether the data generating processes were consistent with intertemporal budget balance.

More recently, several different researchers have devised and implemented tests of the intertemporal budget constraint in a variety of different contexts. Hamilton and Flavin (1986), Hansen, Roberds, and Sargent (1987),[1] Trehan and Walsh (1988), Wilcox (1989), Hakkio and Rush (forthcoming), and Haug (forthcoming) develop and implement tests related to the government's budget constraint. Campbell and Shiller (1987) test present-value models of stock and bond prices that are formally equivalent to tests of intertemporal budget balance.

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[1]Hansen, Roberds, and Sargent's empirical results are contained in a handout circulated when their paper was presented at a Federal Reserve System conference in April 1988.

BHARAT TREHAN *is senior economist at the Federal Reserve Bank of San Francisco.* CARL E. WALSH *is associate professor of economics, University of California, Santa Cruz, and visiting scholar at the Federal Reserve Bank of San Francisco.*

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With the exceptions of Hamilton and Flavin (1986) and Wilcox (1989), all the cited papers develop their tests by exploiting the presence, under intertemporal budget balance, of a cointegrating relationship linking net-of-interest expenditures, revenues, interest payments, and the outstanding stock of debt. These tests generally require the auxiliary assumptions of a constant expected real rate of interest and difference stationarity of the revenue and expenditure processes.[2]

In this paper we extend this work in two directions. First, we relax the requirement that expenditures and revenues be difference stationary, while maintaining the assumption of a constant expected real rate of interest, and show that the cointegration test continues to be valid as long as a quasi difference of the net-of-interest deficit is stationary. The formulation we employ has an added advantage in that it leads naturally to an error correction specification for the joint adjustment of the revenue and expenditure processes. Specifically, if the interest-inclusive deficit is stationary, intertemporal budget balance holds, and this measure of the deficit is the appropriate error correction term. Second, we examine what happens if the expected real rate of interest is not a constant. We show that the cointegration test is generally no longer valid, but the test developed in Trehan and Walsh (1988) is still applicable. Specifically, we show that, as long as the expected real rate of interest is positive, intertemporal budget balance holds if the *inclusive*-of-interest deficit is stationary. We then empirically examine two budget balance questions that have been of concern recently. Does the size of the federal government's deficits imply that the intertemporal budget balance constraint is being violated? Do the recent U.S. current account deficits imply that foreigners now hold "too large" an amount of U.S. assets?

While several recent studies have attempted to assess the consistency of the time path of federal government expenditures, revenues, and debt with the assumption of intertemporal budget balance, different researchers have arrived at different conclusions. For instance, Trehan and Walsh (1988) were unable to reject the hypothesis of intertemporal budget balance using U.S. data from 1890 to 1986; Haug (forthcoming) reached the same conclusion using quarterly U.S. data from 1960 to 1986. By contrast, Wilcox (1989), Hakkio and Rush (forthcoming), and Hansen, Roberds, and Sargent (1987) conclude that the postwar U.S. data are inconsistent with this hypothesis. One obvious problem in reconciling these results is the different sample periods employed by different researchers. In addition, these papers also employ different tests. For instance, Hansen, Roberds, and Sargent maintain the auxiliary assumption that the expected real rate of interest was constant over their sample (1948–1986), while Hakkio and Rush allow for a stochastic real rate as long as it is stationary. Wilcox avoids assumptions about the expected real rate process by discounting the government's debt back to a fixed reference date using ex-post real rates and examining the behavior of the resulting discounted debt series.

To allow for direct comparison with existing tests, we apply the test developed in

[2]Not all papers have assumed a constant expected real rate. See Wilcox (1989) and Hakkio and Rush (forthcoming). We discuss their tests below. While Hamilton and Flavin (1986) do not discuss their test in these terms, their approach can also be expressed in terms of cointegration.

this paper to a postwar U.S. data set that has already been used in several published studies. Since we are able to reject the null hypothesis of no intertemporal budget balance using postwar data, we argue that the various assumptions about the real interest rate implicit in the previous tests have also played a role in producing the conflicting results that have been reported.

A parallel situation occurs in the analysis of net international indebtedness. Intertemporal balance of payments equilibrium requires that the current net stock of foreign assets equal the discounted present value of current and expected future current account deficits. We show that the approach developed in this paper can be used to test whether the data are consistent with intertemporal balance of payments equilibrium.

While the assumption of intertemporal current account balance is widely used to derive restrictions on the behavior of asset holdings, exchange rates, etc., we are unaware of any empirical tests that are similar to those carried out for the government's budget balance. Krugman (1988) provides some discussion of the issue of sustainability, but his conclusions are based on a simple simulation model. We use data on the net international investment position of the United States over the period 1946–1987 to test for sustainability, and are able to reject the hypothesis that the recent U.S. current account deficits have been "too large."

The rest of the paper is organized as follows. In section 2, we derive a necessary and sufficient condition for intertemporal budget balance under the assumption that the expected real rate of interest is constant and compare this to other tests of present value relationships. In section 3 we relax the constant expected real rate assumption and show that stationarity of the inclusive-of-interest deficit implies that intertemporal budget balance holds as long as the expected real rate of interest is positive. Section 4 contains our test of the government's budget balance, using data constructed by Hamilton and Flavin (1986) for the period 1960 to 1984. In section 5, we use data over 1946 to 1987 to test for intertemporal current account balance. Section 6 concludes.

2. CONSTANT EXPECTED REAL RATE

In this section, we set out a simple budget relationship and derive the restrictions that must be satisfied for sustainability. We call a budget process sustainable if the expected present discounted value of the implied future stock of debt converges to zero. Requiring that budget processes be sustainable rules out Ponzi schemes in which debt is continually rolled over. In the absence of this condition, the government (for instance) could cut current taxes, while leaving current expenditures as well as all future taxes and expenditures unchanged.[3]

[3]For a dynamically efficient economy with zero population growth, McCallum (1984) shows that a violation of this condition is inconsistent with optimizing behavior on the part of the holders of the debt. A referee has suggested that this inconsistency may not arise if the economy is dynamically inefficient. In any case, the empirical evidence in Abel et al. (1989) suggests that the U.S. economy is dynamically efficient.

Consider the general budget identity

$$s_t - s_{t-1} = r_t s_{t-1} + d_t \tag{1}$$

where s_t is the real stock of outstanding debt (assets if negative) at the end of period t, r_t is the real rate of interest during the period t, and d_t is the period t net-of-interest "deficit." Equation (1) describes the evolution over time of the stock variable s.

Let I_{t-1} denote the information set of private agents at the beginning of period t. We assume that r_t is stochastic with $E(r_{t+i} \mid I_{t-1}) = r$, for all $i \geq 0$. R_t will be used to denote the realized gross rate of return $1 + r_t$, and its expected value will be denoted by R. Further, we assume that while s_{t-1} is in I_{t-1}, neither r_t nor d_t are in I_{t-1}.

Taking expectations of (1) conditional on the information set I_{t-1} and recursively eliminating future values of the stock of debt yields the intertemporal budget constraint:

$$s_{t-1} = -\sum_{j=0}^{\infty} R^{-(j+1)}E(d_{t+j}|I_{t-1}) + \lim_{j\to\infty} R^{-(j+1)}E(s_{t+j}|I_{t-1}) \ . \tag{2}$$

Under the hypothesis of present value budget balance, the last term in (2) must equal zero:

$$\lim_{j\to\infty} R^{-(j+1)}E(s_{t+j}|I_{t-1}) = 0 \ . \tag{3}$$

If condition (3) is satisfied, the discounted value of the expected future stock of debt converges to zero as the time horizon goes to infinity. Equation (2) then implies that the current outstanding real stock of debt, s_{t-1}, must equal the present discounted value of current and expected future net-of-interest surpluses, $-\Sigma R^{-(j+1)}E(d_{t+j} \mid I_{t-1})$.

If the econometrician has observations on the past history of the joint process $z_t = (s_t, d_t)$, equation (3) can be shown to impose restrictions on the behavior of z_t.[4] The particular restriction on z that we use to generate a test of (3) is an implication of the following proposition:

PROPOSITION 1. *If the evolution of* s_t *is given by* (1) *with* $E(r_{t+1} \mid I_{t-1}) = r$ *for all* $i \geq 0$, *and* $(1 - \lambda L)d_t$ *is a mean zero stationary stochastic process with* $0 \leq \lambda < R$, then (3) *holds if and only if there exists a linear combination of* d_t *and* s_{t-1} *that is stationary.*

[4]The definition of the information set is important. For example, suppose the econometrician's information set is given by $\Omega_{t-1} = \{d_{t-1}, d_{t-2} \ldots\}$, a subset of I_{t-1}. Hansen, Roberds, and Sargent (1987) show that, with respect to the information set Ω, intertemporal budget balance places no testable restrictions on the process characterizing the deficit d_t. That is, given a moving average representation for d_t that fails to satisfy intertemporal budget balance, there exists another moving average representation that satisfies the present value constraint.

Proof. Suppose that intertemporal budget balance holds, and let the deficit process be given by

$$(1 - \lambda L)d_t = A(L)\epsilon_t,^5 \tag{4}$$

where

$$A(L) = \sum_0^\infty \alpha_i L^i, \sum_0^\infty |\alpha_i|^2 < \infty ,$$

and ϵ_t is a q-dimensional martingale difference sequence that generates the time t information set. That is, $I_t = \{\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \ldots\}$. The parameter λ satisfies the restriction $0 \leq \lambda < R$. Using (4) to evaluate (2) when intertemporal budget balance holds, we obtain

$$s_{t-1} = -E\left[\frac{\lambda d_{t-1} + A(L)\epsilon_t}{R} + \frac{\lambda^2 d_{t-1} + \lambda A(L)\epsilon_t + A(L)\epsilon_{t+1}}{R^2} + \ldots |I_{t-1}\right]$$

which reduces to

$$s_{t-1} = -\frac{\lambda}{R - \lambda} d_{t-1} - \frac{1}{R - \lambda} E\left[\sum_0^\infty R^{-j} A(L)\epsilon_{t+j}|I_{t-1}\right] . \tag{5}$$

Using a formula in Sargent (1987) under the assumption that expectations are formed by means of linear projections, (5) becomes

$$s_{t-1} = \frac{-\lambda}{R - \lambda} d_{t-1} - \frac{1}{R - \lambda} B(L)\epsilon_{t-1} ,$$

where

$$B(L) = \frac{L^{-1}(A(L) - A(R^{-1}))}{1 - R^{-1}L^{-1}} .$$

Finally, using (4) again, we have

$$s_{t-1} = -\frac{1}{R - \lambda}(d_t - A(L)\epsilon_t) - \frac{1}{R - \lambda} B(L)\epsilon_{t-1} \tag{6}$$

or

$$s_{t-1} = -\mu d_t + B'(L)\epsilon_t \tag{6'}$$

[5]The requirement that $(1-\lambda)d_t$ be of mean zero can be relaxed slightly; a deterministic term can appear as long as it implies a surplus equal to the interest payment on the initial stock of debt. See Trehan and Walsh (1988) for details.

where $\mu = \frac{1}{R - \lambda}$ and $\Sigma_{j=0}^{\infty} |b_j'|^2 < \infty$. Thus s_{t-1} and d_t are cointegrated with cointegrating vector (1 μ). That is, there exists a linear combination of s and d given by $s_{t-1} + \mu d_t$ that is stationary.

Conversely, assume that d_t and s_{t-1} are cointegrated so that we can write

$$s_{t-1} - \alpha d_t = C(L)\epsilon_t \tag{7}$$

for some constant α and stationary moving average process $C(L)\epsilon_t$.[6] Quasi-difference (7) to get

$$(1 - \lambda L)s_{t-1} = \alpha(1 - \lambda L)d_t + (1-\lambda L)C(L)\epsilon_t$$

or, substituting for $(1 - \lambda L)d_t$,

$$(1 - \lambda L)s_{t-1} = [\alpha A(L) + (1 - \lambda L)C(L)]\epsilon_t = D(L)\epsilon_t \ .$$

It is straightforward to show that

$$\lim_{j\to\infty} R^{-(j+1)}E(s_{t+j}|I_{t-1}) = \lim_{j\to\infty}\left(\frac{\lambda}{R}\right)^{j+1} s_{t-1}$$

$$+ \lim_{j\to\infty} \sum_{t=0}^{j} \frac{\lambda^i}{R^{j+1}} E(D(L)\epsilon_{t+j+1-i}|I_{t-1}) = 0 \ ,$$

for $0 \leq \lambda < R$. Thus, budget balance holds if s and d are cointegrated.

Proposition 1 provides a simple test of intertemporal budget balance in the case where the conditional expectation of the real rate is constant. If d is a stationary stochastic process ($0 \leq \lambda < 1$), budget balance is satisfied if and only if s is also stationary. If d is nonstationary ($1 \leq \lambda <$ R), s must also be nonstationary and there must exist a linear combination of s and d that is stationary.

The usefulness of Proposition 1 depends, in part, on the generality of the process that d is allowed to follow. In fact, our assumption that $(1-\lambda L)d_t$ is stationary encompasses several cases that are of interest. First, if $\lambda = 1$ (that is, if d is difference stationary), equation (6′) reduces to

$$rs_{t-1} = -d_t + rB'(L)\epsilon_t \ .$$

But $d_t + rs_{t-1}$ is just the deficit *inclusive* of interest (evaluated at the expected real rate of interest), so that the test for budget balance reduces to a test for the sta-

[6]Note that the ϵ_t process generates the time t information set. Therefore, both s_{t-1} and $(1-\lambda L)d_t$ can be expressed in terms of $\{\epsilon_t, \epsilon_{t-1}, \ldots\}$.

tionarity of the deficit inclusive of interest.[7] This is the test developed in Trehan and Walsh (1988).

Another interesting case occurs when the exclusive-of-interest deficit is set to cover a fraction of the expected interest payment on the outstanding debt. Consider the following stochastic version of an example in Wilcox (1989). Let

$$d_t = -\beta r s_{t-1} + v_t \tag{8}$$

where $0 < \beta < 1$ and v_t is white noise. Since we are still maintaining the hypothesis that $E(r_{t+i} \mid I_{t-1}) = r$ for all i, let ϕ_t denote $r_t - r$; ϕ_t is a mean zero, serially uncorrelated process. Using (1), it is straightforward to show that

$$(1 - \lambda L)d_t = (1 - RL)v_t - \beta r \phi_{t-1} s_{t-2} , \tag{9}$$

where $\lambda = R - \beta r < R$, so $(1 - \lambda L)d_t$ is a stationary process (Granger and Hallman (1988) and the conditions for Proposition 1 are satisfied. Here d_t and s_{t-1} are cointegrated by assumption since $d_t + \beta r s_{t-1} = v_t$ is stationary. To see that intertemporal budget balance holds, use (8) in (1) to obtain

$$s_t = (1 + r_t - \beta r)s_{t-1} + v_t = \lambda s_{t-1} + \phi_t s_{t-1} + v_t$$

so that $E(s_{t+j} | I_{t-1}) = \lambda^{j+1} s_{t-1}$, and

$$\lim_{j\to\infty} R^{-(j+1)} E(s_{t+j} | I_{t-1}) = \lim_{j\to\infty} \left(\frac{\lambda}{R}\right)^{j+1} s_{t-1} = 0 .$$

Setting $\lambda = 0$ in Proposition 1 above gives us the case considered by Hamilton and Flavin (1986). They draw upon Hamilton and Whiteman's (1985) result that if q differences are sufficient to reduce d to stationarity, then intertemporal budget balance implies that q differences will reduce s to stationarity. Specifically, Hamilton and Flavin find d_t to be stationary, and therefore argue that stationarity of s_t implies that budget balance holds.

An alternative approach to testing present value budget balance has been developed by Campbell and Shiller (1987) and Hansen, Roberds, and Sargent (1987). They note that (2) and (3) imply

$$r s_{t-1} + d_{t-1} = -\sum_{j=0}^{\infty} R^{-j} E[(1 - L)d_{t+j} \mid \Gamma_{t-1}] \tag{10}$$

[7]In our empirical tests we use the actual deficit inclusive of interest, $d_t + r_t s_{t-1}$. This does not invalidate our test as can be seen by noting that $d_t + r_t s_{t-1}$ differs from $d_t + r s_{t-1}$ by $(r_t - r)s_{t-1}$. Under the assumptions of this section $(r_t - r)$ is a white noise process, so its product with s_{t-1} will be stationary. See Granger and Hallman (1988).

where Γ_{t-1} is the information set consisting of lagged observations on s and d. They next assume the VAR representation obtained by inverting the Wold moving average representation for the joint process

$$y_t' = [rs_t + d_t,\ (1 - L)d_t]$$

is of finite order. With this additional assumption, testable cross-equation restrictions on the VAR representation for y_t can be derived.

Write the bivariate VAR system for y_t as

$$y_t = \mathrm{B(L)}y_{t-1} + u_t \tag{11}$$

where B(L) is a 2 × 2 matrix of polynomials in the lag operator L of maximum order $p < \infty$. Now stack the system (11) to form

$$z_t = Az_{t-1} + v_t \tag{12}$$

where $z_t' = [rs_t + d_t, rs_{t-1} + d_{t-1}, \ldots, rs_{t-p+1} + d_{t-p+1}, (1 - L)d_t, \ldots, (1 - L)d_{t-p+1}]$.

Define $g' = (1\ 0 \ldots 0)$ and $h' = (0 \ldots 0\ 1\ 0 \ldots 0)$ such that $g' z_t \equiv rs_t + d_t$ and $h'z_t \equiv (1 - L)d_t$. Now equation (10) implies $g'z_t = rs_t + d_t = -\Sigma_{j=0}^{\infty} R^{-j}E[(1 - L)d_{t+1+j}|\Gamma_t]$, and (12) implies $E[(1 - L)d_{t+1+j}|\Gamma_t] = h'A^{j+1}z_t$. Thus

$$g'z_t = -\sum_{j=0}^{\infty} R^{-j}h'A^{j+1}z_t \equiv -h'A(1 - R^{-1}A)^{-1}z_t\,. \tag{13}$$

which holds for general z_t if and only if

$$g'(1-R^{-1}A) = -h'A\,. \tag{14}$$

Since (14) is an implication of intertemporal budget balance under the assumption of a constant expected real interest rate, it can be tested by estimating the VAR system for y_t. This test is implemented for the government deficit by Hansen, Roberds, and Sargent (1987).

3. VARIABLE REAL RATE

What if the assumption that the expected real rate of interest is constant does not provide a good characterization of the data generating process? In this case, sustainability (or intertemporal budget balance) no longer implies cointegration between the stock of debt and the net-of-interest deficit, and these variables could potentially even be of different orders of integration. Given a strictly positive expected real rate of interest, sustainability requires

$$\lim_{j\to\infty} E({}_t\rho_{t+j}^{-1} s_{t+j} | I_{t-1}) = 0$$

where

$${}_t\rho_{t+j} = \prod_{i=0}^{j} R_{t+i}$$

is the j-period discount factor from time t to $t + j$. In this case, calculations similar to those underlying (5) lead to

$$s_{t-1} = -E\left\{ \left(\frac{1}{{}_t\rho_t} + \frac{\lambda}{{}_t\rho_{t+1}} + \frac{\lambda^2}{{}_t\rho_{t+2}} + \ldots \right) d_t + \left(\frac{1}{{}_t\rho_{t+1}} + \frac{\lambda}{{}_t\rho_{t+2}} + \ldots \right) A(L)\epsilon_{t+1} + \ldots | I_{t-1} \right\}$$

which shows that a constant coefficient cannot be recovered from a regression of s_{t-1} on d_t.

By contrast, the test based on the stationarity of the deficit inclusive of interest continues to be valid. Stationarity of $r_t s_{t-1} + d_t$ ensures that the outstanding stock of debt grows at most according to a linear trend. That is, if $r_t s_{t-1} + d_t = (1-L)s_t$ is stationary around an average value k, then

$$\lim_{j\to\infty} E(s_{t+j}|I_{t-1}) = s_{t-1}^{*} + (j + 1)k ,$$

where s_t^* depends on t but not on j.[8] The discount rate series has the property that

$$E\left(\frac{{}_t\rho_{t+j}}{{}_t\rho_{t+j-1}} |I_{t-1} \right) = 1 + E(r_{t+j}|I_{t-1}) > 1 .$$

Hence, $\{{}_t\rho_{t+j}\}$ grows exponentially. Thus, $\{s_{t+j}\}$ is of smaller order than $\{{}_t\rho_{t+j}\}$; that is to say,

$$\lim_{j\to\infty} E({}_t\rho_{t+j}^{-1} s_{t+j} | I_{t-1}) = 0.$$

[8] s_t^* is simply s_t minus a temporary component. For example, if $(1-L)s_t = \epsilon_t - b\epsilon_{t-1} + k$, $s_t^* = s_t - b\epsilon_t$.

Thus, we have

PROPOSITION 2. *If* R_t *is a stochastic process strictly bounded below by* $1 + \delta$ $(\delta > 0)$ *in expected value and* $(1 - L)s_t$ *is a stationary process, then intertemporal budget balance is satisfied.*

Hakkio and Rush (forthcoming) and Wilcox (1989) develop tests of intertemporal budget balance without requiring that the expected value of the real interest rate be constant. Hakkio and Rush assume that the real interest rate is stationary, and, under the assumption that the expenditure and revenue processes are random walks, show that intertemporal budget balance requires that inclusive-of-interest government expenditures be cointegrated with revenues.

A related test of intertemporal budget balance when the real interest rate is variable has been proposed by Wilcox (1989). He examines the process followed by the stock of debt discounted back to a fixed point in time. Intertemporal budget balance holds if and only if the discounted debt series is stationary with mean zero. Wilcox's test requires a two-step procedure. After using ex-post realized real rates of return to construct a discounted debt series, he tests for stationarity. If nonstationarity is rejected, he tests for a zero mean in both univariate and multivariate representations of the discounted debt process.

To summarize the results of this and the previous section, we have shown that stationarity of the inclusive-of-interest deficit is necessary and sufficient for intertemporal budget balance when the expected real rate of interest is constant and $(1 - L)d_t$ is stationary. A generalization of this condition is that d_t and s_t be cointegrated, a condition that continues to hold if the assumption on d_t is relaxed so that $(1-\lambda L)d_t$ is stationary, as long as $0 \leq \lambda < R$.

But the cointegration test itself does not generalize to the case where the expected real rate of interest is allowed to vary. A similar conclusion holds with respect to the VAR cross-equation restrictions implied by cointegration. By contrast, stationarity of the inclusive-of-interest deficit is sufficient to imply that intertemporal budget balance holds, as long as the expected real rate of interest is positive. It is worth emphasizing that this last result does not rely on any assumptions about the individual s and d processes.

4. EXAMPLE 1: FEDERAL BUDGET DEFICITS

As mentioned in the introduction, there are a number of studies that examine whether the recent federal budget deficits violate intertemporal budget balance. The results are generally contradictory. This section begins with a review of this work. Our survey reveals that tests that require the assumption of a constant real rate generally lead to a rejection of intertemporal budget balance. However, these studies are also marked by the use of different sample periods. To avoid complicating the issue even further, we use a sample period that has been used before. We find that the two tests discussed above lead to conflicting results, even after we control for sample period. The two tests can differ either if $(1 - \lambda L)d_t$ is nonstationary or if the expected real rate of interest is not constant. Since $(1 - L)d_t$ appears stationary, we conclude that the government's budget does not violate intertemporal budget balance, but that the assumption of a constant real rate is at odds with the data.

Hamilton and Flavin test for the presence of unit roots in d_t and s_t, and conclude that both processes are stationary. However, as shown by Kremers (1988) and Wilcox (1989), their results are tenuous in that they are sensitive to the precise lag length employed in the Dickey-Fuller test. Hansen, Roberds, and Sargent derive the VAR-based test discussed above and show that the cross-equation restrictions implied by this condition are rejected by quarterly data over the period 1948–1986. Trehan and Walsh (1988) derive the stationarity test under the assumption of a constant real rate and show that it is equivalent to the cointegration test. They also show that the assumption of intertemporal budget balance is not rejected by the data over 1890–1986. Hakkio and Rush (forthcoming) test for cointegration between government expenditures (inclusive of interest) and revenues, under the assumption that the expected real rate is stationary. Using quarterly data, they find that while expenditures and revenues appear to be cointegrated over the period from 1950 to 1988, this is not the case for sample periods beginning in 1964. However, stationarity restrictions on the real rate process may not be good approximations to the true data generating process; Huizinga and Mishkin (1986), for example, report evidence of shifts in the real rate process in the 1920s and 1980s. Wilcox (1989) uses ex-post real rates to discount Hamilton and Flavin's debt series back to 1960 and shows that the unconditional expected value of this series is positive, thereby violating intertemporal budget balance. One potential difficulty with this approach when applied to relatively short data series is that the estimated mean may be sensitive to the exact set of variables included in the multivariate representations of the discounted debt process. This, of course, is only a small sample problem. Further, because realized real rates were negative during parts of the 1970s, Wilcox's discounted series appears to trend upwards beginning in 1974 (see his fig. 1, p. 300), and we suspect this contributes to his rejection of intertemporal budget balance (compare with our series in the change in debt, Figure 1).[9]

Clearly, there is some conflict in these test results. To eliminate at least one source of conflict, we use a data set that has already been used in several of these studies.[10] Specifically, data on the government deficit are from Hamilton and Flavin and consist of annual observations on the real net-of-interest surplus and outstanding stock of U.S. federal debt from 1960 to 1984. Hamilton and Flavin make a variety of adjustments to the conventional measures of the surplus and the debt to obtain quantities that more accurately reflect the government's net liabilities. The reader is referred to their paper for details. From (1), we obtain the deficit inclusive of interest as the first difference of the series on real debt.

[9]A referee has pointed out that Abel et al. (1989) show negative safe rates of return are possible in a dynamically efficient economy. However, their example concerns an economy with nonstorable output and no investment. If output is storable, however, a negative safe rate would be inconsistent with consumer optimizing behavior.

[10]In addition to Hamilton and Flavin (1986), these include Kremers (1988) and Wilcox (1989).

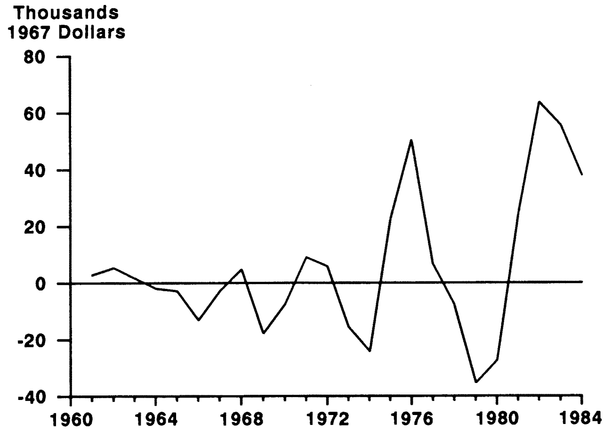

FIG. 1. The Change in the Market Value of the U.S. Debt

We plot this series in Figure 1. Intertemporal budget balance requires that this series be mean stationary, that is, it should exhibit a tendency to return to a constant mean. The interest inclusive deficit in Figure 1 appears to fluctuate around a constant mean value that is close to zero. In addition, the series appears to be heteroskedastic: the amplitude of the swings in the series has increased over time.

To formally test for the presence of nonstationarity, we first employ the Dickey-Fuller test for unit roots. In the simplest case, assume that the variable of interest, say X_t, evolves according to $X_t = \beta X_{t-1} + e_t$, where $\{e_t\}$ is a sequence of normal, independent random variables with mean 0 and variance σ^2. Critical values for the Dickey-Fuller test statistic for the null hypothesis that $\beta = 1$ are reported in Fuller (1976). A constant, time trend, and lagged values of the first difference of X_t may also be included in the estimated regression, depending on the process X_t is assumed to follow.

The results from the unit root tests are contained in Table 1. Equation (1) reproduces the results of Kremers (1988) and Wilcox (1989), showing that the computed value of the test statistic is too small to reject the null hypothesis of a unit root in the stock of debt process at any reasonable level of significance. However, since the estimate of the drift term is insignificant, we also present the results of the same regression excluding the constant [equation (2)]. Once again, we are unable to reject the presence of a unit root. Similarly, equation (3) reproduces Hamilton and Flavin's result for the deficit process, showing that we can reject the presence of a unit root at the 10 percent level. Equation (4) excludes the insignificant drift term, and demonstrates that a unit root in d_t can be rejected at the 1 percent level.[11] Finally, equations (5) and (6) repeat these exercises for the first difference of the stock of debt. In both cases, we can reject the hypothesis of a unit root at the 1 percent level.

[11]The finding that d_t is stationary implies that Proposition 1's requirement that $(1 - \lambda L)d_t$ be stationary is easily satisfied.

TABLE 1

TESTS FOR GOVERNMENT BUDGET BALANCE

(a) Dickey-Fuller Tests

	Variable	Sample	Statistic	Constant	Lags
1.	s_t	63–84	−0.37	Yes	2
2.	s_t	63–84	−0.74	No	2
3.	d_t	62–84	−2.87†	Yes	1
4.	d_t	63–84	−2.97**	No	1
5.	$(1 - L)s_t$	63–84	−4.04**	Yes	1
6.	$(1 - L)s_t$	63–84	−4.01**	No	1

(b) Phillips Tests

	Variable	Sample	Statistic	Constant
1.	s_t	61–84	−0.31	No
2.	d_t	61–84	−2.39*	No
3.	$(1 - L)s_t$	62–84	−2.06*	No

NOTES: In each case, the null hypothesis is that the estimated equation contains a unit root.
†denotes significant at 10%; *denotes significant at 5%; **denotes significant at 1%.

Under the hypothesis of a constant expected real rate, the presence of a unit root in s_t and its apparent absence in d_t would be inconsistent with intertemporal budget balance, while the stationarity of the first difference of s_t would accord with budget balance.

Given the small size of the sample, it is useful to examine the magnitude of the estimated coefficients. Since small samples are likely to lead to relatively large estimated standard errors, our inability to reject the unit-root null may actually reflect the data's inability to differentiate between point estimates that are really quite far apart. We obtain an estimated coefficient of .98 (compared to a value of 1.0 under the null) for the debt stock process, and coefficients of 0.30 and 0.27 for the exclusive-of-interest and inclusive-of-interest deficit processes, respectively. The magnitudes of these coefficients are generally in accord with the results of our unit root tests.

However, before these results are accepted, it is also necessary to determine whether the error terms from the estimated equations satisfy the assumptions of the Dickey-Fuller test. For instance, Kremers (1988) showed that Hamilton and Flavin's finding that the stock of debt was stationary was based upon an equation whose residuals showed significant first-order serial correlation. Including an additional lag of the first difference of the stock of debt eliminated this correlation, but also overturned their conclusion that the debt process was stationary. We used the LM test suggested by Pagan (1984) to test equations (2), (4), and (6) (that is, those versions that force the drift term to zero) for autocorrelation of orders 1 through 4. We were unable to reject the null of no autocorrelation at the 50 percent significance level for either d_t or $(1-L)s_t$, while for s_t the smallest marginal significance level we obtained was .16.

In addition, we tested for normality of the error process using the Jarque-Berra (1980) statistic. The largest of our computed test statistics had a marginal significance level of 0.36, suggesting that non-normality is not a problem for these regressions.

Finally, we used Engle's (1982) procedure to test for autoregressive conditional heteroskedasticity (ARCH) of orders 1 through 4. The results suggest that heteroskedasticity may be a problem for some of these regressions. The computed test statistic (under the null hypothesis of no ARCH of order 2) has a marginal significance level of less than .05 for equation (4), around .15 for equation (2), and around .3 for equation (5). To allow for heteroskedasticity in testing for unit roots in the debt and deficit processes, we also calculate the values of the Phillips' (1987) test statistic, which is robust to the presence of both conditional heteroskedasticity and autoregressive error terms.

The results (shown in part b of Table 1) reveal that we cannot reject nonstationarity of the debt process at the 10 percent level. However, nonstationarity of the exclusive-of-interest deficit process can be rejected at 5 percent. Similarly, the hypothesis that the first difference of the debt process is nonstationary is rejected at 5 percent. Thus, the results from the Phillips test are consistent with those from the Dickey-Fuller test.[12]

Our finding that we cannot reject nonstationarity of the debt stock process but can reject nonstationarity of the exclusive-of-interest deficit process at the 5 percent level is inconsistent with intertemporal budget balance in a world in which the expected real rate of interest is constant. Tests such as those of Hansen, Roberds, and Sargent that are based on restrictions on the VAR representation of $rs_{t-1} + d_t$ and $(1-L)d_t$ will reject intertemporal budget balance because the two are integrated of different orders. Yet our test results also consistently reject nonstationarity of $(1-L)s_t$, which (from Proposition 2) is still sufficient to ensure sustainability. We interpret this evidence as suggesting that the real rate is not constant, but that the stochastic process describing the evolution of the first difference of the stock of debt over this period is sustainable. In other words, the inclusive-of-interest deficit appears to be stationary, an outcome that implies that the government's budget is balanced in present value terms.

Finally, what about the possibility that these results are sample-specific? It turns out that similar results are obtained using the data set from Trehan and Walsh (1988). Specifically, our analysis reveals that for annual data over 1890–1986, we cannot reject the null hypothesis of a unit root in the debt process, but we can reject this hypothesis for the net-of-interest deficit. And we can reject the null of a unit root for the inclusive of interest deficit as well. Since the test that d_t and s_{t-1} are cointegrated and the test that $(1-L)s_t$ is stationary cannot lead to different outcomes in a world of constant expected real interest rates, it appears that the cointegration tests reject intertemporal budget balance (for both our data samples)

[12]Schwert (1987) shows that the Phillips test tends to reject the null hypothesis of a unit root too often if the process under consideration contains moving average components. To get around this problem, he recommends using a relatively large number of autocovariances to compute the test statistic. Since the sample is already so short, the results in Table 1(b) were obtained by using a relatively small number of autocovariances. Specifically, we used Schwert's formula for l_4 (page 88). However, we also repeated the test using his l_{12} formula. These results reveal that we can reject the hypothesis of a unit root in the d_t process at the 5 percent level, and in the $(1 - L)s_t$ process at the 10 percent level. While these results are consistent with those presented above, it is worth pointing out that they are based on a sample of about fifteen observations only.

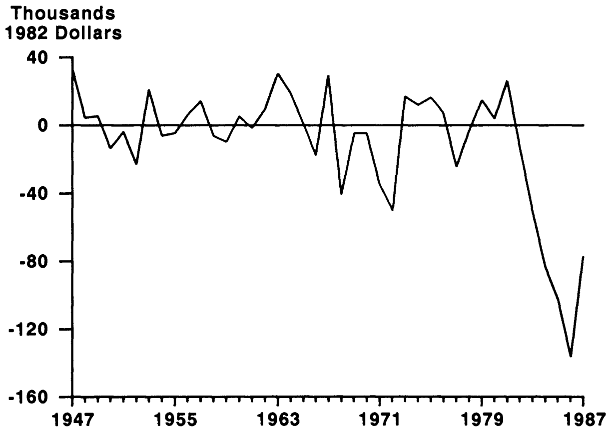

FIG. 2. The Change in the U.S. Net International Investment Position

because the assumption of constant expected real rates conflicts with the data generating process.[13]

5. EXAMPLE 2: U.S. INTERNATIONAL INDEBTEDNESS

In this section, we examine data on the current account from 1946 to 1987 to determine whether the large current account deficits of recent years imply that foreigners now hold an unsustainably large amount of U.S. debt. The variables in equation (1) can be reinterpreted quite easily to apply to this case, with s_t being the net stock of domestic assets held by foreigners. As discussed above, intertemporal budget balance requires that the present discounted value of s_t equal zero as t goes to infinity; for the case at hand this means that the present discounted value of domestic assets held by foreigners should converge to zero.

Our test for intertemporal budget balance requires that the change in the stock of net domestic assets held by foreigners be stationary. Accordingly, we use data on the net international investment position of the United States, obtained from various issues of the *Survey of Current Business*. These data take into account changes in the value of asset holdings arising from capital flows, as well as those arising from exchange rate changes. We have used the GNP deflator to convert nominal to real values. The resulting series is plotted in Figure 2. Note that if the series in Figure 2 had been terminated in 1981, there could be little doubt concerning its stationarity. However, the addition of the years 1982 to 1987 makes any conclusion drawn from a simple inspection more problematic.

[13]A referee has suggested that the nonconstancy of expected real rates can be verified by estimating regressions employing ex-post measures of the expected real rate. However, preliminary examination of the data reveals that the "expected" real rate time series we obtain using survey measures of inflation are quite different from those we obtain using ex-post inflation forecasts employing regression techniques. Since resolving this issue is beyond the scope of this paper, we decided against using regression-based techniques to make inferences about real interest rates.

TABLE 2

TESTS FOR SUSTAINABILITY OF U.S. INTERNATIONAL INDEBTEDNESS

(a) Dickey-Fuller Tests

	Variable	Sample	Statistic	Constant	Lags
1.	$(1 - L)s_t$	46–87	−2.21*	No	0

(b) Phillips Tests

	Variable	Sample	Statistic	Constant
1.	$(1 - L)s_t$	46–87	−2.54*	No

NOTES: In each case, the null hypothesis is that the estimated equation contains a unit root.
*denotes significant at 5%.

The results from our test are presented in Table 2. The Dickey-Fuller test rejects the null hypothesis of nonstationarity for the change in the external investment position of the United States. The estimated coefficient on the first lag is 0.74, compared to a value of 1 under the null hypothesis. Diagnostic tests for serial correlation and normality do not suggest any problems. Once again, we tested for autocorrelation of orders 1 through 4; the lowest marginal significance level we obtained was .36. The marginal significance level of the Jarque-Berra statistic for normality was .26. However, the results from the ARCH test are more ambiguous. Specifically, in testing for first-order ARCH, we obtained an $F(1,36)$ statistic of 4.02, which has a marginal significance level of 0.05. Accordingly, Table 2b presents the results from the Phillips test. Here again the results from the Phillips test agree with those from the Dickey-Fuller test.[14,15] Thus, despite the large reduction in the U.S. net asset position during the 1982–87 period, the past history of the first-differenced process is consistent with the assumption that it will return to its mean.

6. CONCLUSIONS AND IMPLICATIONS

In this paper we have extended existing tests of intertemporal budget balance in two ways. We have shown that if expected real rates are constant, the existence of a stationary linear combination of the stock of debt and the net-of-interest deficit is necessary and sufficient for intertemporal budget balance—as long as a quasi difference of the net-of-interest deficit is stationary. If the first difference of the net-of-interest deficit is stationary, the relevant linear combination is just the inclusive-of-

[14]These results are also obtained by using Schwert's l_4 formula. We obtain the same result if we use his l_{12} formula, that is, we can still reject nonstationarity of the first difference of the net-international-investment-position process at 5 percent.

[15]In a previous version of the paper, we used data on the current account balance to shed light on the question of balance-of-payments equilibrium. We found that we could not reject nonstationarity of the current account balance. (The computed test statistics were all positive.) While the current account balance is similar to the first difference of the data on the net international investment position, there is an important difference between the two. Data on the external investment position are explicitly adjusted for the effect of exchange rate changes, while no such changes are made to the data on the current account. We believe that these adjustments should not be ignored, especially in light of the large exchange rate swings of recent years. Consequently, we are inclined to accept the results in Table 2 obtained from the data on the external investment position of the United States.

interest deficit (or the first difference of the stock of debt). Further, we have shown that stationarity of the inclusive-of-interest deficit is sufficient to ensure intertemporal budget balance, as long as the expected real rate is positive.

We have also presented two different applications of the test developed here. First, we examined data on the federal government's budget and found that a stationary linear combination of the stock of debt and the net-of-interest deficit does not exist. However, the first difference of the stock of debt is stationary. We interpret this finding to imply that the deficit process is consistent with sustainability, but that the assumption of a constant expected real rate is a bad approximation to the data. Second, we used data on the external investment position of the United States to determine whether foreigners now hold an unsustainably large amount of U.S. assets. Once again, we found that the data can reject this hypothesis.

In interpreting these results, however, the shortness of our samples (twenty-four years in one case, forty-one in the other) should be borne in mind. Short samples are likely to make it more difficult to reject nonstationarity if mean reversion only manifests itself over long periods. However, the data are able to reject nonstationarity of the interest-inclusive federal deficit and the change in net international investment, and the results for the federal deficit are consistent with those obtained earlier using almost one hundred years of U.S. data.

The test that we have developed here has the advantage that it provides a straightforward way to impose intertemporal budget balance in empirical studies. While recent theoretical models impose intertemporal budget balance when studying intertemporal optimization, this generally has not been the case in empirical studies. Our analysis here shows that in studying government expenditure and financing decisions, for instance, intertemporal budget balance can be imposed quite easily by estimating a vector-error-correction model that includes the inclusive-of-interest deficit as the error-correction term. We are currently engaged in just such a research project. Similar applications exist in open-economy macroeconomics. For instance, this approach would allow one to study how trade flows and exchange rates adjust to maintain intertemporal budget balance.

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