Simple Sustainable Forward Guidance at the ELB*

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Abstract

Forward guidance plays an important role in monetary policy analysis at the effective lower bound (ELB) for nominal interest rates. Yet in discretionary environments, forward guidance is generally assumed to lack credibility. I analyze whether discretionary policymakers may still make credible promise about future policy. I show that promises to keep the nominal rate at zero for a fixed number of periods after an ELB episode ends are sustainable if the promise is not for too many periods. However, the length of forward guidance that minimizes the present value of losses at the ELB may not be sustainable.

1 Introduction

The current era of very low interest rates has raised troubling questions for all central banks, but particularly for those that target inflation. Do the dangers of hitting the effective lower bound (ELB) for short-term interest rates call for increasing inflation targets as insurance against returning to the ELB? Does inflation targeting still provide an inadequate framework

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for monetary policy? Or does the presence of the ELB imply inflation targeting should be replaced by some other policy framework, such as price-level targeting?

The discussion of the issues surrounding these questions – and on the consequences of the ELB more generally – have reached two conclusions. First, in an environment in which the central bank is able to credibly commit to future actions, the costs of the ELB are small. For example, this is the conclusion of the work by Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006) and Nakov (2008). A central bank able to commit to future actions is not unduly constrained when its current policy rate is at its lower bound; making promises about the future path of the policy rate is sufficient to allow policymakers to influence economic activity effectively. If commitment is the appropriate way to understand the monetary policy environment, then the ELB does not call for any reform of inflation targeting or for raising the average inflation target.

Second, if a central bank is able to commit to a policy framework such as inflation targeting but implements policy within that regime in a discretionary fashion, then the ELB can be very costly, as shown for example by Adam and Billi (2007). This conclusion leads naturally to the proposal of Blanchard, Dell’Ariccia, and Mauro (2010) to raise the average inflation target, making it less likely that the ELB will be encountered. It also leads to proposals to replace inflation targeting with alternatives policy regimes, such as price-level targeting, in which discretionary policy is able to mimic some of the advantages of commitment, as shown by Vestin (2006).

Finding policy regimes that can limit the adverse effects of the ELB is important, as episodes of very low interest rates cannot, as they once were, be viewed as extremely rare

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1 Reifschneider (2016) demonstrates the effectiveness of credible forward guidance (together with balance sheet policies) using the FRB/US model. Levin, López-Salido, Nelson, and Yun (2010) argue that forward guidance may be less effective in the face of large and persistent shocks that drive the economy to the ELB.

2 Recent work on price-level and nominal income targeting includes Billi (2013), Giannoni (2014), Billi (2017).
events. Figure 1 shows histograms of U.S. short-term interest rates. The top panel is based on the monthly effective federal funds rate from January 1960 to March 2018, while the lower panel is for the 3-month Treasury bill rate since 1934. Both show that a large fraction of months have seen rates below 25 basis points. For the shorter sample based on the funds rate, 12% of months have seen the funds rate at or below 25 basis points. For the longer period, the 3-month T-bill rate fell below 25 basis points in 17% of all months.\footnote{This histogram is misleading in the sense that, to take the top panel, all the months at or below 25 basis points occurred consecutively between December 2008 and December 2015.}

If policy is inevitably discretionary in nature, the occurrence of frequent episodes at the ELB is a strong argument for raising inflation targets or adopting price-level targeting. Most of the literature that has focused on the monetary policy consequences of the ELB has treated the credibility of the central bank as either complete, as in commitment equilibria, or totally absent, as in analyses of discretion. In the one case, future promises are fully believed and subsequently delivered on and the ELB is not costly. In the latter case, the public
places no weight on any promises the central bank might make and the ELB is very costly. Promises – forward guidance – are thus either extremely powerful, as in work on the forward guidance puzzle by DelNegro, Giannoni, and Patterson (2012), Cochrane (2017), and McKay, Nakamura, and Steinsson (2016), or completely powerless in a discretionary environment.⁴

Forward guidance has frequently been analyzed using simple analytical frameworks that have helped provide insights into the consequences of the ELB and the role of forward guidance. For example, Eggertsson and Woodford (2003) introduced the assumption that each period there is a fixed probability of exiting the ELB. This approach has been used by Eggertsson (2011), Christiano, Eichenbaum, and Rebelo (2011), Braun, Körber, and Waki (2012), and McKay, Nakamura, and Steinsson (2016), among others. Alternatively, several authors have considered perfect foresight equilibria in which the ELB will bind for a known number of periods. For example, Werning (2011), Carlstrom, Fuerst, and Paustian (2012), Cochrane (2017), and Kiley (2016) use such a framework. Under either approach, the assumption has been that the ELB is a one-off occurrence. Once the economy exits from the ELB, it never returns. In this case, announcements can never be credible absent a commitment technology. Under discretion, there is no benefit to fulfilling promises made during an ELB episode; any credibility gained from fulfilling promises is of no future use.

The situation changes if the economy may encounter the ELB again. This, of course, is the presumption of work examining the role of the inflation target or the policy regime in reducing the probability of or mitigating the effects of future ELB episodes. But if the economy may return to the ELB, a rational central bank may have an incentive to fulfill past promises, even under discretion. Doing so brings a future benefit of credibility should the ELB again bind. In fact, Nakata (2017) has shown that the fully optimal Ramsey policy can be sustained if there is only a slight probability the ELB will occur in the future. This is an

⁴Exceptions include the papers by Bodenstein, Hebden, and Nunes (2012) and Nakata (2017) which are discussed below.
important result and implies that pure discretion is not the appropriate benchmark against which to evaluate proposals to switch to price level targeting or to raise the inflation target. Nakata result is important, but the optimal Ramsey policy may be difficult to communicate in practice, and appropriately steering expectations in ways that sustain the Ramsey policy may be difficult precisely because the policymaker is known to be able to defect. It is, therefore, of interest to examine the sustainability of policies that are suboptimal relative to Ramsey but whose simplicity may make them easier to communicate to the public.\footnote{Bilbiie (2017) analyzes simple forward guidance policies in which the duration of the policy is stochastic; after exiting the ELB, the central bank keeps the nominal rate at zero with a constant probability. He is able to obtain closed form solutions and investigates the optimal expected duration of forward guidance in an environment in which future ELB episodes never occur.}

Of course, if promises made during an ELB period are extreme enough, it is unlikely central bank will fulfill them even if the economy may someday return to the ELB. However, as others have noted (Carlstrom, Fuerst, and Paustian (2012), Kiley (2016), McKay, Nakamura, and Steinsson (2016)), forward guidance is very powerful in standard new Keynesian models. This suggests that the central bank may need to make only modest promises at the ELB. If so, the costs of fulfilling them may be correspondingly small. Thus, the power of forward guidance, combined with the possibility of a return to the ELB, may lead even a discretionary policymaker to make and keep promises. Forward guidance may be sustainable.\footnote{In the presence of endogenous state variables, current policy choices can affect the incentives faced by future policymakers, thereby generating a channel through which the policymaker can effectively influence expectations about future policy. For example, Jeanne and Svensson (2007) have investigated how generating a large increase in the government’s nominal debt can create an incentive for future inflation. Thus, a government’s concerns about its balance sheet can provide a mechanism for current policy to influence future policy choices. This channel is absent in the present paper which employs a basic new Keynesian model in which there are no endogenous state variables.}

In this paper, I investigate a simple form of forward guidance and ask whether, in a discretionary environment, a policymaker can still make credible promises about future policy. If so, the stark contrast typically drawn between the consequences of the ELB under discretion and under commitment may be too exaggerated. And if this is true, the case against inflation
targeting and the arguments for raising the inflation target or switching to price-level targeting are weakened. Effective and sustainable forward guidance would reduce the need for these alternatives. Their merits would need to be based on considerations other than their effects in reducing the probability of encountering the ELB or their superior performance (relative to discretion) at the ELB, a point also made by Loisel (2008).

If future promises are credible even in a discretionary environment, the sharp distinction between discretion and commitment is blurred and credibility is no longer an all or nothing property of policy actions. Two literatures have developed approaches that allow for partial credibility. The first follows the stochastic planning problem analyzed by Roberds (1987) and Schaumburg and Tambaletti (2007), and includes the related work on loose commitment by Debortoli and Nunes (2010), Bodenstein, Hebden, and Nunes (2012), and Debortoli, Maih, and Nunes (2014). The stochastic planning approach assumes a policymaker is able to commit to future policies, but each period there is an exogenous probability a new policymaker will be appointed. Under loose commitment, there is a fixed probability each period that the policymaker reoptimizes. In either case, promises are discounted to reflect the likelihood that the current policymaker will be replaced or reoptimize.\(^7\)

The second literature, which I follow, builds on notion of sustainable plans under discretion developed by Chari and Kehoe (1990) and Stokey (1991) and employed by Ireland (1997), Kurozumi (2008), Kurozumi (2012) and Nakata (2017).\(^8\) That is, I assume the absence of any commitment technology. A past promise might be honored, but only if doing so is the best strategy for the policymaker at the time the promise needs to be honored. Kurozumi (2008) has investigated whether the optimal commitment policy in the basic new Keynesian model is sustainable under discretion. He shows that the optimal sustainable policy falls

\(^7\)An early example of a model in which equilibrium was affected by the probability of a future change in policy maker was provided by Ball (1995). In his model, however, the new policy maker was drawn from a distribution of policy makers who differed in their preferences.

\(^8\)This literature builds on Abreu (1988). See also Levine, McAdam, and Pearlman (2008).
between that of optimal discretion and optimal commitment, but it converges over time to
the optimal commitment policy if the policymaker’s discount rate is not too large. Kurozumi
(2012) shows that a regime of flexible inflation targeting is sustainable, but only if the cen-
tral banker places more weight – but not too much weight – on inflation stability than is
reflected in social welfare. That is, the central banker must be a Rogoff (1985) conservative,
but not too conservative. In contrast, Loisel (2008) shows a trigger strategy equilibrium can
support a reputational equilibrium that overcomes discretionary inflation and stabilization
biases without needing to delegate to a conservative central banker.

What has not been examined is whether announcements of the type associated with date-
based forward guidance can form part of a sustainable policy plan. This gap in the existing
literature is one this paper fills.

The rest of the paper is organized as follows. Section 2 follows Nakata (2017) in modifying
the framework of Eggertsson and Woodford (2003) to allow for a positive probability that
after exiting an ELB episode the economy may again encounter a binding ELB constraint, and
equilibrium under pure discretion, which serves as a benchmark of comparison for forward
guidance policies, is examined. In section 3, the effects of a promise to keep the nominal
rate at zero after an ELB episode ends are studied. The case of one-period forward guidance
is considered first, while promises to keep the nominal rate at zero for several periods after
an ELB episode ends are then investigated, together with the optimal length of forward
guidance. Forward guidance is very powerful in the basic new Keynesian model, and Gabaix
(2016), McKay, Nakamura, and Steinsson (2016), and McKay, Nakamura, and Steinsson
(2017) have proposed discounted Euler equations that reduce the power of forward guidance.
The robustness of the results to employing a discounted Euler equation is considered in
section 4, while conclusions are summarized in section 5.
2 Recurring episodes at the ELB

The basic model adopts the two-state Markov structure of Eggertsson and Woodford (2003), modified following Nakata (2017) to allow for a positive probability of returning to the ELB in the future. Private sector behavior is described by the standard linear new Keynesian model represented by

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t), \tag{1} \]

and

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{2} \]

together with a specification of monetary policy. For convenience the ELB on the nominal interest rate is taken to be zero, so

\[ i_t \geq 0. \tag{3} \]

(1) is the Euler condition for intertemporal consumption choice, where \( x_t \) is the output gap, \( \pi_t \) the inflation rate, \( i_t \) is the nominal interest rate, and \( r_t \) is an exogenous stochastic process. (2) is the reduced form equation of inflation that can be derived from a time-dependent model of price adjustment such as the Calvo model.\(^9\)

\(^9\) A number of authors (Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006), Adam and Billi (2007), Nakov (2008), Levin, López-Salido, Nelson, and Yun (2010), Billi (2017)) have examined stochastic equilibria in new Keynesian models subject to occasionally binding lower bounds on the nominal interest rate. In these models, the economy can pass into, out of, and back into periods during which the lower bound constraint is binding. However, this literature has not investigated specific examples of forward guidance. Work on assessing the empirical effects of forward guidance include Campbell, Evans, Fisher, and Justiniano (2012), Campbell (2016) and Swanson (2016).

\(^{10}\) The underlying nonlinear model that leads to the reduced form equations employed here is so well known that providing details on it seems unnecessary. See, for example, chapter 11 of Walsh (2017), which provides an extended discussion of the ELB. See Eggertsson and Singh (2016) for a justification for the use of the log linear approximation at the ELB.
The monetary authority desires to minimize

\[
L_t = \frac{1}{2} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left( \pi_{t+j}^2 + \lambda x_{t+j}^2 \right).
\]

(4)

The policy environment is one of discretion; there is no formal mechanism that allows the policymaker to commit to future policy actions.

The shock \( r_t \) in (1) follows a two-state Markov process. In the state \( z \), \( r_t = r_z < 0 \) and \( i_z = 0 \); in state \( n \), \( r_t = \beta^{-1} - 1 \equiv \rho > 0 \). If \( r_t = r_z \), then \( r_{t+1} = r_z \) with probability \( q \) and \( r_{t+1} = \rho \) with probability \( 1 - q \). If \( r_t = \rho \), then \( r_{t+1} = \rho \) with positive probability \( s \), \( 0 < s \leq 1 \), and \( r_{t+1} = r_z \) with probability \( 1 - s \). Thus, \( 1 - s \) is the probability of reverting to the ELB.

The previous literature building on the analytical structure of Eggertsson and Woodford (2003) set \( s = 1 \), implying that once the economy exits from the ELB, it never returns. Similarly, the literature that treats the ELB as binding for a fixed number of periods after which it never binds again similarly assumes there is never any return to the ELB (see, for example, Werning (2011), Cochrane (2017) and Kiley (2014)). When \( s < 1 \), the economy can experience repeated episodes in both state \( z \) and state \( n \). Let \( \pi_j \) and \( x_j \) denote equilibrium inflation and the output gap in state \( j = z, n \); let \( i_n \) denoted the nominal interest rate in state \( n \).

In a discretionary policy environment, when will a policymaker find it is incentive compatible to fulfill past promises that were made when the ELB was binding? If \( s = 1 \) so that economy never returns to the ELB, optimal discretion can deliver \( \pi_n = x_n = 0 \). Thus, any promise made at the ELB that involves either inflation or the output gap deviating from zero would incur a larger loss than simply implementing the optimal discretionary policy. Promises made at the ELB that would imply non-zero values for \( x \) or \( \pi \) will never be honored by a policymaker acting with discretion to minimize (4). Absent a commitment technology,
the only credible policy upon exit is to set \( i_n = \rho \), consistent with \( x_n = \pi_n = 0 \).\(^{11}\) But if \( s < 1 \), a policymaker may find it optimal to honor past promises. Doing so may entail lower losses in future states in which the ELB is again binding.

To be more specific, let \( L^d_j \) denote the present value of losses in state \( j \) under the optimal period-by-period discretionary policy that, in each period, minimizes the policymaker’s loss function, taking expectations and future policy as given. Let \( L^o_j \) be the present value of losses when the economy is in state \( j \) under an arbitrary policy \( o \). The policy \( o \) may involve promises made in the past about policy actions in the current state. Such a policy is sustainable if \( L^o_j \leq L^d_j \) for all \( j \). That is, continuing to implement policy \( o \), including any promises made in the past, constitutes a sustainable plan if the present value of losses obtained by implementing the policy is, in every state, less than that obtained by reverting to the policy \( d \). A sustainable policy is time-consistent; the policymaker has no incentive to switch from the policy and adopt the discretionary policy.\(^{12}\) In other words, any contingent sequence of inflation, the output gap, and the nominal interest rate that satisfies (1) - (3) for every \( t \geq 0 \) is called sustainable if, for each \( t \geq 0 \), the present discounted value of losses is less than the present value of losses under the optimal, time-consistent discretionary policy. Thus, policies for which the current period’s loss exceeds that obtained under the discretionary policy may still be sustainable if future losses under the policy are less than those under discretion.

To assess the sustainability of forward guidance policies, it is necessary to first determine equilibrium under optimal discretion. If there is a positive probability of returning to the ELB, private agents, in forming expectations about future inflation and the output gap, will

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\(^{11}\)A policy that simply sets \( i_t = \rho \) does not ensure \( x = \pi = 0 \) is the unique, stationary equilibrium under rational expectations. A rule of the form \( i_t = \pi + \phi \pi_t \) with \( \phi > 1 \) would do so.

\(^{12}\)The concept of a sustainable policy plans was first introduced by Chari and Kehoe (1990). Stokey (1991) defines a pair of strategies (for the government and private sector) that is compatible with a competitive equilibrium in the private sector, given the government’s strategy, and for which the government has no incentive to alter its strategy as a credible policy. See Nakata (2017) for a formal treatment of sustainability in the context of the Markov structure I employ.
place positive weight on the equilibrium inflation and output gap that occurs at the ELB. Consequently, equilibrium away from the ELB now depends on equilibrium at the ELB. And the converse also holds.

The policymaker under pure discretion faces a static decision problem each period that involves minimizing

$$l_t = \frac{1}{2} (\pi_t^2 + \lambda x_t^2),$$  \hspace{1cm} (5)

subject to (1), (2), and (3), taking expectations and future policy as given. Away from the ELB, optimal policy is characterized by a targeting rule of the form\(^{13}\)

$$\kappa \pi_t + \lambda x_t = 0.$$  \hspace{1cm} (6)

Using superscript \(d\) to denote the equilibrium under discretion, inflation \(\pi_n^d\) and the output gap \(x_n^d\) when the ELB is not binding solve

$$\pi_n^d = \beta \left[ s \pi_n^d + (1 - s) \pi_z^d \right] + \kappa x_n^d,$$  \hspace{1cm} (7)

and

$$\kappa \pi_n^d + \lambda x_n^d = 0,$$  \hspace{1cm} (8)

where expected inflation is equal to \(s \pi_n^d + (1 - s) \pi_z^d\). Equilibrium must also satisfy the non-negative constraint on \(i_t\); this requires that

$$i_n^d = \rho + \left[ s \pi_n^d + (1 - s) \pi_z^d \right] + \sigma (1 - s) \left( x_z^d - x_n^d \right) \geq 0,$$  \hspace{1cm} (9)

\(^{13}\)In most of the literature using this model, policy after the ELB episode ends is characterized by a simple instrument rule rather than by optimal discretion. In the present context, \(\pi_n = x_n = 0\) is also the locally unique stationary equilibrium if the nominal rate is given by \(i_n = \rho + \phi \pi_n\) once the ELB constrains no longer binds, with \(\phi > 1\). The choice of \(\phi\), as long as it exceeds \(1\), plays no role in affecting equilibrium at the ELB or away from the ELB when the ELB episode is a one-off event. This is not true with recurring episodes at the ELB.
where this last equation is obtained by solving the Euler condition (1) in state $n$.

When $s = 1$, $\pi^d_n = x^d_n = 0$ constitutes an equilibrium under discretion when the ELB is nonbinding. When $s < 1$, it is no longer feasible to achieve a zero inflation rate and output gap, as neither expected inflation, $s\pi^d_n + (1 - s)\pi^d_z$, nor the expected output gap, $sx^d_n + (1 - s)x^d_z$, equal zero. As long as some probability is assigned to that possibility the economy will return to the ELB, expected inflation and the expected output gap when not at the ELB will depend on $x^d_z$ and $\pi^d_z$.

The dependence of equilibrium away from the ELB on the equilibrium at the ELB can be made explicit by solving (7) and (8) for $x_n$ and $x_n$, obtaining

$$\pi^d_n = (1 - s)\left[\frac{\beta \lambda}{\lambda (1 - \beta s) + \kappa^2}\right] \pi^d_z,$$

and

$$x^d_n = -(1 - s)\left[\frac{\beta \kappa}{\lambda (1 - \beta s) + \kappa^2}\right] \pi^d_z.$$

With deflation at the ELB, $\pi^d_z < 0$. Then, when $s < 1$, optimal discretion implies $\pi^d_n < 0$ and $x^d_n > 0$ such that $\kappa \pi^d_n + \lambda x^d_n = 0$. Away from the ELB, $\pi^d_n$ and $x^d_n$ also depend on $q$, because the probability of remaining at the ELB affects $\pi^d_z$.

As $\pi^d_z$ becomes more negative, or as the probability of reverting $1 - s$ becomes larger, $x^d_n$ becomes larger, and to generate this expansion, the policymaker reduces the nominal rate $i^d_n$. However, equilibrium away from the ELB requires that $i^d_n$ satisfy the non-negativity constraint (3); it could be that the central bank’s first-order condition (8) under the optimal discretionary policy would require the nominal interest rate to be negative. The value of $i^d_n$

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14If the central bank reverts to a simple rule such as $i_t = r_t + \phi \pi_t$ once the ELB episode ends, equilibrium also involves $\pi_n < 0$ and $x_n > 0$. However, unlike the optimal discretionary targeting rule in (8), the simple rule does not generally ensure the optimal tradeoff between deflation and real expansion is achieved.
consistent with (1) can be written as

\[ i_n^d = \rho + \pi_n^d + (1 - s) \left[ \sigma \left( x_z^d - x_n^d \right) + \left( \pi_z^d - \pi_n^d \right) \right].\] (10)

With \( x_z^d < x_n^d \) and \( \pi_z^d < \pi_n^d \), the term in brackets is negative. Thus, a large probability of returning to the ELB (a small \( s \)) can lead to a violation of the nonnegativity constraint on \( i_n^d \). If \( q \) is large, the ELB episode is expected to be of long duration, and this reduces \( \pi_z^d \) and \( x_z^d \), contributing to a fall in \( \pi_n^d \) and \( x_n^d \) and reducing the nominal interest rate when the economy is not at the ELB. In this case, there may be regions of the parameter space in which the nonnegativity constraint on \( i_n^d \) is violated.

At the ELB, equilibrium is characterized by (11) and (12):

\[ \pi_z^d = \beta \left[ q \pi_z^d + (1 - q) \pi_n^d \right] + \kappa x_z^d \] (11)

\[ x_z^d = \left[ q x_z^d + (1 - q) x_n^d \right] + \left( \frac{1}{\sigma} \right) \left[ q \pi_z^d + (1 - q) \pi_n^d + r_z \right]. \] (12)

Equations (7) - (12) can be solved jointly to obtain equilibrium inflation and the output gap in states \( z \) and \( n \) and the nominal rate in state \( n \). To do so, the baseline calibration is given in Table 1, which is based on the values employed by Eggertsson and Woodford (2003) and used more recently by Nakata (2017) and McKay, Nakamura, and Steinsson (2016). The loss function (4) is interpreted as derived from a second-order approximation of the welfare of the representative household around the economy’s efficient equilibrium; in this case Woodford (2003) showed that \( \lambda = \kappa / \theta \), where \( \theta \) is the price elasticity of demand faced by individual firms. Using Woodford’s value of \( \theta = 7.88 \) implies \( \lambda = 0.003 \). If inflation is expressed at an annual rate, then \( \lambda^a = 16 \lambda = 0.048 \).

The final two parameters of the model are \( s \) and \( q \). To discipline the calibration of the
two transition probabilities when \( s < 1 \), I employ the evidence based on figure 1. Define

\[
P = \begin{bmatrix} s & 1 - q \\ 1 - s & q \end{bmatrix}.
\]

The steady-state fractions of time spend away from the ELB and at the ELB are given by the diagonal elements of \( \lim_{T \to \infty} P^T \). To discipline the calibration of these frequencies, I use the data employed in figure 1) to match the fractions to either the January 1960 to January 2017 federal funds sample frequencies (88% of the time away from the ELB, 12% of the time at the ELB) or the longer January 1934 to January 2017 3-month treasury rate sample (83% and 17% in the two states respectively). Figure 2 shows the combinations of \( s \) and \( q \) that match these two fractions. Eggertsson and Woodford (2003) and McKay, Nakamura, and Steinsson (2017) assume \( q = 0.9 \), and for this value of \( q \), \( s = 0.9861 \) (indicated by an x in the figure) implies a steady-state frequency at the ELB of 12%, while \( s = 0.9794 \) (indicated by the o) implies a steady-state frequency of 17%. These two values of \( s \), together with \( q = 0.9 \), will be employed in the baseline calibration. To also assess the effects of a smaller continuation probability at the ELB, calibrations based on \( q = 0.85 \) will also be used. The associated values of \( s \) for the two samples for this lower value of \( q \) are indicated in the figure and given in Table 2.\(^{15}\)

Optimal discretion may result in a binding ELB constraint when \( r_t = \rho \). Then can occur when \( \pi_2^d \) is sufficiently negative that expected inflation when away from the ELB is negative,

\(^{15}\)As discussed in Eggertsson (2011), the economy experiences what he describes as deflationary black hole if \( q \) rises above 0.9. As \( q \to 1 \), it enters what Braun, Körber, and Waki (2012) characterize as a type 2 equilibrium. As discussed in the next section, I restrict attention to values of \( q \leq 0.90 \) to be consistent with equilibria in which \( x_2 < 0, \pi_2 < 0, \) and \( i_s > 0 \) under optimal discretion.
leading $\pi_n^d$ to also be negative. To maintain the targeting rule (8) characterizing optimal discretion would require a positive output gap. If $x_n^d$ is large and negative, achieving a positive value for $x_n^d$ may force $i_n^d$ down to zero. Denote the value of $i_n^d$ as $\bar{i}_n^d(s, q)$ to highlight its dependence on $s$ and $q$. Define $\Omega = \{s, q \text{ s.t. } i_n^d(s, q) \geq 0\}$. In the subsequent analysis, attention is restricted to $(s, q) \in \Omega$. Figure 3 shows the level of the nominal rate consistent with optimal discretion for ranges of $s$ and $q$. For the standard value of $q = 0.90$, the non-negativity constraint on $i_n^d$ is binding only for $s > 0.975$. Thus, for both the baseline calibrations used for $s$ and $q$ (shown by the $x$ and $o$ markers on the $q = 0.9$ line), $i_n^d > 0$.

Figures 4 shows equilibrium inflation (upper panel) and the output gap (lower panel) under discretion as a function of $q$ when the ELB is binding and when it isn’t for $s = 1.0$ and $s = 0.9692$. This latter value of $s$ is chosen as it is the smallest value from table 2. Recall that $\pi_n^d = x_n^d = 0$ for $s = 1$. The output gap under discretion rises as the likelihood of returning to the ELB rises ($s$ falls below 1). With reversion to the ELB more likely, expected inflation...
when the ELB does not bind falls, and in response, $i_n^d$ is reduced to increase the output gap and maintain the targeting criterion (6). Note, however, that for the values of $s$ shown, the impact of a higher probability of another ELB episode has a relatively minor effect on the equilibrium, especially for smaller values of $q$.

Let $L_k^d$ for $k = z, n$ be the present discounted value of losses in state $k$ under pure discretion. Then $L_z^d$ and $L_n^d$ satisfy the following valuation equations:

$$L_z^d = \frac{1}{2} \left[ \left( \pi_z^d \right)^2 + \lambda \left( x_z^d \right)^2 \right] + \beta q L_z^d + \beta (1 - q) L_n^d,$$  \hspace{1cm} (13)$$

and

$$L_n^d = \frac{1}{2} \left[ \left( \pi_n^d \right)^2 + \lambda \left( x_n^d \right)^2 \right] + \beta s L_n^d + \beta (1 - s) L_z^d.$$  \hspace{1cm} (14)$$

Following Billi (2011), losses are expressed in terms of their steady-state consumption equiv-

Figure 3: Nominal interest rate under discretion when ELB is nonbinding. $x$ (o) indicates ($q, s$) combinations consistent with alternative calibrations of the fraction of time at ELB.
Figure 4: Inflation and the output gap under optimal discretion as a function of $q$ for $s = 1$ (solid lines) and $s = 0.9687$ (dashed lines). Values at the ELB indicated by $x$; values away from the ELB indicated by $o$. Upper panel: Inflation. Lower panel: output gap.

Thus, a loss of $L_z$ is equivalent to a $100\mu_z$ percent reduction in steady-state consumption.\(^\text{16}\)

If $r_z = -2\%$ (expressed at an annual percentage rate) and the parameter values given in Table 1 with $q = 0.90$ and $s = 1$ as in Eggertsson and Woodford (2003), the equilibrium output gap and inflation rate at the ELB are $x_z = -0.1434$ and $\pi_z = -0.0263$ (–14.34\% and –10.53\% respectively, when inflation is expressed at an annual rate). This translates into a consumption-equivalent loss of $\mu_z = 5.04\%$.

The importance of the calibration of $q$ for these welfare losses is apparent in Figure 5, which shows $L^d_z$ as a function of $s$ and $q$.\(^\text{17}\)

\(^{16}\)See Billi (2011). Billi (2017) uses this measure to evaluation nominal GDP targeting and price-level targeting. To calculate $\mu_z$, I set $\omega = 0.75$ and $\eta = 2$. These values are consistent with the values of $\kappa$ and $\lambda$ given in Table 1.

\(^{17}\)The figure shows loss for $q \leq 0.90$ to avoid values of $q$ that imply $i_n < 0$.
duration at the ELB increases. An increase in $s$, in contrast, lowers the loss as a larger $s$ means the economy reverts less frequently to the ELB. When $s = 1$, $L_n^d = 0$ for all $q$. Loss at the ELB is very sensitive to $q$ when $s = 1$; it falls from 5.04% of steady-state consumption when $q = 0.9$ to less than 1% of steady-state consumption when $q = 0.89$, and to 0.06% when $q = 0.85$. Loss at the ELB increases for a given $q$ as the probability of returning to the ELB rises ($s$ falls). When $q = 0.9$, $\mu_z$ rises from 5.04% to 7.94% as $s$ falls from 1 to 0.99, and rises to 8.78% when $s$ equals the value that matches the 1960-2017 fraction of quarters at the ELB ($s = 0.9861$); it is only 0.18% of steady-state consumption when the 1960-2017 sample is matched by setting $q = 0.85$ and $s = 0.9791$.

3  **Sustainability of forward guidance**

The results under optimal discretion can now be used to assess the sustainability of forward guidance. Suppose the central bank announces that it will keep the nominal rate at zero for $k$
periods after the the ELB constraint no longer binds. In period \( k + 1 \), assuming the economy has not returned to the binding ELB, the central bank implements the optimal discretionary policy given by (6). Keeping the nominal rate at zero after an ELB episode has been shown to be part of an optimal commitment policy by Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Nakov (2008) and Werning (2011). The case in which the nominal rate is keep at its effective lower bound for one period \((k = 1)\) is considered first, before generalizing to the case of \( k > 1 \). To preview the results, loss at the ELB is minimized for \( k > 0 \); however if the expected duration of ELB episodes is short \((q \text{ small})\) and the periods away from the ELB are long \((s \text{ large})\), the value of \( k \) that minimizes the present value of losses at the ELB may not be sustainable.

3.1 Keeping the nominal rate at zero for one period

With one-period forward guidance \((k = 1)\), the economy can be in one of three states: at the ELB (state \( z \)), in an exit period with (3) no longer a binding constraint but the nominal rate kept at zero as promised under forward guidance (state \( e \)), or after the forward guidance period has ended, the ELB constraint is nonbinding, and optimal discretion is implemented (state \( n \)). Let superscript \( fg \) indicate outcomes under the forward guidance policy. Losses in the three states are \( L^z_{fg}, L^e_{fg}, \text{ and } L^n_{fg} \). No forward guidance policy would be adopted if it led to a larger loss at the ELB, so \( L^e_{fg} \leq L^n_{n} \) is a necessary condition for a welfare improving policy of forward guidance. However, such a policy will not be sustainable if the present value of the loss obtained by implementing the promised policy in the exit period exceeds the present value of the loss under discretion, i.e., if \( L^e_{fg} > L^n_{n} \). If this condition held, then as soon as the economy exited from the ELB, the policymaker would defect and adopt the optimal time-consistant policy. Private agents, understanding the incentives faced by the policymaker would attach no credibility to the forward guidance provided at the ELB.
The policy would also not be sustainable if $L_{fg}^e > L_{dn}^d$. However, this cannot be the case if $L_{fg}^e < L_{dn}^d$. The reason is that if the economy remains away from the ELB, the forward guidance policy and the optimal discretionary policy both implement the targeting criterion given by the first order condition $\kappa \pi_n + \lambda x_n = 0$. Since expected future inflation and the output gap are closer to their optimal values of zero under forward guidance (as $\pi^f_{fg}$ and $x^f_{fg}$ will be smaller in absolute value than $\pi^d_z$ and $x^d_z$), a better outcome is achieved under the forward guidance policy. Thus, $L_{fg}^e < L_{dn}^d$ implies $L_{fg}^e < L_{dn}^d$. Only a comparison of the present value of losses in the exit period needs to be made to determine the policy’s sustainability.

Equilibrium now involves three inflation rates and three output gaps, corresponding to the situation at the ELB, during the exit period, and when the economy remains away from the ELB. It is also necessary to solve for the nominal interest rate when away from the ELB to ensure it is non-negative. The seven equilibrium conditions are as follows:

$$
\pi^f_{fg} = \beta \left[ q \pi^f_{fg} + (1 - q) \pi^f_{fg} \right] + \kappa x_{fg}
$$

(16)

$$
x^f_{fg} = \left[ q x^f_{fg} + (1 - q) x^f_{fg} \right] + \left( \frac{1}{\sigma} \right) \left[ q \pi^f_{fg} + (1 - q) \pi^f_{fg} + r_z \right]
$$

(17)

$$
\pi^f_{eg} = \beta \left[ s \pi^f_{eg} + (1 - s) \pi^f_{fg} \right] + \kappa x^f_{eg}
$$

(18)

$$
x^f_{eg} = \left[ s x^f_{eg} + (1 - s) x^f_{eg} \right] + \left( \frac{1}{\sigma} \right) \left[ s \pi^f_{eg} + (1 - s) \pi^f_{fg} + \rho \right]
$$

(19)

$$
\pi^f_{ng} = \beta \left[ s \pi^f_{ng} + (1 - s) \pi^f_{ng} \right] + \kappa x^f_{ng}
$$

(20)
\[ \kappa \pi_n^{fg} + \lambda x_n^{fg} = 0. \]  
\tag{21} 

\[ x_n^{fg} = \left[ s x_n^{fg} + (1 - s) x_z^{fg} \right] - \left( \frac{1}{\sigma} \right) \left[ i_n - \left( s \pi_n^{fg} + (1 - s) \pi_z^{fg} \right) - \rho \right] \]  
\tag{22} 

The last equation reflects the assumption that if the economy remains away from the ELB, the optimal time-consistent policy can be implemented as long as \( i_n > 0 \).

To determine how much the promise to keep \( i_e = 0 \) improves over discretion, the present value of losses at the ELB, in the exit period, and in subsequent periods if the economy remains away from the ELB must be calculated. \( L_z^{fg}, \ L_e^{fg}, \) and \( L_n^{fg} \) satisfy the following three conditions:

\[ L_z^{fg} = \frac{1}{2} \left[ \left( \pi_z^{fg} \right)^2 + \lambda \left( x_z^{fg} \right)^2 \right] + \beta q L_z^{fg} + \beta (1 - q) L_e^{fg} \]

\[ L_e^{fg} = \frac{1}{2} \left[ \left( \pi_e^{fg} \right)^2 + \lambda \left( x_e^{fg} \right)^2 \right] + \beta s L_n^{fg} + \beta (1 - s) L_z^{fg} \]

\[ L_n^{fg} = \frac{1}{2} \left[ \left( \pi_n^{fg} \right)^2 + \lambda \left( x_n^{fg} \right)^2 \right] + \beta s L_n^{fg} + \beta (1 - s) L_z^{fg}. \]

The gain from credible forward guidance is defined as

\[ G = L_z^{d} - L_z^{fg}, \]

where \( L_z^{d} \) is the loss at the ELB under optimal discretion. If \( G > 0 \), then the loss is larger under discretion than with forward guidance. Figure 6 shows \( G \), expressed in terms of its steady-state consumption equivalence using (15); it is positive through the range of \( s \) and \( q \) such that \( i_n^{d} > 0 \), indicating that losses are smaller with forward guidance. This reflects the well-known result that promising to keep the nominal rate at zero after the ELB constraint
is no longer binding improves outcomes at the ELB by raising expectations of inflation and the output gap after exiting the ELB. Not surprisingly, the gain increases with $q$, that is, the lower the probability of exiting the ELB, and therefore the longer the expected duration of an episode at the ELB, the greater is the gain from forward guidance. In contrast, the gain decreases with $s$, as more frequent returns to the ELB (a lower $s$) increases the gain from forward guidance.

To assess the sustainability of a promise to keep the nominal interest at zero during the exit period, the present value of losses in the exit period must be less than that obtained by switching to the optimal discretionary policy. That is, sustainability requires that $L_{fg} < L_{dn}$. Define the temptation to defect as

$$T \equiv L_{fg}^{f} - L_{dn}^{d}.$$  

If $T > 0$, the policy of forward guidance is not sustainable. After the initial period in which
\( i_e = 0 \), equilibrium under both the forward guidance and optimal discretion must satisfy (20) - (21). Hence, with 1-period forward guidance, outcomes differ in state \( n \) only because the equilibrium at the ELB differs under the two policies. It follows that if the gain is positive, then \( L_{fg}^n < L_d^n \); only the present value of losses in the exit period need to be assessed to determine whether there will be a temptation to defect from the promised policy of setting \( i_e = 0 \).

The present discounted value of losses that affect the gain from forward guidance and the temptation to defect are shown for \( q = 0.90 \) and 0.85 in figure 7. Previously, it was verified that temptation is positive for \( s = 1 \), in which case forward guidance is unsustainable. Across the range of values of \( s \) shown, the loss from discretion in state \( z \) (solid black line) exceeds that achieved under the forward guidance policy (dashed red line). Temptation is measured by the difference between \( L_{fg}^n \) and \( L_d^n \), and these two losses are shown by the dotted black and dot-dashed red lines, respectively. Except for \( s \geq 0.9999 \), \( L_{fg}^n \) is smaller than \( L_d^n \), implying 1-period forward guidance is sustainable. When \( s = 1 \), the standard result that forward guidance is not sustainable is obtained as temptation is small but positive for all \( q \). When \( s = 0.9999 \), however, temptation is negative. Thus, if there is even a remote probability of a future ELB episode, a promise to maintain the nominal interest rate at zero for one period after the ELB constraint is relaxed is a sustainable policy.

Table 3 reports the gain from 1-period forward guidance and the temptation to defect to optimal discretion. Results are shown for the values of \( q \) and \( s \) given for the values from Table 2 that match the 1960-2017 or the 1934-2017 frequency at the ELB. Results are also shown for \( s = 0.9999 \) and \( s = 1 \). Not surprisingly, the gains from keeping the nominal rate at zero are positive, and they are increasing as \( q \), and therefore the expected duration of ELB episodes, increase. For given \( q \), moving down a row is associated with a fall in \( s \). The gain is not monotonic in \( s \). For \( q = 0.85 \), the gain increases as \( s \) falls, while for \( q = 0.90 \), moving
from $s = 1$ to $s = 0.9999$ increases the gain from forward guidance, while the gain then falls as $s$ is reduced to 0.9861 to match the 1960-2017 frequency and falls further for $s = 0.9794$, the value matching the 1934-2017 frequency. Only for $s = 1$, the case normally considered in the Eggertsson-Woodford framework, is forward guidance not sustainable.

Figure 8 illustrates why forward guidance is sustainable as long as $s$ is even slightly below 1. The figure shows equilibrium inflation (top panel) and the output gap (lower panel) in the exit period and in subsequent periods away from the ELB under one-period forward guidance as a function of $s$ for $q = 0.90$. Also shown are the outcomes under optimal discretion. The dashed lines represent outcomes under discretion; outcomes under forward guidance in the exit period are shown by the solid lines and for subsequent periods away from the ELB by the dot-dashed lines. Note that except when $s$ approaches to 1, inflation in the exit period is closer to zero than it is under optimal discretion. Thus, rather than forward guidance producing worse inflation outcomes when it comes time to implement the promise to keep
the nominal rate at zero, forward guidance produces better inflation outcomes. The reason is that the improved outcomes under forward guidance at the ELB imply expected inflation is closer to zero upon exiting than it is under discretion. As \( s \to 1 \), \( \pi^{fg}_e \) turns positive, but it is still smaller in absolute value than \( \pi^d_n \) except for \( s \) extremely close to 1 (i.e., only for the case in which a return to the ELB is very unlikely). As a consequence, the loss due to inflation deviating from zero is smaller for one-period forward guidance for all \( s < 0.9997 \).

Loss also depends on the output gap. From the lower panel of figure 8, the forward guidance policy does lead to a larger, positive output gap in the exit period than achieved with optimal discretion. However, the baseline calibration in Table 1 assigns a weight of \( \lambda = 0.003 \) to output gap volatility in the loss function. Therefore, the better inflation performance outweighs the poorer output gap performance and accounts for why the standard presumption – that the central banker would not have an incentive to fulfill promises once
the ELB episode ends – does not hold.

Nakata (2017) showed that the fully optimal Ramsey policy is sustainable for values of $s$ even slightly below 1. The policy considered here – one extra period of a zero nominal rate – is inferior to the Ramsey policy. Despite this, it too is sustainable. Thus, even forward guidance that takes a simple form and may therefore be easier to communicate to the public can be supported as a sustainable policy in the absence of a commitment mechanism. This implies that the standard comparison of pure discretionary policies at the ELB with commitment policies is too limited. Even in the absence of an ability to commit to future actions, the promises of a discretionary policymaker can be credible. Forward guidance in the form of a pledge to keep the nominal interest rate at zero for one period after exiting from the ELB is a sustainable policy in an otherwise discretionary regime as long as $s$ is strictly less than 1.

Forward guidance is sustainable because the output and inflation costs of deviating from pure discretion in the exit period are small in the sense that the deviation of $\pi_e^{fg}$ and $x_e^{fg}$ from their counterparts under discretion turn out to be small and, in the case of inflation, outcomes are better (i.e., inflation is closer to zero) under forward guidance. Hence, the cost of fulfilling the promised forward guidance is also small and is dominated by the benefit of improved performance at the ELB; as a result, the present discounted losses under forward guidance are less than under optimal discretion except for $s = 1$.

### 3.2 Multi-period promises

The previous section consider forward guidance that kept the nominal interest rate at zero for one period after the ELB constraint is relaxed. Suppose the central bank promises to keep the nominal rate at zero for $k \geq 1$ periods after exiting an ELB episode. Assume that the economy has remained away from the ELB for the full $k$ periods, policy reverts to the optimal discretion targeting criterion given by (6). Whenever the economy returns to the
binding ELB, the process starts over.

With $k$-period guidance, it is necessary to solve for the equilibrium for $k + 2$ periods: at the ELB, for each of the $k$ periods in which the ELB is no longer binding but the nominal rate is kept at zero, and for the period once policy returns to optimal discretion after the period of forward guidance has ended. The probability that the ELB constraint remains non-binding for the entire $k$ periods for which the central bank promises to keep the nominal rate at zero is $s^{k-1}$. For the smallest value of $s$ considered in Table 2 (0.9691), the probability the economy remains away from the ELB for 8 quarters is 80%, while for $s = 0.9999$, it is 99.93%. Forward guidance is never sustainable when $s = 1$, but the previous section found that 1-period forward guidance was sustainable for $s = 0.9999$. I therefore present results for $k = 0$ (discretion) to $k = 8$ for $s = 0.9999$ for $q = 0.90$ and $q = 0.85$. Results are also reported for $(q, s) = (0.90, 0.9861)$ and $(q, s) = (0.85, 0.9791)$, combinations from Table 2 that match the frequency of quarters at the ELB for the 1960-2017 period.\footnote{Results are similar for the values of $q$ and $s$ that match the 1934-2017 frequencies.}

Table 4 shows results for $s = 0.9999$ when $q = 0.90$ (panel a) and for $q = 0.85$ (panel b). Each row presents the present value of losses at the ELB, during the first period after an ELB episode ends, and when the economy has remained away from the ELB for $k + 1$ periods. Also reported is the gain from forward guidance and the temptation to defect from the promised forward guidance policy. The column labeled $L_z$ shows the present value of losses at the ELB. When $q = 0.90$, a credible promise to keep the nominal interest rate at zero for up to six periods after exiting the ELB episode significantly improves over discretion. The lowest loss, however, is achieved with a promise to keep the nominal rate at zero for four periods beyond the end of the binding ELB constraint. The final column provides evidence on the sustainability of forward guidance. For $k \leq 6$, forward guidance is sustainable. For $k > 6$, the gain is negative and so a promise of $k = 6$ would clearly not be sustainable.
When $q$ is smaller, the expected duration of ELB episodes is shorter. Consequently, the costs of the ELB are lower and the future benefit of fulfilling a promise to keep the nominal rate at zero upon exiting the ELB is smaller. As shown in panel (b) of Table 4, this has two consequences. First, the length of forward guidance that minimizes loss at the ELB falls, with $L_z$ minimized for $k = 3$. But second, a policy that promises to keep the nominal rate at zero for three periods after the ELB no longer binds is not sustainable. The present value of the loss in the exit period exceeds the present value of the loss under discretion. The optimal sustainable policy sets $k = 2$. Loss could be reduced by 25% if forward guidance were extended by one period, but a promise to keep the nominal rate at zero for 3 periods after exiting the binding ELB is not sustainable.

Tables 5 reports results for $q = 0.90$ and $q = 0.85$, but in this case, $s$ is set to the values given in Table 2 that are consistent with the fraction of quarters at the ELB during the
Figure 10: The gain (top panel) and temptation to defect to optimal discretion (lower panel) as a function of $k$ for $q = 0.85$. Note: for $s = 1$ and $0.9999$, temptation is multiplied by 100 to make it visible.

1960-2017 period.\(^\text{19}\) Comparing Table 5 with Table 4 illustrates that the lower value of $s$ increases the loss associated with the ELB constraint. It does so because a fall in $s$ increases the probability of returning to the ELB. For $q = 0.90$, the optimal $k$ is still 4, and a promise to keep the nominal rate at zero for 4 periods is sustainable; it is 3 periods when $q = 0.85$. Figure 9 shows the gain and the temptation when $q = 0.90$ for $s = 0.9999$ (dashes) and $s = 0.9861$ (solid). Also shown for comparison is the case for $s = 1$ (crosses). Figure 10 shows gain and temptation for $q = 0.85$. Not surprisingly, there is very little difference in the gain from forward guidance when $s = 0.9999$ or $s = 1$. What does differ is the sustainability of forward guidance. When $s = 1$, forward guidance is not sustainable as $T > 0$ for all $k$. For $s = 0.9999$, $T$ is negative (though small) for $k < 7$.

\(^{19}\)Table 6, panel (a), only shows outcomes for $k = 0, 1, ..., 5$. For the parameter values used in that panel, $i_n < 0$ for $k > 5$.  

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Figure 11: Inflation (upper panel) and the output gap (lower panel) under discretion, $k = 1$, and $k = 4$ for $q = 0.90$ and $s = 0.9861$.

Figure 11 illustrates the time path of inflation and the output gap if it exits the ELB period at time 0 and remains away from the ELB for at least 8 periods. Results are shown for optimal discretion (i.e., $k = 0$) and the policies associated with $k = 1$ and $k = 4$. The latter policy is, from panel (a) of Table 5, the optimal forward guidance policy. Under discretion, inflation remains negative even after the ELB period because, with $s < 1$, the private sector faces a future episode of deflation. One-period forward guidance lowers the present value of loss at the ELB from 8.83% of steady-state consumption to 7.14% (a 19% reduction); the output gap is slightly higher while at the ELB and lower after the end of the zero nominal rate period. When $k = 4$, the welfare costs of the ELB are almost totally eliminated (they fall by over 99%) and inflation never falls below zero.

For $k \leq 4$, forward guidance is sustainable because of the significant effect forward guidance has in raising inflation and the output gap at the ELB. As a consequence, it also leads both inflation and the output gap to be closer to zero when the economy is away from the
ELB than is achieved by discretion. The equilibrium outcomes for inflation and the output
gap for $k = 0$ (discretion) and for the optimal sustainable values of $k$ for $(q,s) = (0.90, 0.9861)$
and $(q,s) = (0.85, 0.9791)$. are shown in Table 6. Even though the output gap is much larger
during the exit period under forward guidance, Table 6 showed that $L_{fg}$ is only a small
fraction of the loss experienced in the absence of forward guidance.

Multi-period promises improve outcomes significantly relative to pure discretion, and such
promises (as long as they are not for too many periods) can be sustainable. Even though it
has been assumed that there is no commitment mechanism and that the central bank will
renee on past promises whenever the expected present value of losses exceeds that obtained
under discretion, optimal forward guidance is sustainable. A central bank that cannot commit
can still credibly promise to keep interest rates at zero beyond the end of the ELB episode.

4 A discounted Euler equation

McKay, Nakamura, and Steinsson (2016) and McKay, Nakamura, and Steinsson (2017) have
argued that the basic Euler equation given by (1) implies implausibly large effects of forward
guidance, and these large effects arise because expected future output has a one-to-one ef-
fect on current output. The power of forward guidance implies that only modest promises
concerning future policy can significantly reduce the welfare costs at the ELB. Improving
outcomes at the ELB also acts to improve outcomes away from the ELB when $s < 1$, thereby
leading to lower losses in the exit period than is achieved under discretion. Thus, the power
of forward guidance may account for the finding that even multi-period promises are sus-
tainable. To investigate this possibility, the discounted Euler equation proposed by McKay,
Nakamura, and Steinsson (2016) is employed.

Based on an incomplete markets model that leads to precautionary savings on the part
of households, they propose a discounted Euler equation that takes the form

\[ x_t = \delta E_t x_{t+1} - \left( \frac{\chi}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t) , \]  

(23)

with \( 0 < \delta \leq 1 \) and \( 0 < \chi \leq 1 \).\(^{20}\) In their base calibration, they set \( \delta = 0.97 \) and \( \chi = 0.75 \). With these values, together with the same parameter values used by Eggertsson and Woodford (2003), they find the output gap is \(-2.88\%\) at the ELB, significantly less that the \(-14.43\%\) obtained with the standard Euler equation. While they consider only the case in which the economy never returns to the ELB once it exits, similar effects carry over to the case in which \( s < 1 \). Because both inflation and the output are not as negative at the ELB as with the standard Euler equation, expected inflation and output would also be higher for any \( s < 1 \) when the economy is not at the ELB if (23) holds. This means, in turn, that under discretion the nominal interest rate is not as low when the economy is away from the ELB. As a consequence, \( i_n > 0 \) for even small \( s \) and large \( q \), unlike the case found using the standard undiscounted Euler equation (1).

The discounted Euler equation implies the consequences of the ELB and the strength of forward guidance policies are muted. However, the basic findings on sustainability are robust to replacing (1) with (23). Table 7 shows that the optimal number of periods to promise to keep the nominal interest rate at zero after exiting an ELB episode is still four (three) when \( q = 0.90 (0.85) \) and \( s \) is chosen to match the 1960-2017 ELB frequency. In each case, the optimal length of forward guidance is sustainable. Paralleling Table 6, Table 8 shows the inflation and output gap outcomes under discretion and the optimal length of forward guidance with the discounted Euler equation. The muted effects at the ELB with the discounted Euler equation accounts for the result that in panel (a) of Table 7, unlike the case shown in Table 5, the nominal rate on exit remains positive for \( k = 6, 7, \) and 8.

\(^{20}\)Gabaix (2016) develops a behavioral NK model that also implies a discounted Euler equation.
5 Summary and conclusions

Recent research has emphasized the adverse consequences for the economy when the central bank’s policy instrument is constrained by an effective lower bound on the short-term nominal interest rate and policy is implemented in a time-consistent, discretionary manner. These adverse effects stand in contrast to the situation in which the central bank is able to implement the optimal but time-inconsistent commitment policy. Under the presumption that discretion is the more realistic assumption about policy, proposals for reforming inflation targeting policy frameworks have emphasized changes that either make it less likely the ELB will be encountered or that call for establishing alternative regimes, such as price-level targeting, that can cause expectations to move in a manner that promotes stabilization and mimics a commitment policy regime.

Proposed reforms presume that the ELB will be encountered again in the future. Yet analytical analysis of policy at the ELB typically assumes that once the economy exits the ELB, it never again encounters the ELB.\footnote{As noted earlier, the exception is Nakata (2017).} If this is the case, then any promises about future policy – that is, forward guidance – lack credibility. Once the economy is out of the ELB period, there is no incentive for the policymaker to implement the policies that were promised in the past.

But if the economy may revert to the ELB, then promises made during an ELB episode may be credible even in the absence of a commitment mechanism. If the promised policy actions improve outcomes when at the ELB, it may be rational for the central bank to fully implement those promises because, while doing so generates a cost, it also brings an expected future benefit. Future promises may be sustainable.

The main focus of the paper is on the sustainability of date-based forward guidance under discretion. A simple form of such guidance was considered: a pledge to keep the nominal

\footnote{As noted earlier, the exception is Nakata (2017).}
interest rate at zero for $k$ periods after exiting the ELB period. When there is no chance of returning to the ELB, such policies are not sustainable. However, if there is even the slightest chance of returning to the ELB, forward guidance policies may be sustainable. For example, the promise to keep the nominal rate at zero for one period after the ELB constraint is relaxed is sustainable if the probability of another ELB episode is as little as 0.1% per quarter. For multiple-period forward guidance, policies that promise to keep the nominal rate at zero for too long are unsustainable. However, in a calibrated version of the model, the optimal length of forward guidance was sustainable unless ELB episodes were expected to be short. Thus, if future episodes at the ELB are both likely and likely to be long, the promises of policymakers operating in an environment of discretion may be credible, despite the absence of any formal mechanism to ensure commitment.

**References**


Billi, R. M. (2013): “Nominal GDP Targeting and the Zero Lower Bound: Should We Abandon Inflation Targeting?,”.


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### Table 1: Benchmark Values

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<tr>
<th>Parameter</th>
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### Table 2: Benchmark Values for $s$

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### Table 3: One-period Forward Guidance

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<td>0.0913</td>
<td>−0.0640</td>
</tr>
<tr>
<td>1934-2017</td>
<td>0.1140</td>
<td>−0.0895</td>
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### Notes: Losses expressed as percent of steady-state consumption.
Table 4: Multiperiod forward guidance: $s = 0.9999$

<table>
<thead>
<tr>
<th>panel (a): $q = 0.90$</th>
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<tbody>
<tr>
<td>$k$</td>
<td>$L_z$</td>
<td>$L_e$</td>
<td>$L_n$</td>
<td>Gain</td>
<td>Temptation</td>
</tr>
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<td>0.0498</td>
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<td>0.0000</td>
</tr>
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<td>0.0301</td>
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<td>-0.0197</td>
</tr>
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<tr>
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<td>0.0044</td>
<td>0.0033</td>
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<td>0.0073</td>
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<td>-0.0364</td>
</tr>
<tr>
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<td>0.0283</td>
<td>2.1849</td>
<td>-0.0097</td>
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<td>0.0390</td>
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<td>0.1667</td>
<td>0.1306</td>
<td>-8.2358</td>
<td>0.1169</td>
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<table>
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<td>Temptation</td>
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<td>0.0002</td>
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Note: Losses expressed as percent of steady-state consumption.
Table 5: Multiperiod forward guidance

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<th>$L_n$</th>
<th>Gain</th>
<th>Temptation</th>
</tr>
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<td>2.8712</td>
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<td>6.4829</td>
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</table>

Note: Losses expressed as percent of steady-state consumption.

<table>
<thead>
<tr>
<th>k</th>
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<th>$L_e$</th>
<th>$L_n$</th>
<th>Gain</th>
<th>Temptation</th>
</tr>
</thead>
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<td>5.2770</td>
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<td>5.3103</td>
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Note: Losses expressed as percent of steady-state consumption.
Table 6: Discretion and forward guidance

\[ q = 0.90, \ s = 0.9861, \]

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \pi_z )</th>
<th>( \pi_e )</th>
<th>( \pi_n )</th>
<th>( x_z )</th>
<th>( x_e )</th>
<th>( x_n )</th>
</tr>
</thead>
<tbody>
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<td>-0.829</td>
<td>-11.870</td>
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<tr>
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<td>0.026</td>
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<td>1.995</td>
<td>-0.044</td>
</tr>
</tbody>
</table>

\[ q = 0.85, \ s = 0.9791 \]

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \pi_z )</th>
<th>( \pi_e )</th>
<th>( \pi_n )</th>
<th>( x_z )</th>
<th>( x_e )</th>
<th>( x_n )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.171</td>
<td>-2.361</td>
<td>0.284</td>
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<tr>
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<td>0.141</td>
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<td>1.967</td>
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</table>

Notes: Inflation at annual rates; output gap in percent.
Table 7: Discounted Euler

\[ q = 0.90 \quad s = 0.9861 \]

<table>
<thead>
<tr>
<th>( k )</th>
<th>( L_z )</th>
<th>( L_e )</th>
<th>( L_n )</th>
<th>Gain</th>
<th>Temptation</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.2636</td>
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<td>0.0000</td>
</tr>
<tr>
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<td>0.1735</td>
<td>0.1511</td>
<td>-0.0901</td>
</tr>
<tr>
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<td>0.1543</td>
<td>0.0921</td>
<td>0.0919</td>
<td>0.2880</td>
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</tr>
<tr>
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<td>0.0297</td>
<td>0.0292</td>
<td>0.3931</td>
<td>-0.2340</td>
</tr>
<tr>
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<td>0.0033</td>
<td>0.0019</td>
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<td>0.0400</td>
<td>0.0364</td>
<td>0.3810</td>
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</tr>
<tr>
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<td>0.1847</td>
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<tr>
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</table>

\[ q = 0.90 \quad s = 0.9791 \]

<table>
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<th>( L_e )</th>
<th>( L_n )</th>
<th>Gain</th>
<th>Temptation</th>
</tr>
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<tbody>
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<td>-0.0338</td>
</tr>
<tr>
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<td>0.0193</td>
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<tr>
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<tr>
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</table>

Note: Losses expressed as percent of steady-state consumption.
Table 8: Discretion and forward guidance, Discounted Euler

\[ q = 0.90, \ s = 0.9861 \]

<table>
<thead>
<tr>
<th></th>
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<th>( \pi_e )</th>
<th>( \pi_n )</th>
<th>( x_z )</th>
<th>( x_e )</th>
<th>( x_n )</th>
</tr>
</thead>
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<td>-0.186</td>
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<td>0.004</td>
</tr>
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</table>

\[ q = 0.85, \ s = 0.9791 \]

<table>
<thead>
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<th>( \pi_e )</th>
<th>( \pi_n )</th>
<th>( x_z )</th>
<th>( x_e )</th>
<th>( x_n )</th>
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</table>

Note: Inflation at annual rates; output gap in percent.