Simple Sustainable Forward Guidance at the ELB*

Carl E. Walsh†

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Abstract

Forward guidance plays an importnat role in models of monetary policy at the effective lower bound (ELB) for nominal interest rates. Yet in common frameworks used to study equilibrium at the ELB, forward guidance is not credible. In this paper, I ask whether policy makers lacking the ability to commit may, nevertheless, still make credible announcements about future policy. They can if there is a positive probability of hitting the ELB in the future. Nakata (2014) has shown that with even a small probability of future ELB episodes, the optimal Ramsey policy can be sustained. However, such policies are potentially difficult to communicate to the public. I examine the sustainability of simple promises, such as a promise to keep the nominal interest rate at zero for a fixed number of periods after the ELB episode ends. I show that such promises are sustainable, as long as the promise is not for too many periods. If the expected duration of ELB episodes is short and the expected duration away from the ELB is long, the length of forward guidance that minimizes the present value of losses at the ELB may not be sustainable.

1 Introduction

The current era of very low interest rates has raised troubling questions for all central banks, but particularly for those that target inflation. Do the dangers of hitting the effective lower

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†Department of Economics, University of California, Santa Cruz, walshc@ucsc.edu.

bound (ELB) for short-term interest rates call for increasing inflation targets as insurance against returning to the ELB? Does inflation targeting still provide an inadequate framework for monetary policy? Or does the presence of the ELB imply inflation targeting should be replaced by some other policy framework, such as price-level targeting?

The discussion of the issues surrounding these questions – and on the consequences of the ELB more generally – have reached two conclusions. First, in an environment in which the central bank is able to credibly commit to future actions, the costs of the ELB are small. For example, this is the conclusion of the work by Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006) and Nakov (2008).[1] A central bank able to commit to future actions is not unduly constrained when its current policy rate is at its lower bound; making promises about the future path of the policy rate is sufficient to allow policymakers to influence economic activity effectively. If commitment is the appropriate way to understand the monetary policy environment, then the ELB does not call for any reform of inflation targeting or for raising the average inflation target.

Second, if a central bank is able to commit to a policy framework such as inflation targeting but implements policy within that regime in a discretionary fashion, then the ELB can be very costly, as shown for example by Adam and Billi (2007). This conclusion leads naturally to the proposal of Blanchard, Dell'Ariccia, and Mauro (2010) to raise the average inflation target, making it less likely that the ELB will be encountered. It also leads to proposals to replace inflation targeting with alternatives policy regimes, such as price-level targeting, in which discretionary policy is able to mimic some of the advantages of commitment, as shown by Vestin (2006).[2]

Finding policy regimes that can limit the adverse effects of the ELB is important, as episodes of very low interest rates cannot, as they once were, be viewed as extremely rare events. Figure 1 shows histograms of U.S. short term interest rates. The top panel is based on the monthly effective federal funds rate from January 1960 to January 2017, while the lower panel is for the 3-month Treasury bill rate since 1934. Both show that a large fraction of months have seen rates below 25 basis points. For the shorter sample based on the funds

[1] Reifschneider (2016) demonstrates the effectiveness of credible forward guidance (together with balance sheet policies) using the FRB/US model. Levin, López-Salido, Nelson, and Yun (2010) argue that forward guidance may be less effective in the face of large and persistent shocks that drive the economy to the ELB.

[2] Recent work on price-level and nominal income targeting includes Billi (2013), Giannoni (2014), Billi (2015). Bodenstein (2017) show that discretionary policies that focus on stabilizing inflation and the change in the output gap (speed limit policies as analyzed in Walsh (2003)) can outperform flexible inflation targeting and price level targting. targeting

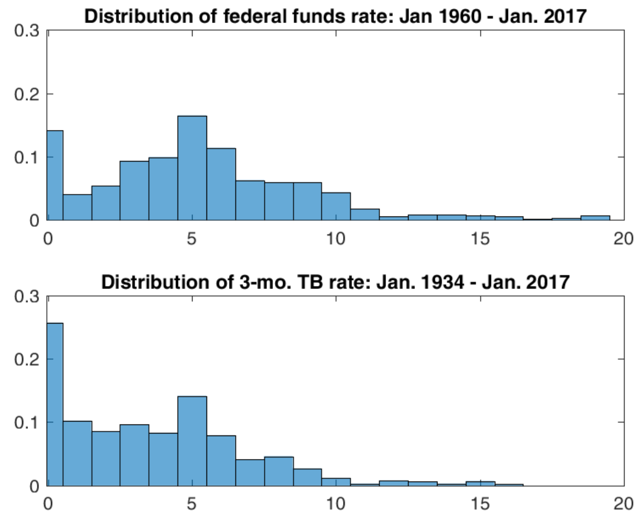

Figure 1: Histogram of U.S. interest rates. Upper panel: federal funds rate. Lower panel: 3-month T-Bill rate.

rate, 13% of months have seen the funds rate at or below 25 basis points. For the longer period, the 3-month T-bill rate fell below 25 basis points in 17% of all months.[3]

If policy is inevitably discretionary in nature, the occurrence of frequent episodes at the ELB is a strong argument for raising inflation targets or adopting price-level targeting. If commitment provides a better description of policy, then forward guidance about future policy can serve to mitigate significantly the effects of the ELB. Most of the literature that has focused on the monetary policy consequences of the ELB has treated the credibility of the central bank as either complete, as in commitment equilibria, or totally absent, as in analyses of discretion. In the one case, future promises are fully believed and subsequently delivered on. In the latter case, the public places no weight on any promises the central bank might make. Such promises – forward guidance – are thus either extremely powerful, as in work on the forward guidance puzzle by DelNegro, Giannoni, and Patterson (2012), Cochrane (2013), and McKay, Nakamura, and Steinsson (2016b), or completely powerless in a discretionary environment.[4]

Forward guidance has frequently been analyzed using simple analytical frameworks that

[3] This histogram is misleading in the sense that, to take the top panel, all the months at or below 25 basis points occurred consecutively between December 2008 and December 2015.

[4] Exceptions include Bodenstein, Hebden, and Nunes (2012) and Nakata (2014) which are discussed below.

have helped provide insights into the consequences of the ELB and the role of forward guidance. For example, Eggertsson and Woodford (2003) introduced the assumption that each period there is a fixed probability of exiting the ELB. This approach has been used by Eggertsson (2011), Christiano, Eichenbaum, and Rebelo (2011), Braun, Körber, and Waki (2012), and McKay, Nakamura, and Steinsson (2016b), among others. Alternatively, several authors have considered perfect foresight equilibria in which the ELB will bind for a known number of periods. For example, Werning (2011), Carlstrom, Fuerst, and Paustian (2012), Cochrane (2013), and Kiley (2016) use such a framework. Under either approach, the assumption has been that the ELB is a one-off occurrence. Once the economy exits from the ELB, it never returns. In this case, announcements can never be credible absent a commitment technology. Under discretion, there is no benefit to fulfilling promises made during an ELB episode; any credibility gained from fulfilling promises is of no future use.

The situation changes if the economy may encounter the ELB again. This, of course, is the presumption of work examining the role of the inflation target or the policy regime in reducing the probability of or mitigating the effects of future ELB episodes. But if the economy may return to the ELB, a rational central bank may have an incentive to fulfill past promises, even under discretion. Doing so brings a future benefit of credibility should the ELB again bind. In fact, Nakata (2014) has shown that the fully optimal Ramsey policy can be sustained if there is only a slight probability the ELB will occur in the future. This is an important result and implies that pure discretion is not the appropriate benchmark against which to evaluate proposals to switch to price level targeting or to raise the inflation target.

Nakata's results show that a policymaker who could deviate may nevertheless find it optimal to follow the optimal Ramsey policy, but such a policymaker may find it difficult to communicate this policy. And appropriately steering expectations in ways that sustain the Ramsey policy may be difficult precisely because the policymaker is known to be able to defect. It is, therefore, of interest to examine the sustainability of policies that are suboptimal relative to Ramsey but whose simplicity may make them easier to communicate to the public.[5]

Of course, if promises made during an ELB period are extreme enough, it is unlikely

[5] Bilbiie (2017) analyzes simple forward guidance policies in which the duration of the policy is stochastic; after exiting the ELB, the central bank keeps the nominal rate at zero with a constant probability. He is able to obtain closed form solutions and investigates the optimal expected duration of forward guidance in an environment in which future ELB episodes never occur.

under discretion that a central bank will fulfill them even if the economy may someday return to the ELB. However, as others have noted (Carlstrom, Fuerst, and Paustian (2012), Kiley (2016), McKay, Nakamura, and Steinsson (2016b)), forward guidance is very powerful in standard new Keynesian models. This suggests that the central bank may need to make only modest promises at the ELB. If so, the costs of fulfilling them may be correspondingly small. Thus, the power of forward guidance, combined with the possibility of a return to the ELB, may lead even a discretionary policymaker to make and keep promises. Forward guidance may be sustainable.

In this paper, I investigate a simple form of forward guidance and ask whether a policymaker who is unable to commit can still make promises about future policy that it will in fact be rational to fulfill. If so, the stark contrast between the consequences of the ELB under discretion and under commitment may be too exaggerated. And if this is true, the case against inflation targeting and the arguments for raising the inflation target or switching to price-level targeting are weakened. Effective and sustainable forward guidance would reduce the need for these alternatives. Their merits would need to be based on considerations other then their effects in reducing the probability of encountering the ELB or their superior performance (relative to discretion) at the ELB, a point also made by Loisel (2008).

Pure discretionary and optimal commitment are extreme alternatives. One implies a complete absence of credibility to fulfill promises; the other involves complete credibility. If future promises are credible even in a discretionary environment, the sharp distinction between discretion and commitment is blurred and credibility is no longer an all or nothing property of policy actions. Two literatures have developed approaches that allow for partial credibility. The first follows the stochastic planning problem analyzed by Roberds (1987), and includes the work by Schaumburg and Tambalotti (2007), Debortoli and Nunes (2010), Bodenstein, Hebden, and Nunes (2012), and Debortoli, Maih, and Nunes (2014). The second builds on notion of sustainable plans developed by Chari and Kehoe (1990) and Stokey (1991) and employed by Ireland (1997), Kurozumi (2008), Kurozumi (2012) and Nakata (2014).[6]

[6] In the presence of endogenous state variables, current policy choices can affect the incentives faced by future policymakers, thereby generating a channel through which the policymaker can effectively influence expectations about future policy. For example, Jeanne and Svensson (2007) have investigated how generating a large increase in the government's nominal debt can create an incentive for future inflation. Thus, a government's concerns about its balance sheet can provide a mechanism for current policy to influence future policy choices. This channel is absent in the present paper which employs a basic new Keynesian model in which there are no endogenous state variables.

The stochastic planning approach of Roberds (1987) and Schaumburg and Tambalotti (2007) assumes a policymaker is able to commit to future policies, but each period there is an exogenous probability a new policymaker will be appointed. Future policymakers are not constrained by the promises made by their predecessors, so promises are discounted to reflect the likelihood that the current policymaker will be replaced.[7] If the current policymaker will, with certainty, not be around to implement any promises, pure discretion emerges. At the other extreme, if the current policymaker remains in office forever with certainty, promises are completely credible.

Closely related to the imperfect credibility that arises with stochastic changes in the policymaker is the notion of loose commitment developed by Debortoli and Nunes (2010) in analyzing fiscal policy and that has been applied to monetary policy issues in Bodenstein, Hebden, and Nunes (2012), Dennis (2014) and Debortoli, Maih, and Nunes (2014). Under loose commitment, there is a fixed probability each period that the policymaker reoptimizes. Because the policymaker may reoptimize in the future, any promises made are discounted, as past promises are ignored when the policymaker reoptimizes.

In contrast to this literature, I follow Chari and Kehoe (1990), Stokey (1991) and the work by Ireland (1997), Kurozumi (2008) and Kurozumi (2012) in focusing on sustainable plans under discretion.[8] That is, I assume the absence of any commitment technology. A past promise might be honored, but only if doing so is the best strategy for the policymaker at the time the promise needs to be honored. Kurozumi (2008) has investigated whether the optimal commitment policy in the basic new Keynesian model is sustainable under discretion. He shows that the optimal sustainable policy falls between that of optimal discretion and optimal commitment, but it converges over time to the optimal commitment policy if the policymaker's discount rate is not too large. Kurozumi (2012) shows that a regime of flexible inflation targeting is sustainable, but only if the central banker places more weight – but not too much weight – on inflation stability than is reflected in social welfare. That is, the central banker must be a Rogoff (1985) conservative, but not too conservative. In contrast, Loisel (2008) shows a trigger strategy equilibrium can support a reputational equilibrium that overcomes discretionary inflation and stabilization biases without needing to delegate to a conservative central banker.

[7] An early example of a model in which equilibrium was affected by the probability of a future change in policy maker was provided by Ball (1995). In his model, however, the new policy maker was drawn from a distribution of policy makers who differed in their preferences.

[8] This literature builds on Abreu (1988). See also Levine, McAdam, and Pearlman (2008).

What has not been examined is whether announcements of the type associated with date-based forward guidance can form part of a sustainable policy plan. This gap in the existing literature is one this paper hopes to fill. The rest of the paper is organized as follows. Section 2 follows Nakata (2014) in modifying the framework of Eggertsson and Woodford (2003) to allow for a positive probability that after exiting an ELB episode the economy may again encounter a binding ELB constraint. Equilibrium under pure discretion, which serves as a benchmark of comparison for forward guidance policies, is examined in section 3. The analysis then considers alternative forms of forward guidance. In section 4, the effects of a promise to keep the nominal rate at zero after an ELB episode ends are studied. The case of one-period forward guidance is considered first, while promises to keep the nominal rate at zero for several periods after an ELB episode ends are then investigated, together with the optimal length of forward guidance. The robustness of the results to some modifications of the model are discussed in section 5, while conclusions are summarized in section 6.

2 Recurring episodes at the ELB

The basic model adopts the two-state Markov structure of Eggertsson and Woodford (2003), modified following Nakata (2014) to allow for a positive probability of returning to the ELB in the future.[9] Private sector behavior is described by the standard linear new Keynesian model represented by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right), \tag{1}$$

and

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \tag{2}$$

together with a specification of monetary policy. For convenience the ELB on the nominal interest rate is taken to be zero, so

$$i_t \geq 0. \tag{3}$$

[9] A number of authors (Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006), Adam and Billi (2007), Nakov (2008), Levin, López-Salido, Nelson, and Yun (2010), Billi (2015)) have examined stochastic equilibria in new Keynesian models subject to occasionally binding lower bounds on the nominal interest rate. In these models, the economy can pass into, out of, and back into periods during which the lower bound constraint is binding. However, this literature has not investigated specific examples of forward guidance. Work on assessing the empirical effects of forward gudiance include Campbell, Evans, Fisher, and Justiniano (2012), Campbell (2016) and Swanson (2016).

(1) is the Euler condition for intertemporal consumption choice, where x_t is the output gap, π_t the inflation rate, i_t is the nominal interest rate, and r_t is an exogenous stochastic process. (2) is the reduced form equation of inflation that can be derived from a time-dependent model of price adjustment such as the Calvo model.[10]

The monetary authority desires to minimize

$$L_t = \frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda x_{t+j}^2\right). \tag{4}$$

The policy environment is one of discretion; there is no formal mechanism that allows the policymaker to commit to future policy actions.

The shock r_t in (1) follows a two-state Markov process. In the state z, $r_t = r_z < 0$ and $i_z = 0$; in state n, $r_t = \beta^{-1} - 1 \equiv \rho > 0$. If $r_t = r_z$, then $r_{t+1} = r_z$ with probability q and $r_{t+1} = \rho$ with probability $1 - q$. If $r_t = \rho$, then $r_{t+1} = \rho$ with positive probability s, $0 < s \leq 1$, and $r_{t+1} = r_z$ with probability $1 - s$. Thus, $1 - s$ is the probability of reverting to the ELB. The previous literature building on the analytical structure of Eggertsson and Woodford (2003) set $s = 1$, implying that once the economy exits from the ELB, it never returns. Similarly, the literature that treats the ELB as binding for a fixed number of periods after which it never binds again similarly assumes there is never any return to the ELB (see, for example, Werning (2011), Cochrane (2013) and Kiley (2014)). When $s < 1$, the economy can experience repeated episodes in both state z and state n. Let π_j and x_j denote equilibrium inflation and the output gap in state $j = z, n$; let i_n denoted the nominal interest rate in state n.

In a discretionary policy environment, when will a policymaker find it is incentive compatible to fulfill past promises that were made when the ELB was binding? If $s = 1$ so that economy never returns to the ELB, optimal discretion can deliver $\pi_n = x_n = 0$. Thus, any promise made at the ELB that involves either inflation or the output gap deviating from zero would incur a larger loss than simply implementing the optimal discretionary policy. The policymaker has no incentive to do anything except renege on past promises. Any promises made at the ELB that would imply non-zero values for x or π will never be honored by a policymaker acting with discretion to minimize (4). Absent a commitment technology, the

[10] The underlying nonlinear model that leads to the reduced form equations employed here is so well known that providing details on it seems unnecessary. See, for example, chapter 11 of Walsh (2017), which provides an extended discussion of the ELB. See Eggertsson and Singh (2016) for a justification for the use of the log linear approximation at the ELB.

only credible policy upon exit is to set $i_n = \rho$, consistent with $x_n = \pi_n = 0$.[11] But if $s < 1$, a policymaker may find it optimal to honor past promises. Doing so may entail lower losses in future states in which the ELB is again binding.

To be more specific, let L_j^d denote the present value of losses in state j under the optimal period-by-period discretionary policy that, in each period, minimizes the policymaker's loss function, taking expectations and future policy as given. Let L_j^o be the present value of losses when the economy is in state j under an arbitrary policy o. The policy o may involve promises made in the past about policy actions in the current state. Such a policy is sustainable if $L_j^o \leq L_j^d$ for all j. That is, continuing to implement policy o, including any promises made in the past, constitutes a sustainable plan if the present value of losses obtained by implementing the policy is, in every state, less than that obtained by reverting to the policy d. A sustainable policy is time-consistent; the policymaker has no incentive to switch from the policy and adopt the discretionary policy.[12] In other words, any contingent sequence of inflation, the output gap, and the nominal interest rate that satisfies (1) - (3) for every $t \geq 0$ is called sustainable if, for each $t \geq 0$, the present discounted value of losses is less than the present value of losses under the optimal, time-consistent discretionary policy. Thus, policies for which the current period's loss exceeds that obtained under the discretionary policy may still be sustainable if future losses under the policy are less than those under discretion.

3 Optimal discretion

To assess the sustainability of forward guidance policies, it is necessary to first determine equilibrium under optimal discretion. If there is a positive probability of returning to the ELB, private agents, in forming expectations about future inflation and the output gap, will place positive weight on the equilibrium inflation and output gap that occurs at the ELB. Consequently, equilibrium away from the ELB now depends on equilibrium at the ELB. And the converse also holds.

[11] A policy that simply sets $i_t = \rho$ does not ensure $x = \pi = 0$ is the unique, stationary equilbrium under rational expectations. A rule of the form $i_t = \pi + \phi\pi_t$ with $\phi > 1$ would do so.

[12] The concept of a sustainable policy plans was first introduced by Chari and Kehoe (1990). Stokey (1991) defines a pair of strategies (for the government and private sector) that is compatible with a competitive equilibrium in the private sector, given the government's strategy, and for which the government has no incentive to alter its strategy as a *credible policy*. See Nakata (2014) for a formal treatment of sustainability in the context of the Markov structure I employ.

The policymaker under pure discretion faces a static decision problem each period that involves minimizing

$$l_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right), \tag{5}$$

subject to (1), (2), and (3), taking expectations and future policy as given. Optimal policy is characterized by a targeting rule of the form[13]

$$\kappa\pi_t + \lambda x_t = 0. \tag{6}$$

Using superscript d to denote the equilibrium under discretion, inflation π_n^d and the output gap x_n^d when the ELB is not binding solve

$$\pi_n^d = \beta\left[s\pi_n^d + (1-s)\,\pi_z^d\right] + \kappa x_n^d, \tag{7}$$

and

$$\kappa\pi_n^d + \lambda x_n^d = 0, \tag{8}$$

where expected inflation is equal to $s\pi_n^d + (1-s)\,\pi_z^d$. Equilibrium must also satisfy the non-negative constraint on i_t; this requires that

$$i_n^d = \rho + \left[s\pi_n^d + (1-s)\,\pi_z^d\right] + \sigma\,(1-s)\left(x_z^d - x_n^d\right) \geq 0, \tag{9}$$

where this last equation is obtained by solving the Euler condition (1) in state n.

When $s = 1$, $\pi_n^d = x_n^d = 0$ constitutes an equilibrium under discretion when the ELB is nonbinding. When $s < 1$, it is no longer feasible to achieve a zero inflation rate and output gap, as neither expected inflation, $s\pi_n^d + (1-s)\,\pi_z^d$, nor the expected output gap, $sx_n^d + (1-s)\,x_z^d$, equal zero. As long as some probability is assigned to that possibility the economy will return to the ELB, expected inflation and the expected output gap when not at the ELB will depend on x_z^d and π_z^d.

The dependence of equilibrium away from the ELB on the equilibrium at the ELB can

[13] In most of the literature using this model, policy after the ELB episode ends is characterized by a simple instrument rule rather than by optimal discretion. In the present context, $\pi_n = x_n = 0$ is also the locally unique stationary equilibrium if the nominal rate is given by $i_n = \rho + \phi\pi_n$ once the ELB constrain no longer binds, with $\phi > 1$. The choice of ϕ, as long as it exceeds 1, plays no role in affecting equilibrium at the ELB or away from the ELB when the ELB episode is a one-off event.

be make explicit by solving (7) and (8) for π_n and x_n, obtaining[14]

$$\pi_n^d = (1-s)\left[\frac{\beta\lambda}{\lambda(1-\beta s)+\kappa^2}\right]\pi_z^d,$$

and

$$x_n^d = -(1-s)\left[\frac{\beta\kappa}{\lambda(1-\beta s)+\kappa^2}\right]\pi_z^d,$$

With deflation at the ELB, $\pi_z^d < 0$. Then, when $s < 1$, optimal discretion implies $\pi_n^d < 0$ and $x_n^d > 0$ such that $\kappa\pi_n^d + \lambda x_n^d = 0$. Away from the ELB, π_n^d and x_n^d also depend on q, because the the probability of remaining at the ELB affects π_z^d.

As π_z^d becomes more negative, or as the probability of reverting $1-s$ becomes larger, x_n^d becomes larger, and to generate this expansion, the policymaker reduces the nominal rate i_n^d. However, equilibrium away from the ELB requires that i_n^d satisfy the non-negativity constraint (3); it could be that the central bank's first-order condition (8) under the optimal discretionary policy would require the nominal interest rate to be negative. The value of i_n^d consistent with (1) can be written as

$$i_n^d = \rho + \pi_n^d + (1-s)\left[\sigma\left(x_z^d - x_n^d\right) + \left(\pi_z^d - \pi_n^d\right)\right]. \tag{10}$$

With $x_z^d < x_n^d$ and $\pi_z^d < \pi_n^d$, the term in brackets is negative. Thus, a large probability of returning to the ELB (a small s) can lead to a violation of the nonnegativity constraint on i_n^d. If q is large, the ELB episode is expected to be of long duration, and this reduces π_z^d and x_z^d, contributing to a fall in π_n^d and x_n^d and reducing the nominal interest rate when the economy is not at the ELB. In this case, there may be regions of the parameter space in which the nonnegativity constraint on i_n^d is violated.

At the ELB, equilibrium is characterized by (11) and (12):

$$\pi_z^d = \beta\left[q\pi_z^d + (1-q)\,\pi_n^d\right] + \kappa x_z^d \tag{11}$$

$$x_z^d = \left[q x_z^d + (1-q)\,x_n^d\right] + \left(\frac{1}{\sigma}\right)\left[q\pi_z^d + (1-q)\,\pi_n^d + r_z\right]. \tag{12}$$

[14] If the central bank reverts to a simple rule such as $i_t = r_t + \phi\pi_t$ once the ELB episode ends, equilibrium also involves $\pi_n < 0$ and $x_n > 0$. However, unlike the optimal discretionary targeting rule in (8), the simple rule does not generally ensure the optimal tradeoff between deflation and real expansion is achieved. The Appendix derives the value of ϕ that replicates the outcomes that occur under optimal discretion.

Equations (7) - (12) can be solved jointly to obtain equilibrium inflation and the output gap in states z and n and the nominal rate in state n. To do so, the baseline calibration is given in Table 1, which is based on the values employed by Eggertsson and Woodford (2003) and used more recently by Nakata (2014) and McKay, Nakamura, and Steinsson (2016b). The loss function (4) is interpreted as derived from a second-order approximation of the welfare of the representative household around the economy's efficient equilibrium; in this case Woodford (2003) showed that $\lambda = \kappa/\theta$, where θ is the price elasticity of demand faced by individual firms. Using Woodford's value of $\theta = 7.88$ implies $\lambda = 0.003$. If inflation is expressed at an annual rate, then $\lambda^a = 16\lambda = 0.048$.

Table 1: Benchmark Values

Parameter	Values (quarterly rates)
β	0.99
σ	2
κ	0.02
r_z	-0.005
ρ	$\beta^{-1} - 1 = 0.01$
λ	0.003

The final two parameters of the model are s and q. To discipline the calibration of the two transition probabilities when $s < 1$, I employ the evidence based on figure 1. Define

$$P \equiv \begin{bmatrix} s & 1-q \\ 1-s & q \end{bmatrix}.$$

The steady-state fractions of time spend away from the ELB and at the ELB are given by the diagonal elements of $\lim_{T\to\infty} P^T$. I match these fractions to either the 1960-2017 sample frequencies (87% of the time away from the ELB, 13% of the time at the ELB) or the longer 1934-2017 sample period (83% and 17% in the two states respectively). Figure 2 shows the combinations of s and q that match these two fractions. Eggertsson and Woodford (2003) and McKay, Nakamura, and Steinsson (2016a) assume $q = 0.9$, and for this value of q, $s = 0.9858$ (indicated by an x in the figure) implies a steady-state frequency at the ELB of 13%, while $s = 0.9792$ (indicated by the o) implies a steady-state frequency of 17%. These two values of s, together with $q = 0.9$, will be employed in the baseline calibration. To also assess the effects of a smaller continuation probability at the ELB, calibrations based on

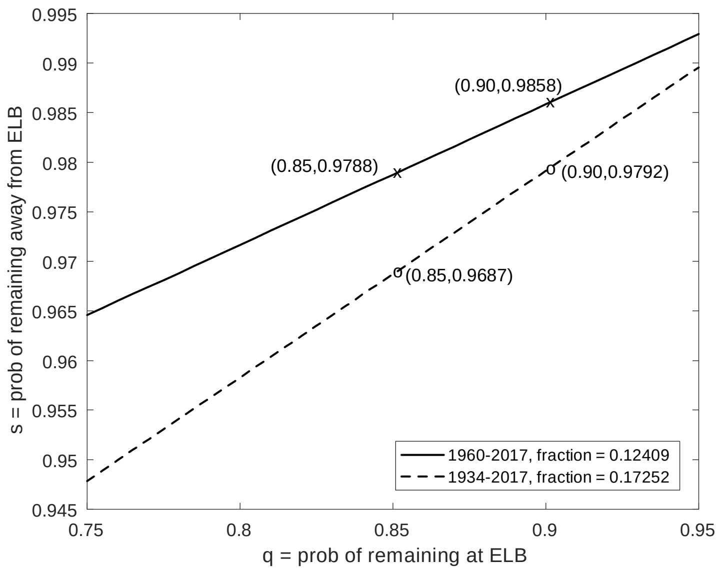

Figure 2: The solid (dashed) line shows (q, s) combinations consistent with the economy being at the ELB 17% (12%) of the time. For $q = 0.9$ ($q = 0.85$), the 1934-2017 period is matched when $s = 0.979$ ($s = 0.9687$), while the 1960-2017 period is matched when $s = 0.9858$ ($s = 0.9788$).

$q = 0.85$ will also be used. The associated values of s for the two samples for this lower value of q are indicated in the figure and given in Table 2.[15]

Table 2: Benchmark Values for s

	$q = 0.85$	$q = 0.90$
1960-2017	0.9788	0.9858
1934-2017	0.9687	0.9792

Optimal discretion may result in a binding ELB constraint when $r_t = \rho$. Then can occur when π_z^d is sufficiently negative that expected inflation when away from the ELB is negative, leading π_n^d to also be negative. To maintain the targeting rule (8) characterizing optimal discretion would requires a positive output gap. If x_z^d is large and negative, achieving a positive value for x_n^d may force i_n^d down to zero. Denote the value of i_n^d as $i_n^d(s, q)$ to

[15] As discussed in Eggertsson (2011), the economy experiences what he describes as deflationary black hole if q rises above 0.9. As $q \to 1$, it enters what Braun, Körber, and Waki (2012) characterize as a type 2 equilibrium. As discussed in the next section, I restrict attention to vaues of $q \leq 0.90$ to be consistent with equilibria in which $x_z < 0$, $\pi_z < 0$, and $i_n > 0$ under optimal discretion.

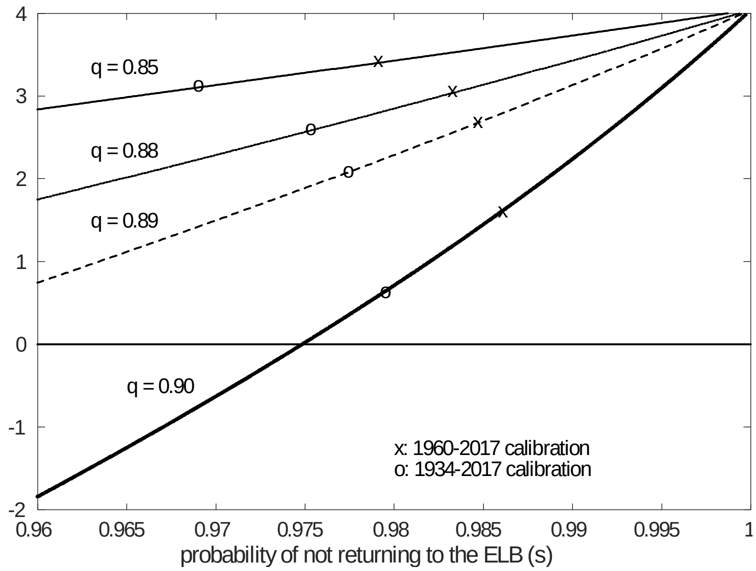

Figure 3: Nominal interest rate under discretion when ELB is nonbinding. x (o) indicates (q s) combinations consistent with fraction of time at ELB based on 1960-2017 (1934-2017) sample.

highlight its dependence on s and q. Define $\Omega = \left\{s, q \text{ s.t. } i_n^d(s,q) \geq 0\right\}$. In the subsequent analysis, attention is restricted to $(s,q) \in \Omega$. Figure 3 shows the level of the nominal rate consistent with optimal discretion for ranges of s and q. For the standard value of $q = 0.90$, the non-negativity constraint on i_n^d is binding only for $s > 0.975$. Thus, for both the baseline calibrations used for s and q (shown by the x and o markers on the $q = 0.9$ line), $i_n^d > 0$.

Figures 4 shows equilibrium inflation (upper panel) and the output gap (lower panel) under discretion as a function of q when the ELB is binding and when it isn't for $s = 1.0$ and $s = 0.9687$. This latter value of s is chosen as it is the smallest value from table 2. Recall that $\pi_n^d = x_n^d = 0$ for $s = 1$. The output gap under discretion rises as the likelihood of returning to the ELB rises (s falls below 1). With reversion to the ELB more likely, expected inflation when the ELB does not bind falls, and in response, i_n^d is reduced to increase the output gap and maintain the targeting criterion (6). Note, however, that for the values of s shown, the impact of a higher probability of another ELB episode has a relatively minor effect on the equilibrium, especially for smaller values of q.

Let L_k^d for $k = z, n$ be the present discounted value of losses in state k under pure

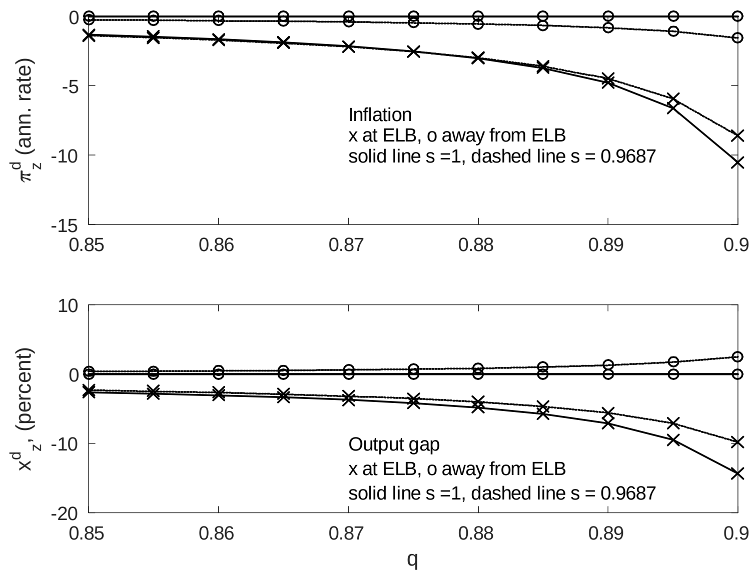

Figure 4: Inflation and the output gap under optimal discretion as a function of q for $s = 1$ (solid lines) and $s = 0.9687$ (dashed lines). Values at the ELB indicated by x; values away from the ELB indicated by o. Upper panel: Inflation. Lower panel: output gap.

discretion. Then L_z^d and L_n^d satisfy the following valuation equations:

$$L_z^d = \frac{1}{2}\left[\left(\pi_z^d\right)^2 + \lambda\left(x_z^d\right)^2\right] + \beta q L_z^d + \beta(1-q)L_n^d, \tag{13}$$

and

$$L_n^d = \frac{1}{2}\left[\left(\pi_n^d\right)^2 + \lambda\left(x_n^d\right)^2\right] + \beta s L_n^d + \beta(1-s)L_z^d. \tag{14}$$

Following Billi (2011), losses are expressed in terms of their steady-state consumption equivalence given by

$$\mu_z = (1-\beta)\left[\frac{\omega\theta\left(1+\eta\theta\right))}{(1-\omega)(1-\omega\beta)}\right] L_z. \tag{15}$$

Thus, a loss of L_z is equivalent to a $100\mu_z$ percent reduction in steady-state consumption.[16] If $r_z = -2\%$ (expressed at an annual percentage rate), using the parameter values given in Table 1 and setting $q = 0.90$ and $s = 1$ as in Eggertsson and Woodford (2003), the

[16] See Billi (2011). Billi (2015) uses this measure to evaluation nominal GDP targeting and price-level targeting. To calculate μ_z, I set $\omega = 0.75$ and $\eta = 2$. These values are consistent with the values of κ and λ given in table 1.

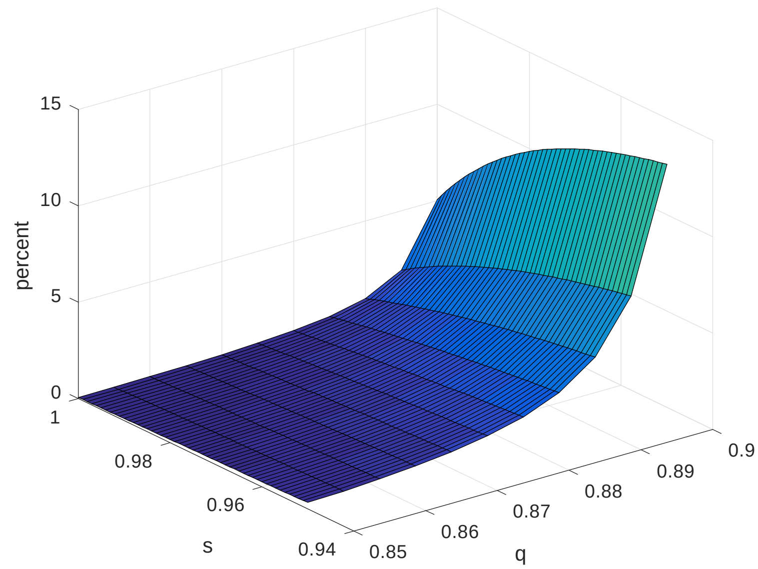

Figure 5: Present value of losses at the ELB: optimal discretion.

equilibrium output gap and inflation rate at the ELB are $x_z = -0.1434$ and $\pi_z = -0.0263$ (-14.34% and -10.53% respectively, when inflation is expressed at an annual rate). This translates into a consumption-equivalent loss of $\mu_z = 5.04\%$.

The importance of the calibration of q for these welfare losses is apparent in Figure 5, which shows L_z^d as a function of s and q.[17] Loss increases with q, given s, as the expected duration at the ELB increases. An increase in s, in contrast, lowers the loss as a larger s means the economy reverts less frequently to the ELB. When $s = 1$, $L_n^d = 0$ for all q. Loss at the ELB is very sensitive to q when $s = 1$; it falls to less than 1% of steady-state consumption when $q = 0.89$, and to 0.05% when $q = 0.85$. Loss at the ELB increases for a given q as the probability of returning to the ELB rises (s falls). When $q = 0.9$, μ_z rises from 5.04% to 7.94% as s falls from 1 to 0.99, and rises to 8.83% when s equals the value that matches the 1960-2017 fraction of quarters at the ELB ($s = 0.9858$); it is only 0.18% of steady-state consumption when the 1960-2017 sample is matched by setting $q = 0.85$ and $s = 0.9788$.

[17] The figure shows loss for $q \leq 0.90$ to avoid values of q that imply $i_n < 0$.

4 Sustainability of forward guidance

The results under optimal discretion can now be used to assess the sustainability of forward guidance. Suppose the central bank announces that it will keep the nominal rate at zero for k periods after the the ELB constraint no longer binds. In period $k+1$, assuming the economy has not returned to the binding ELB, the central bank implements the optimal discretionary policy given by (6). Keeping the nominal rate at zero after an ELB episode has been shown to be part of an optimal commitment policy by Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Nakov (2008) and Werning (2011). The case in which the nominal rate is keep at its effective lower bound for one period ($k = 1$) is considered first, before generalizing to the case of $k > 1$. To preview the results, loss at the ELB is minimized for $k > 0$; however if the expected duration of ELB episodes is short (q small) and the periods away from the ELB are long (s large), the value of k that minimizes the present value of losses at the ELB may not be sustainable.

4.1 Keeping the nominal rate at zero for one period

With one-period forward guidance ($k = 1$), the economy can be in one of three states: at the ELB (state z), in an exit period with (3) no longer a binding constraint but the nominal rate kept at zero as promised under forward guidance (state e), or after the forward guidance period has ended, the ELB constraint is nonbinding, and optimal discretion is implemented (state n). Let superscript fg indicate outcomes under the forward guidance policy. Losses in the three states are L_z^{fg}, L_e^{fg}, and L_n^{fg}. No forward guidance policy would be adopted if it led to a larger loss at the ELB, so $L_z^{fg} \leq L_z^d$ is a necessary condition for a welfare improving policy of forward guidance. However, such a policy will not be sustainable if the present value of the loss obtained by implementing the promised policy in the exit period exceeds the present value of the loss under discretion, i.e., if $L_e^{fg} > L_n^d$. If this condition held, then as soon as the economy exited from the ELB, the policymaker would defect and adopt the optimal time-consistant policy. Private agents, understanding the incentives faced by the policymaker would attach no credibility to the forward guidance provided at the ELB.

The policy would also not be sustainable if $L_n^{fg} > L_n^d$. However, this cannot be the case if $L_e^{fg} < L_n^d$. The reason is that if the economy remains away from the ELB, the forward guidance policy and the optimal discretionary policy both implement the targeting criterion given by the first order condition $\kappa\pi_n + \lambda x_n = 0$. Since expected future inflation and the output gap are closer to their optimal values of zero under forward guidance (as π_z^{fg} and

x_z^{fg} will be smaller in absolute value than π_z^d and x_z^d), a better outcome is achieved under the forward guidance policy. Thus, $L_e^{fg} < L_n^d$ implies $L_n^{fg} < L_n^d$. Only a comparison of the present value of losses in the exit period needs to be made to determine the policy's sustainability.

Equilibrium now involves three inflation rates and three output gaps, corresponding to the situation at the ELB, during the exit period, and when the economy remains away from the ELB. It is also necessary to solve for the nominal interest rate when away from the ELB to ensure it is non-negative. The seven equilibrium conditions are as follows:

$$\pi_z^{fg} = \beta\left[q\pi_z^{fg} + (1-q)\pi_e^{fg}\right] + \kappa x_z \tag{16}$$

$$x_z^{fg} = \left[q x_z^{fg} + (1-q)x_e^{fg}\right] + \left(\frac{1}{\sigma}\right)\left[q\pi_z^{fg} + (1-q)\pi_e^{fg} + r_z\right] \tag{17}$$

$$\pi_e^{fg} = \beta\left[s\pi_n^{fg} + (1-s)\pi_z^{fg}\right] + \kappa x_e^{fg} \tag{18}$$

$$x_e^{fg} = \left[s x_n^{fg} + (1-s)x_z^{fg}\right] + \left(\frac{1}{\sigma}\right)\left[s\pi_n^{fg} + (1-s)\pi_z^{fg} + \rho\right] \tag{19}$$

$$\pi_n^{fg} = \beta\left[s\pi_n^{fg} + (1-s)\pi_z^{fg}\right] + \kappa x_n^{fg} \tag{20}$$

$$\kappa\pi_n^{fg} + \lambda x_n^{fg} = 0. \tag{21}$$

$$x_n^{fg} = \left[s x_n^{fg} + (1-s)x_z^{fg}\right] - \left(\frac{1}{\sigma}\right)\left[i_n^d - \left(s\pi_n^{fg} + (1-s)\pi_z^{fg}\right) - \rho\right] \tag{22}$$

The last equation reflects the assumption that if the economy remains away from the ELB, the optimal time-consistent policy can be implemented as long as $i_n > 0$.

To determine how much the promise to keep $i_e = 0$ improves over discretion, the present value of losses at the ELB, in the exit period, and in subsequent periods if the economy remains away from the ELB must be calculated. L_z^{fg}, L_e^{fg}, and L_n^{fg} satisfy the following three conditions:

$$L_z^{fg} = \frac{1}{2}\left[\left(\pi_z^{fg}\right)^2 + \lambda\left(x_z^{fg}\right)^2\right] + \beta q L_z^{fg} + \beta(1-q)L_e^{fg}$$

$$L_e^{fg} = \frac{1}{2}\left[\left(\pi_e^{fg}\right)^2 + \lambda\left(x_e^{fg}\right)^2\right] + \beta s L_n^{fg} + \beta(1-s)L_z^{fg}$$

$$L_n^{fg} = \frac{1}{2}\left[\left(\pi_n^{fg}\right)^2 + \lambda\left(x_n^{fg}\right)^2\right] + \beta s L_n^{fg} + \beta(1-s)L_z^{fg}.$$

The gain from credible forward guidance is defined as

$$G \equiv L_z^d - L_z^{fg},$$

where L_z^d is the loss at the ELB under optimal discretion. If $G > 0$, then the loss is larger under discretion than with forward guidance. Figure 6 shows G, expressed in terms of its steady-state consumption equivalence using (15); it is positive through the range of s and q such that $i_n^d > 0$, indicating that losses are smaller with forward guidance. This reflects the well-known result that promising to keep the nominal rate at zero after the ELB constraint is no longer binding improves outcomes at the ELB by raising expectations of inflation and the output gap after exiting the ELB. Not surprisingly, the gain increases with q, that is, the lower the probability of exiting the ELB, and therefore the longer the expected duration of an episode at the ELB, the greater is the gain from forward guidance. In contrast, the gain decreases with s, as more frequent returns to the ELB (a lower s) increases the gain from forward guidance.

To assess the sustainability of a promise to keep the nominal interest at zero during the exit period, the present value of losses in the exit period must be less than that obtained by switching to the optimal discretionary policy. That is, sustainability requires that $L_e^{fg} \leq L_n^d$. Define the temptation to defect as

$$T \equiv L_e^{fg} - L_n^d.$$

If $T > 0$, the policy of forward guidance is not sustainable. After the initial period in which $i_e = 0$, equilibrium under both the forward guidance and optimal discretion must satisfy (20) - (21). Hence, with 1-period forward guidance, outcomes differ in state n only because the equilibrium at the ELB differs under the two policies. It follows that if the gain is positive, then $L_n^{fg} < L_n^d$; only the present value of losses in the exit period need to be assessed to determine whether there will be a temptation to defect from the promised policy of setting $i_e = 0$.

The present discounted value of losses that affect the gain from forward guidance and the

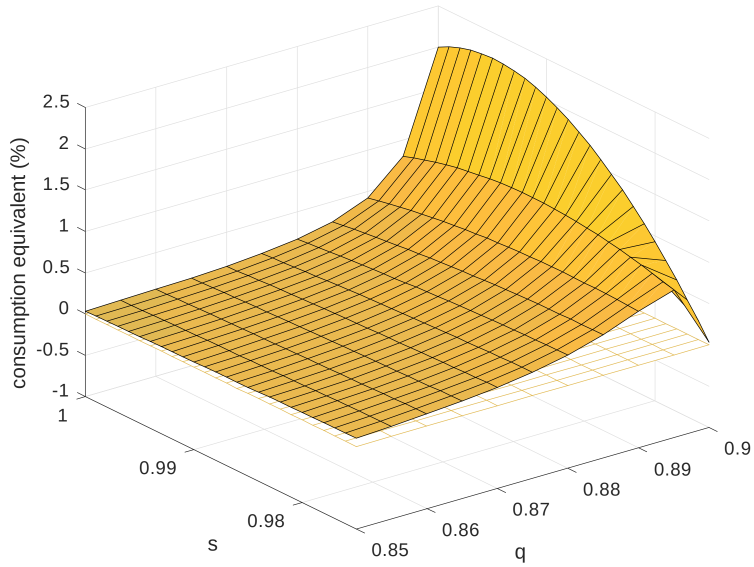

Figure 6: The gain from one-period forward guidance.

temptation to defect are shown for $q = 0.90$ and 0.85 in figure 7. Previously, it was verified that temptation is positive for $s = 1$, in which case forward guidance is unsustainble. Across the range of s values shown, the loss from discretion in state z (solid black line) exceeds that achieved under the forward guidance policy (dashed red line). Temptation is measured by the difference between L_e^{fg} and L_n^d, and these two losses are shown by the dotted black and dot-dashed red lines, respectively. Except for $s \geq 0.9999$, L_e^{fg} is smaller than L_n^d, implying 1-period forward guidance is sustainable. When $s = 1$, the standard result that forward guidance is not sustainable is obtained as temptation is small but positive for all q. When $s = 0.999$, however, temptation is negative. Thus, if there is even a remote probability of a future ELB episode, a promise to maintain the nominal interest rate at zero for one period after the ELB constraint is relaxed is a sustainable policy.

Table 3 reports the gain from 1-period forward guidance and the temptation to defect to optimal discretion. Results are shown for the values of q and s given for the values from Table 2 that match the 1960-2017 or the 1934-2017 frequency at the ELB. Results are also shown for $s = 0.999$ and $s = 1$. Not surprisingly, the gains from keeping the nominal rate at zero are positive, and they are increasing as q, and therefore the expected duration of ELB episodes, increase. For given q, moving down a row is associated with a fall in s. The gain is not monotonic in s. For $q = 0.85$, the gain increases as s falls, while for $q = 0.90$,

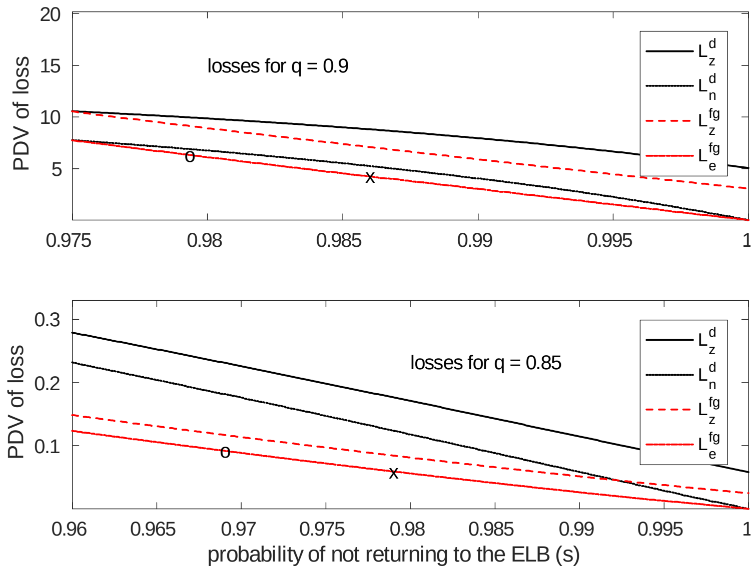

Figure 7: Losses under 1-period forward guidance and discretion (note horizontal axis differ in the two panels).

moving from $s = 1$ to $s = 0.999$ increases the gain from forward guidance, while the gain then falls as s is reduced to 0.9858 to match the 1960-2017 frequency and falls further for $s = 0.9792$, the value matching the 1934-2017 frequency. Only for $s = 1$, the case normally considered in the Eggertsson-Woodford framework, is forward guidance not sustainable.

Table 3: One-period Forward Guidance

	$q = 0.85$		$q = 0.90$	
	$G = L_z^d - L_z^{fg}$	$T = L_e^{fg} - L_n^d$	$G = L_z^d - L_z^{fg}$	$T = L_e^{fg} - L_n^d$
$s = 1$	0.0355	0.0001	2.0026	0.0001
$s = 0.9999$	0.0330	-0.0003	2.0101	-0.0197
1960-2017	0.0923	-0.0651	1.6886	-1.0159
1934-2017	0.1148	-0.0905	0.7985	-0.5551

Losses expressed as percent of steady-state consumption. Bold: one-period forward guidance is sustainable.

Figure 8 illustrates why forward guidance is sustainable as long as s is even slightly below 1. The figure shows equilibrium inflation (top panel) and the output gap (lower panel) in the exit period and in subsequent periods away from the ELB under one-period

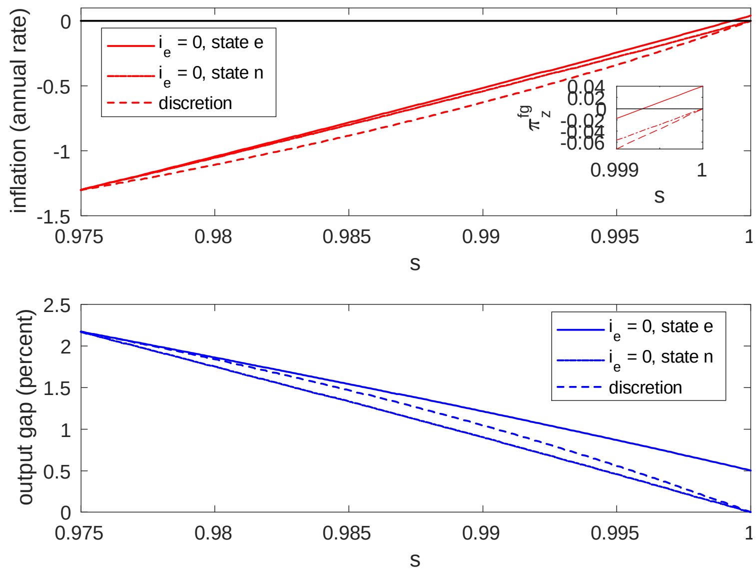

Figure 8: Inflation (top panel) and the output gap (lower panel) after the forward guidance period for $q = 0.90$ when the nominal rate is kept at zero for one period after exiting the ELB (solid lines) and under optimal discretion (dashed line).

forward guidance as a function of s for $q = 0.90$. Also shown are the outcomes under optimal discretion. The dashed lines represent outcomes under discretion; outcomes under forward guidance in the exit period are shown by the solid lines and for subsequent periods away from the ELB by the dot-dashed lines. Note that except when s approaches to 1, inflation in the exit period is closer to zero than it is under optimal discretion. Thus, rather than forward guidance producing worse inflation outcomes when it comes time to implement the promise to keep the nominal rate at zero, forward guidance produces better inflation outcomes. The reason is that the improved outcomes under forward guidance at the ELB imply expected inflation is closer to zero upon exiting than it is under discretion. As $s \to 1$, π_e^{fg} turns positive, but it is still smaller in absolute value than π_n^d except for s extremely close to 1 (i.e., only for the case in which a return to the ELB is very unlikely). As a consequence, the loss due to inflation deviating from zero is smaller for one-period forward guidance for all $s < 0.9997$.

Loss also depends on the output gap. From the lower panel of figure 8, the forward guidance policy does lead to a larger, positive output gap in the exit period than achieved with optimal discretion. However, the baseline calibration in Table 1 assigns a weight of

$\lambda = 0.003$ to output gap volatility in the loss function. Therefore, the better inflation performance outweighs the poorer output gap performance and accounts for why the standard presumption – that the central banker would not have an incentive to fulfill promises once the ELB episode ends – does not hold.

Nakata (2016) showed that the fully optimal Ramsey policy is sustainable for values of s even slightly below 1. The policy considered here – one extra period of a zero nominal rate – is inferior to the Ramsey policy. Despite this, it too is sustainable. Thus, even forward guidance that takes a simple form and may therefore be easier to communicate to the public can be supported as a sustainable policy in the absence of a commitment mechanism. This implies that the standard comparison of pure discretionary policies at the ELB with commitment policies is too limited. Even in the absence of an ability to commit to future actions, the promises of a discretionary policymaker can be credible. Forward guidance in the form of a pledge to keep the nominal interest rate at zero for one period after exiting from the ELB is a sustainable policy in an otherwise discretionary regime as long as s is strictly less than 1.

Forward guidance is sustainable because the output and inflation costs of deviating from pure discretion in the exit period are small in the sense that the deviation of π_e^{fg} and x_e^{fg} from their counterparts under discretion turn out to be small and, in the case of inflation, outcomes are better (i.e., inflation is closer to zero) under forward guidance. Hence, the cost of fulfilling the promised forward guidance is also small and is dominated by the benefit of improved performance at the ELB; as a result, the present discounted losses under forward guidance are less than under optimal discretion except for $s = 1$.

4.2 Multi-period promises

The previous section consider forward guidance that kept the nominal interest rate at zero for one period after the ELB constraint is relaxed. Suppose the central bank promises to keep the nominal rate at zero for $k \geq 1$ periods after exiting an ELB episode. Assume that the economy has remained away from the ELB for the full k periods, policy reverts to the optimal discretion targeting criterion given by (6). Whenever the economy returns to the binding ELB, the process starts over.

With k-period guidance, it is necessary to solve for the equilibrium for $k + 2$ periods: at the ELB, for each of the k periods in which the ELB is no longer binding but the nominal rate is kept at zero, and for the period once policy returns to optimal discretion after the period of forward guidance has ended. The probability that the ELB constraint remains non-binding for the entire k periods for which the central bank promises to keep the nominal rate at zero is s^{k-1}. For the smallest value of s considered in Table 2 (0.9687), the probability the economy remains away from the ELB for 8 quarters is 80%, while for $s = 0.9999$, it is 99.93%. Forward guidance is never sustainable when $s = 1$, but the previous section found that 1-period forward guidance was sustainable for $s = 0.9999$. I therefore present results for $k = 0$ (discretion) to $k = 8$ for $s = 0.9999$ for $q = 0.90$ and $q = 0.85$. Results are also reported for $(q, s) = (0.90, 0.9858)$ and $(q, s) = (0.85, 0.9788)$, combinations from Table 2 that match the frequency of quarters at the ELB for the 1960-2016 period.[18]

The top panel Results for $s = 0.9999$ are presented in Table 4 for $q = 0.90$ and Table 5 for $q = 0.85$. Each table reports the present value of losses at the ELB, during the first period after an ELB episode ends, and when the economy has remained away from the ELB for $k + 1$ periods. Also reported is the gain from forward guidance and the temptation to defect from the promised forward guidance policy. The column labeled L_z shows the present value of losses at the ELB. When $q = 0.90$, a credible promise to keep the nominal interest rate at zero for up to six periods after exiting the ELB episode significantly improves over discretion. The lowest loss, however, is achieved with a promise to keep the nominal rate at zero for four periods beyond the end of the binding ELB constraint. The final column provides evidence on the sustainability of forward guidance. For $k \leq 6$, forward guidance is sustainable. For $k > 6$, the gain is negative and so a promise of $k = 6$ would clearly not be sustainable.

When q is smaller, the expected duration of ELB episodes is shorter. Consequently, the costs of the ELB are lower and the future benefit of fulfilling a promise to keep the nominal rate at zero upon exiting the ELB is smaller. As shown in Table 5, this has two consequences. First, the length of forward guidance that minimizes loss at the ELB falls, with L_z minimized for $k = 3$. But second, a policy that promises to keep the nominal rate at zero for three periods after the ELB no longer binds is not sustainable. The present value of the loss in the exit period exceeds the present value of the loss under discretion. The optimal sustainable policy sets $k = 2$. Loss could be reduced by 25% if forward guidance were extended by one period, but a promise to keep the nominal rate at zero for 3 periods after exiting the binding ELB is not sustainable.

[18] Results are similar for the values of q and s that match the 1934-2016 frequencies.

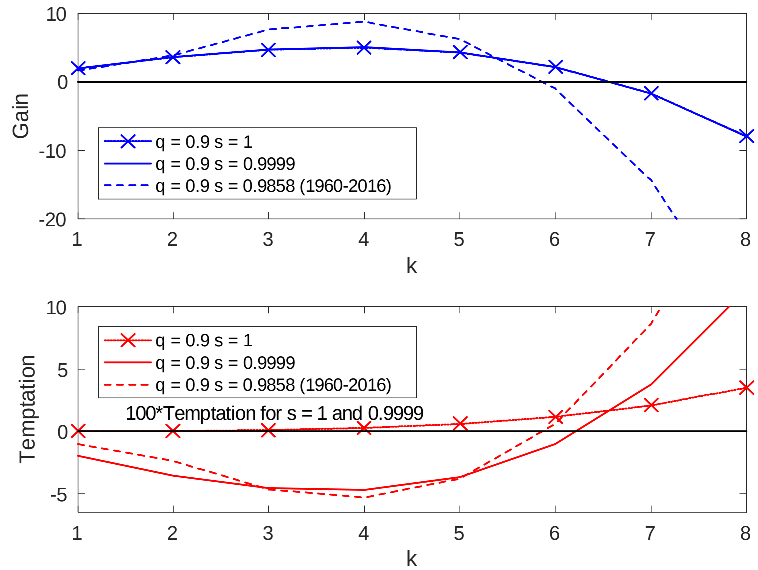

Figure 9: The gain (top panel) and temptation to defect to optimal discretion (lower panel) as a function of k for $q = 0.90$. Note: for $s = 1$ and 0.9999, temptation is multiplied by 100 to make it visible.

Table 4: Multiperiod forward guidance

panel (a): $q = 0.90,\ s = 0.9999$					
k	L_z	L_e	L_n	Gain	Temptation
0	5.0747	0.0498	0.0498	0.0000	0.0000
1	3.0646	0.0301	0.0301	2.0101	−0.0197
2	1.4258	0.0143	0.0140	3.6489	−0.0354
3	0.3345	0.0044	0.0033	4.7402	−0.0454
4	**0.0141**	**0.0029**	**0.0001**	**5.0606**	**−0.0468**
5	0.7472	0.0134	0.0073	4.3276	−0.0364
6	2.8898	0.0401	0.0283	2.1849	−0.0097
7	6.8901	0.0887	0.0676	−1.8154	0.0390
8	13.3105	0.1667	0.1306	−8.2358	0.1169

Losses expressed as percent of steady-state consumption.

Table 5: Multiperiod forward guidance

$q = 0.85$, $s = 0.9999$					
k	L_z	L_e	L_n	Gain	Temptation
0	0.0591	0.0006	0.0006	0.0000	0.0000
1	0.0253	0.0003	0.0002	0.0338	−0.0003
2	**0.0044**	**0.0004**	**0.0000**	**0.0546**	**−0.0002**
3	*0.0033*	*0.0012*	*0.0000*	*0.0558*	*0.0006*
4	0.0306	0.0031	0.0003	0.0285	0.0025
5	0.0973	0.0070	0.0010	−0.0383	0.0064
6	0.2172	0.0137	0.0021	−0.1582	0.0132
7	0.4075	0.0249	0.0040	−0.3484	0.0243
8	0.6894	0.0422	0.0068	−0.6304	0.0416

Losses expressed as percent of steady-state consumption.

Tables 6 and 7 report results for $q = 0.90$ and $q = 0.85$, but in this case, s is set to the values given in Table 2 that are consistent with the fraction of quarters at the ELB during the 1960-2016 period.[19] Comparing Table 6 with Table 4 and Table 7 with Table 5 illustrates that the lower value of s increases the loss associated with the ELB constraint. It does so because a fall in s increases the probability of returning to the ELB. For $q = 0.90$, the optimal k is still 4, and a promise to keep the nominal rate at zero for 4 periods is sustainable. Figure 9 shows the gain and the temptation when $q = 0.90$ for $s = 0.9999$ (dashes) and $s = 0.9858$ (solid). Also shown for comparison is the case for $s = 1$ (crosses). Figure 10 shows gain and temptation for $q = 0.85$. Not surprisingly, there is very little difference in the gain from forward guidance when $s = 0.9999$ or $s = 1$. What does differ is the sustainability of forward guidance. When $s = 1$, forward guidance is not sustainable as $T > 0$ for all k. For $s = 0.9999$, T is negative (though small) for $k < 7$.

[19] Table 6 only shows outcomes for $k = 0, 1, ..., 5$. For the parameter values used in Table 6, $i_n < 0$ for $k > 5$.

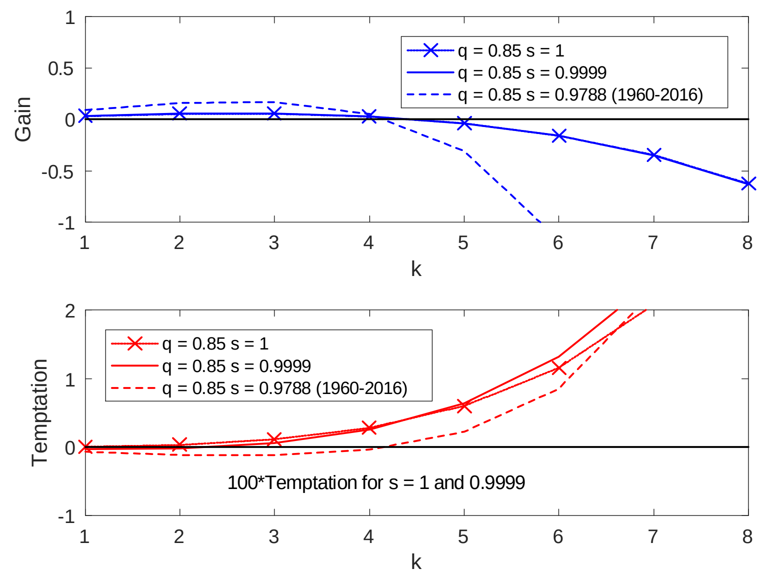

Figure 10: The gain (top panel) and temptation to defect to optimal discretion (lower panel) as a function of k for $q = 0.85$. Note: for $s = 1$ and 0.9999, temptation is multiplied by 100 to make it visible.

Table 6: Multiperiod forward guidance

panel (a): $q = 0.90$, $s = 0.9858$					
k	L_z	L_e	L_n	Gain	Temptation
0	8.8318	5.3134	5.3134	0.0000	0.0000
1	7.1432	4.2975	4.2975	1.6886	-1.0159
2	4.9225	2.9614	2.9614	3.9092	-2.3520
3	2.1179	1.2741	1.2740	6.7139	-4.0392
4	**0.0132**	**0.0107**	**0.0079**	**8.8185**	$\mathbf{-5.3027}$
5	22.4769	13.5642	13.5250	-13.6452	8.2509

Losses expressed as percent of steady-state consumption.

Figure 11 illustrates the time path of inflation and the output gap if it exits the ELB period at time 0 and remains away from the ELB for at least 8 periods. Results are shown for optimal discretion (i.e., $k = 0$) and the policies associated with $k = 1$ and $k = 4$. The latter policy is, from Table 6, the optimal policy of forward guidance. Under discretion, inflation remains negative even after the ELB period because, with $s < 1$, the private sector

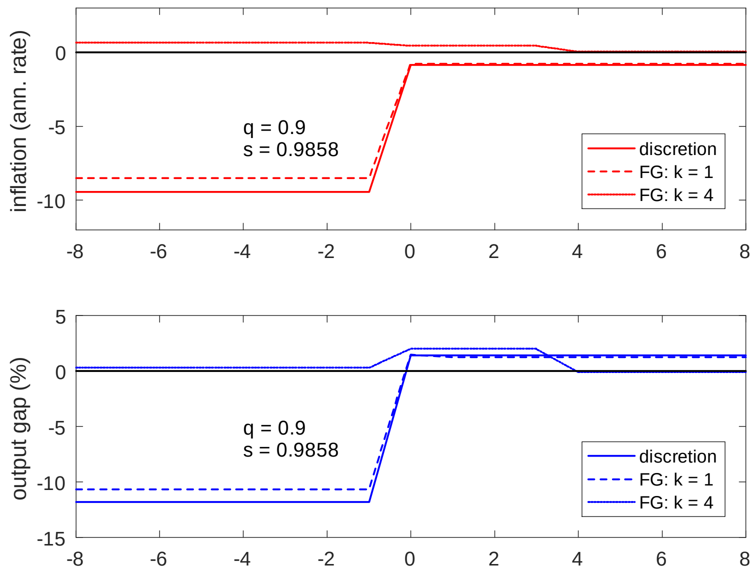

Figure 11: Inflation (upper panel) and the output gap (lower panel) under discretion, $k = 1$, and $k = 4$ for $q = 0.90$ and $s = 0.9858$.

a future episode of deflation. One-period forward guidance lowers the present value of loss at the ELB from 8.83% of steady-state consumption to 7.14% (a 19% reduction); the output gap is slightly higher while at the ELB and lower after the end of the zero nominal rate period. When $k = 4$, the welfare costs of the ELB are almost totally eliminated (they fall by over 99%) and inflation never falls below zero.

Table 7: Multiperiod forward guidance

$q = 0.85,\ s = 0.9788$					
k	L_z	L_e	L_n	Gain	Temptation
0	0.1773	0.1251	0.1251	0.0000	0.0000
1	0.0850	0.0600	0.0600	0.0923	-0.0651
2	0.0177	0.0127	0.0124	0.1596	-0.1124
3	**0.0091**	**0.0075**	**0.0063**	**0.1683**	$\mathbf{-0.1176}$
4	0.1227	0.0903	0.0865	0.0546	-0.0348
5	0.4830	0.3511	0.3408	-0.3056	0.2260
6	1.3476	0.9774	0.9509	-1.1703	0.8523
7	3.2965	2.3911	2.3259	-3.1191	2.2660
8	7.7923	5.6586	5.4975	-7.6150	5.5336

Losses expressed as percent of steady-state consumption.

For $k \leq 4$, forward guidance is sustainable because of the significant effect forward guidance has in raising inflation and the output gap at the ELB. As a consequence, it also leads both inflation and the output gap to be closer to zero when the economy is away from the ELB than is achieved by discretion. The equilibrium outcomes for inflation and the output gap for $k = 0$ (discretion) and for the optimal sustainable values of k for $(q\ s) = (0.90\ 0.9858)$ and $(q\ s) = (0.85\ 0.9788)$. are shown in Table 8. Even though the output gap is much larger during the exit period under forward guidance, Table 8 showed that L_e^{fg} is only a small fraction of the loss experienced in the absence of forward guidance.

Table 8: Discretion and forward guidance

$q = 0.90,\ s = 0.9858,$						
k	π_z	π_e	π_n	x_z	x_e	x_n
0	-9.446	-0.844	-0.844	-11.826	1.406	1.406
4	-4.614	-0.246	-0.412	-5.982	1.740	0.687
$q = 0.85,\ s = 0.9788$						
k	π_z	π_e	π_n	x_z	x_e	x_n
0	-1.3520	-0.1727	-0.1727	-2.3581	0.2878	0.2878
3	0.267	0.2732	0.0341	0.022	1.465	-0.0569

* Inflation at annual rates; output gap in percent.

Multi-period promises improve outcomes significantly relative to pure discretion, and such promises (as long as they are not for two many periods) can be sustainable. Even though it has been assumed that there is no commitment mechanism and that the central bank will renege on past promises whenever the expected present value of losses exceeds that obtained under discretion, optimal forward guidance is sustainable. A central bank that cannot commit can still credibly promise to keep interest rates at zero beyond the end of the ELB episode.

5 A discounted Euler equation

McKay, Nakamura, and Steinsson (2016b) have argued that the basic Euler equation given by (1) implies implausibly large effects of forward guidance, and these large effects arise because expected future output has a one-to-one effect on current output. The power of forward guidance implies that only modest promises concerning future policy can significantly reduce the welfare costs at the ELB. Improving outcomes at the ELB also acts to improve outcomes away from the ELB when $s < 1$, thereby leading to lower losses in the exit period than is achieved under discretion. Thus, the power of forward guidance may account for the finding that even multi-period promises are sustainable. To investigate this possibility, the discounted Euler equation proposed by McKay, Nakamura, and Steinsson (2016b) is employed.

Based on an incomplete markets model that leads to precautionary savings on the part of households, they propose a discounted Euler equation that takes the form

$$x_t = \delta \mathrm{E}_t x_{t+1} - \left(\frac{\chi}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1} - r_t), \tag{23}$$

with $0 < \delta \leq 1$ and $0 < \chi \leq 1$. In their base calibration, they set $\delta = 0.97$ and $\chi = 0.75$. With these values, together with the same parameter values used by Eggertsson and Woodford (2003), they find the output gap is -2.88% at the ELB, significantly less that the -14.43% obtained with the standard Euler equation. While they consider only the case in which the economy never returns to the ELB once it exits, similar effects carry over to the case in which $s < 1$. Because both inflation and the output are not as negative at the ELB as with the standard Euler equation, expected inflation and output would also be higher for any $s < 1$ when the economy is not at the ELB if (23) holds. This means, in turn, that under discretion the nominal interest rate is not as low when the economy is away from the ELB. As a consequence, $i_n > 0$ for even small s and large q, unlike the case found using the standard undiscounted Euler equation (1).

The discounted Euler equation implies the consequences of the ELB and the strength of forward guidance policies are muted. However, the basic findings are robust to replacing (1) with (23). Table 6 shows that the optimal number of periods to promise to keep the nominal interest rate at zero after exiting an ELB episode is still two, and this promise is sustainable.

Table 9: Discounted Euler

	$q = 0.90$			$s = 0.9858$	
k	V_z	V_e	V_n	Gain	Temptation
0	0.4471	0.2690	0.2690	0.0000	0.0000
1	0.2948	0.1774	0.1774	0.1522	-0.0916
2	0.1566	0.0943	0.0942	0.2905	-0.1746
3	0.0410	0.0251	0.0246	0.4060	-0.2438
4	**0.0036**	**0.0036**	**0.0021**	**0.4435**	**-0.2653**
5	0.0658	0.0430	0.0395	0.3813	-0.2259
6	0.2524	0.1587	0.1518	0.1947	-0.1102
7	0.5922	0.3687	0.3562	-0.1451	0.0997
8	1.1192	0.6940	0.6733	-0.6722	0.4251

Table 10: Discretion and forward guidance, Discounted Euler

$q = 0.90,\ s = 0.9858$						
k	π_z	π_e	π_n	x_z	x_e	x_n
0	-2.125	-0.190	-0.190	-2.661	0.316	0.316
4	0.059	0.303	0.005	-0.294	1.467	0.009
$q = 0.85,\ s = 0.9788$						
k	π_z	π_e	π_n	x_z	x_e	x_n
0	-0.734	-0.094	-0.094	-1.280	0.156	0.156
3	0.141	0.197	0.018	-0.086	1.089	-0.003

* Inflation at annual rates; output gap in percent.

6 Summary and conclusions

Recent research has emphasized the adverse consequences for the economy when the central bank's policy instrument is constrained by an effective lower bound on the short-term nominal interest rate and policy is implemented in a time-consistent, discretionary manner. These adverse effects stand in contrast to the situation in which the central bank is able to implement the optimal but time-inconsistent commitment policy. Under the presumption that discretion is the more realistic assumption about policy, proposals for reforming inflation targeting policy frameworks have emphasized changes that either make it less likely the ELB will be encountered or that establish alternative regimes, such as price-level targeting, that can cause expectations to move in a manner that promotes stabilization and mimics a commitment policy regime.

Forward guidance is powerful in models, like the new Keynesian model, in which agents are rational and forward looking. However, if rational agents believe the policymaker will never honor promises about policy in the post-ELB environment, forward guidance lacks credibility. Consequently, if the central bank cannot commit, ELB episodes are likely to be costly, and policy reform should seek to reform flexible inflation targeting to ensure better outcomes at the ELB.

Proposed reforms presume that the ELB will be encountered again in the future. Yet analytical analysis of policy at the ELB typically assumes that once the economy exits the ELB, it never again encounters the ELB.[20] If this is the case, then any promises about future policy – that is, forward guidance – lack credibility. Once the economy is out of the ELB period, there is no incentive for the policymaker to implement the policies that were promised in the past.

But if the economy may revert to the ELB, then promises made during an ELB episode may be credible even in the absence of a commitment mechanism. If the promised policy actions improve outcomes when at the ELB, then it may be rational for the central bank to fully implement those promises because, while doing so generates a cost, it also brings an expected future benefit. Future promises may be sustainable.

I modify the basic model of Eggertsson and Woodford (2003) to allow for both a constant probability of exiting the ELB and a constant probability of returning to the ELB. Unlike the standard analysis, the economy does not achieve zero inflation and a zero output gap once it exits the ELB. With a positive probability of reverting to the ELB, expected inflation and the output gap are no longer zero as in the Eggertsson-Woodford analysis. The equilibrium also depends on the assumption made about post-ELB policy, so a simple instrument rule does not lead to the same equilibrium as obtained optimal discretion. Outcomes at the ELB are much worse if the central bank is expected to adopt a simple but commonly employed instrument rule than if it follows the optimal one-period policy upon exiting.

The main focus of the paper, though, is on the sustainability of forward guidance. Three forms of such guidance were considered: a pledge to keep the nominal interest rate at zero for the initial post-ELB period; a promise to keep the nominal rate at zero for multiple periods after an ELB episode ends; and a promise to deliver a specific inflation rate on exiting

[20] As noted earlier, the exception is Nakata (2014).

the ELB. When there is no chance of returning to the ELB, none of these policies are sustainable. However, if there is even the slightest chance of returning to the ELB, forward guidance policies may be sustainable. For example, the promise to keep the nominal rate at zero for one period after the ELB constraint is relaxed is sustainable if the probability of another ELB episode is as little as 0.1%. For multiple-period forward guidance, policies that promise to keep the nominal rate at zero for too long are unsustainable. However, the optimal number of periods in the calibrated model was only two periods, and this policy is sustainable.

The results obtained here were derived using a very stylized model. The basic model does, however, generalize the framework that has been employed widely in analyzes of the ELB. There are many directions in which the basic model could be extended to determine how robust the reported findings are. The key implication, a result consistent with the findings of Nakata (2014), is likely to be robust: if future episodes at the ELB are likely, then promises made during the ELB period may be credible despite the absence of any mechanism to ensure commitment.

7 Appendix

7.1 Equilibrium under discretion

Under discretion, the policymaker takes expectations as given and chooses π_n and x_n, where the subscript denotes a period in which the ELB constraint is non-binding. Let π' denoted inflation in the following period. Then the policy problem under discretion is to minimize

$$l_t = \frac{1}{2}\left(\pi_n^2 + \lambda x_n^2\right),$$

subject to

$$\pi_n = \beta \mathrm{E}_n \pi' + \kappa x_n.$$

The first-order condition is given by

$$\lambda x_n + \kappa \pi_n = 0.$$

x_n and π_n then jointly solve

$$\lambda x_n + \kappa \pi_n = 0$$

$$\pi_n = \beta\left[s\pi_n + (1-s)\,\pi_z\right] + \kappa x_n$$

Using the FOC to eliminate x_n from the inflation equation, one obtains

$$\left(\lambda + \kappa^2\right)\pi_n = \lambda\beta \mathrm{E}_n \pi' = \lambda\beta\left[s\pi_n + (1-s)\,\pi_z\right],$$

which implies

$$\pi_n = (1-s)\left[\frac{\lambda\beta}{\lambda(1-\beta s) + \kappa^2}\right]\pi_z \tag{24}$$

and

$$x_n = -(1-s)\left[\frac{\kappa\beta}{\lambda(1-\beta s) + \kappa^2}\right]\pi_z. \tag{25}$$

Thus, equilibrium away from the ELB depends on the inflation rate that occurs when the economy is at the ELB. In the normal case with $\pi_z < 0$, inflation is positive and the output gap is negative when the economy is away from the ELB constraint.

7.1.1 Collecting results

$$(1-\beta q)\,\pi_z = \beta(1-q)\,\pi_n + \kappa x_z$$

$$\sigma(1-q)\,x_z = \sigma(1-q)\,x_n + q\pi_z + (1-q)\,\pi_n + r_z$$

$$(1-\beta s)\,\pi_n = \beta(1-s)\,\pi_z + \kappa x_n$$

$$\lambda x_n + \kappa\pi_n = 0$$

$$\begin{bmatrix} 1-\beta q & -\kappa & -\beta(1-q) & 0 \\ -q & \sigma(1-q) & -(1-q) & -\sigma(1-q) \\ -\beta(1-s) & 0 & 1-\beta s & -\kappa \\ 0 & 0 & \kappa & \lambda \end{bmatrix}\begin{bmatrix} \pi_z \\ x_z \\ \pi_n \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} r_z$$

7.1.2 At the ELB

When the ELB constraint is binding, equilibrium is characterized by

$$x_z = \left[qx_z + (1-q)\,x_n\right] + \left(\frac{1}{\sigma}\right)\left[q\pi_z + (1-q)\,\pi_n + r_z\right]$$

$$\pi_z = \beta q \pi_z + \beta (1-q) \pi_n + \kappa x_z.$$

Using (24) and (25) in these two equations, x_z and π_z jointly solve

$$\begin{aligned}
(1-q) x_z &= -(1-q) \left[\frac{\kappa \beta (1-s)}{\lambda (1-\beta s) + \kappa^2} \right] \pi_z \\
&\quad + \left(\frac{q}{\sigma} \right) \pi_z + \left(\frac{1-q}{\sigma} \right) \left[\frac{\lambda \beta (1-s)}{\lambda (1-\beta s) + \kappa^2} \right] \pi_z + \left(\frac{1}{\sigma} \right) r_z
\end{aligned}$$

$$(1-\beta q) \pi_z = \beta (1-q) \left[\frac{\lambda \beta (1-s)}{\lambda (1-\beta s) + \kappa^2} \right] \pi_z + \kappa x_z.$$

Collecting terms:

$$\begin{aligned}
(1-q) \left[\lambda (1-\beta s) + \kappa^2 \right] x_z &= \beta \left(\frac{\lambda}{\sigma} - \kappa \right) (1-q)(1-s) \pi_z \\
&\quad + \left(\frac{q}{\sigma} \right) \left[\lambda (1-\beta s) + \kappa^2 \right] \pi_z \\
&\quad + \left(\frac{1}{\sigma} \right) \left[\lambda (1-\beta s) + \kappa^2 \right] r_z
\end{aligned}$$

$$\left\{ (1-\beta q) \left[\lambda (1-\beta s) + \kappa^2 \right] - \lambda \beta^2 (1-q)(1-s) \right\} \pi_z = \kappa \left[\lambda (1-\beta s) + \kappa^2 \right] x_z.$$

7.2 An instrument rule

In the previous literature, policy in the post-ELB period is often characterized by a simple Taylor-type instrument rule. For example, McKay, Nakamura, and Steinsson (2016b) assume that

$$i_t = \max (0, \rho + \phi \pi_t), \ \phi > 1. \tag{26}$$

Under the policy given by (26), equilibrium when the ELB does not bind satisfies the two equations

$$x_n = [s x_n + (1-s) x_z] - \left(\frac{1}{\sigma} \right) [(\phi - s) \pi_n - (1-s) \pi_z] \tag{27}$$

$$\pi_n = \beta [s \pi_n + (1-s) \pi_z] + \kappa x_n. \tag{28}$$

At the ELB, x_z and π_z must satisfy

$$x_z = [q x_z + (1-q) x_n] + \left(\frac{1}{\sigma} \right) [q \pi_z + (1-q) \pi_n + r_z] \tag{29}$$

$$\pi_z = \beta \left[q\pi_z + (1-q)\,\pi_n \right] + \kappa x_z. \tag{30}$$

These four equations can be solved jointly for x_z, π_z, x_n and π_n.

Equilibrium both when the ELB binds and when it doesn't now depends on the value of the policy response to inflation, ϕ is $s < 1$. This response coefficient is irrelevant when $s = 1$ (as long as the Taylor principle is satisfied).

ϕ can be chosen to replicate exactly outcomes under optimal discretion. This occurs if

$$\phi^* = s + \frac{\left[\beta\sigma(1-s) + \sigma(1-\beta q) + \kappa\right]\left[\lambda(1-\beta s) + \kappa^2\right] - \beta\lambda\sigma(1-s)\left[1 - \beta s + \beta - \beta q\right]}{\beta\lambda\kappa}.$$

For $q = 0.90$, $\phi^* = 2.9264$ when $s = 0.9858$ to match the shorter sample (January 1960 to January 2017) and $\phi^* = 3.021$ when $s = 0.9792$ to match the longer sample (January 1934 to January 2017). For $q = 0.85$, $\phi^* = 3.7434$ ($\phi = 3.888$) when $s = 0.9788$ ($s = 0.9687$) to match the shorter (longer) sample. In all cases, ϕ^* is significantly larger that the standard Taylor rule value of 1.5.

References

Abreu, D. (1988): "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, 56, 383–396.

Adam, K., and R. Billi (2006): "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 38(7), 1877–1905.

Adam, K., and R. M. Billi (2007): "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates," *Journal of Monetary Economics*, 54(3), 728–752.

Ball, L. (1995): "Time Consistent Inflation Policy and Persistent Changes in Inflation," *Journal of Monetary Economics*, 36(2), 329–350.

Bilbiie, F. O. (2017): "Optimal Forward Guidance," pp. 106–112.

Billi, R. (2013): "Nominal GDP Targeting and the Zero Lower Bound: Should We Abandon Inflation Targeting?," (xxx), 1–28.

Billi, R. M. (2011): "Optimal inflation for the US economy," *American Economic Journal: Macroeconomics*, 3(July), 29–52.

Billi, R. M. (2015): "A Note on Nominal GDP Targeting and the Zero Lower Bound," *Riksbank Working Paper*, (270), 1–28.

Blanchard, O., G. Dell'Ariccia, and P. Mauro (2010): "Rethinking macroeconomic policy," *Journal of Money, Credit and Banking*, 42(SUPPL. 1), 199–215.

Bodenstein, M. (2017): "On Targeting Frameworks and Optimal Monetary Policy," pp. 1–86.

Bodenstein, M., J. Hebden, and R. Nunes (2012): "Imperfect Credibility and the Zero Lower Bound," *Journal of Monetary Economics*, 59(2), 135–149.

Braun, R., L. Körber, and Y. Waki (2012): "Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate is Zero," *Federal Reserve Bank of Atlanta Working Paper*, (5a).

Campbell, J. R. (2016): "Forward Guidance and Macroeconomic Outcomes Since the Financial Crisis," in *NBER Macroeconomic Annual*.

Campbell, J. R., C. L. Evans, J. D. M. Fisher, and A. Justiniano (2012): "Macroeconomic Effects of Federal Reserve Forward Guidance," *Brookings Papers on Economic Activity*, (Spring), 1–80.

Carlstrom, C., T. Fuerst, and M. Paustian (2012): "How Inflationary is an Extended Period of Low Interest Rates?," *Federal Reserve Bank of Cleveland Working Paper No. 1202*.

Chari, V. V., and P. J. Kehoe (1990): "Sustainable Plans," *Journal of Political Economy*, 98(4), 783–802.

Christiano, L., M. Eichenbaum, and S. Rebelo (2011): "When is the Government Spending Multiplier Large?," *Journal of Political Economy*, 119(1), 78–121.

Cochrane, J. H. (2013): "The New-Keynesian Liquidity Trap," *NBER Working Paper No. 19476*.

Debortoli, D., J. Maih, and R. Nunes (2014): “Loose Commitment in Medium-Scale Macroeconomic Models: Theory and Applications,” *Macroeconomic Dynamics*, 18(01), 175–198.

Debortoli, D., and R. Nunes (2010): “Fiscal Policy under Loose Commitment,” *Journal of Ecoomic Theory*, 145(3), 1005–1032.

DelNegro, M., M. Giannoni, and C. Patterson (2012): “The Forward Guidance Puzzle,” *Federal Reserve Bank of New York Staff Reports, no. 574*, (October 2012).

Dennis, R. (2014): “Imperfect credibility and robust monetary policy,” *Journal of Economic Dynamics and Control*, 44, 218–234.

Eggertsson, G., and M. Woodford (2003): “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *BPEA*, 34(1), 139–235.

Eggertsson, G. B., and S. R. Singh (2016): “Log-Linear Approximation Versus an Exact Solution At the ZLB in the New Keynesian Model,” *NBER Working Paper 22784*.

Eggertsson, G. B. G. (2011): “What Fiscal Policy Is Effective at Zero Interest Rates?,” *NBER Macroeconomics Annual 2010*, 25(402), 59–112.

Giannoni, M. P. (2014): “Optimal interest-rate rules and inflation stabilization versus price-level stabilization,” *Journal of Economic Dynamics and Control*, 41, 110–129.

Ireland, P. N. (1997): “Sustainable Monetary Policies,” *Journal of Economic Dynamics and Control*, 22(1), 87–108.

Jeanne, O., and L. E. O. Svensson (2007): “Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank,” *American Economic Review*, 97(1), 474–490.

Jung, T., Y. Teranishi, and T. Watanabe (2005): “Optimal Monetary Policy at the Zero-Interest-Rate Bound,” *Journal of Money, Credit, and Banking*, 37(5), 813–835.

Kiley, M. T. (2014): “The Aggregate Demand Effects of Short- and Long-Term Interest Rates,” *International Journal of Central Banking*, 10(4), 69–104.

——— (2016): “Policy Paradoxes in the New Keynesian Model,” *Review of Economic Dynamics*, 21, 1–15.

Kurozumi, T. (2008): "Optimal Sustainable Monetary Policy," *Journal of Monetary Economics*, 55(7), 1277–1289.

——— (2012): "Sustainability, flexibility, and inflation targeting," *Economics Letters*, 114(1), 80–82.

Levin, A., D. López-Salido, E. Nelson, and T. Yun (2010): "Limitations on the Effectiveness of Forward Guidance at the Zero Lower Bound," *International Journal of Central Banking*, 6(1), 143–189.

Levine, P., P. McAdam, and J. Pearlman (2008): "Quantifying and sustaining welfare gains from monetary commitment," *Journal of Monetary Economics*, 55(7), 1253–1276.

Loisel, O. (2008): "Central bank reputation in a forward-looking model," *Journal of Economic Dynamics and Control*, 32(11), 3718–3742.

McKay, A., E. Nakamura, and J. Steinsson (2016a): "The Discounted Euler Equation: A Note," .

——— (2016b): "The Power of Forward Guidance Revisited," *American Econmic Review*, 106(10), 3133–3158.

Nakata, T. (2014): "Reputation and Liquidity Traps," *FRB Finance and Economics Discussion Series 2014-50.*

——— (2016): "Optimal fiscal and monetary policy with occasionally binding zero bound constraints," *Journal of Economic Dynamics and Control*, 73, 220–240.

Nakov, A. (2008): "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 4(2), 73–127.

Reifschneider, D. (2016): "Gauging the Ability of the FOMC to Respond to Future Recessions," *Finance and Economics Discussion Series*, pp. 1–33.

Roberds, W. (1987): "Models of Policy under Stochastic Replanning," *International Economic Review*, 28(3), 731–755.

Rogoff, K. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," *The Quarterly Journal of Economics*, 100(4), 1169–1189.

Schaumburg, E., and A. Tambalotti (2007): "An investigation of the gains from commitment in monetary policy," *Journal of Monetary Economics*, 54(2), 302–324.

Stokey, N. L. (1991): "Credible public policy," *Journal of Economic Dynamics and Control*, 15(4), 627–656.

Swanson, E. T. (2016): "Measuring the Effects of Federal Reserve Forward Guidance and Asset Purchases on Financial Markets," .

Vestin, D. (2006): "Price-Level Targeting versus Inflation Targeting," *Journal of Monetary Economics*, 53(7), 1361–1376.

Walsh, C. E. (2003): "Speed limit policies: the output gap and optimal monetary policy," *American Economic Review*, 93(1), 265–278.

——— (2017): *Monetary Theory and Policy*. The M.I.T. Press, Cambridge, MA, 4th edn.

Werning, I. (2011): "Managing a Liquidity Trap: Monetary and Fiscal Policy," .

Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, NJ.