1. Complete the sentence: a linear transformation is invertible if it is _____ and _____.
   Given a linear transformation $T$ that is invertible, explain how to define its inverse.

2. Is the matrix
   \[
   \begin{bmatrix}
   5 & 3 & -1 \\ 3 & -2 & 4 \\ 7 & 8 & -5
   \end{bmatrix}
   \]
   invertible? If so, find the inverse.

3. • Is the following a subspace of $\mathbb{R}^3$? Prove or disprove:
   \[
   H = \left\{ \begin{bmatrix}
   2x \\ 3y \\ x - y
   \end{bmatrix} : x, y \in \mathbb{R} \right\}
   \]

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   \]

4. Define the rank and nullity of a matrix $A$. What is the relationship between these quantities? Consider the matrix
   \[
   A = \begin{bmatrix}
   5 & 3 & -1 & 2 \\ 3 & -2 & 4 & -1 \\ 7 & 8 & -6 & 5
   \end{bmatrix}
   \]. Calculate the rank and nullity of $A$.

5. Use determinants to calculate the value of $a$ such that the following vectors are linearly dependent.
   \[
   v_1 = \begin{bmatrix}
   2 \\ -4 \\ 1
   \end{bmatrix}, \quad v_2 = \begin{bmatrix}
   -5 \\ 7 \\ -3
   \end{bmatrix}, \quad v_3 = \begin{bmatrix}
   8 \\ a \\ 4
   \end{bmatrix}.
   \]

6. Find a basis for the eigenspace corresponding to the eigenvalue $\lambda = 13$ for the matrix
   \[
   A = \begin{bmatrix}
   \end{bmatrix}.
   \]