1. Write the solution set of the following homogeneous linear system of equations as the span of a set of vectors in $\mathbb{R}^4$:

$$
\begin{align*}
    x_1 + 2x_2 - x_3 + 3x_4 &= 0 \\
    2x_1 - x_2 - 3x_3 + x_4 &= 0 \\
    x_1 - 3x_2 - 2x_3 - 2x_4 &= 0 \\
    4x_1 + 3x_2 - 5x_3 + 7x_4 &= 0
\end{align*}
$$

2. Find an equation relating $a$ and $b$ such that the homogeneous linear system of equations represented by the following augmented matrix has infinitely many solutions if and only if your equation holds:

$$
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    3 & a & 2 & 0 \\
    7 & -3 & b & 0
\end{bmatrix}.
$$

3. Are the three vectors $\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 4 \\ -7 \end{bmatrix}$ linearly dependent or linearly independent? Prove your answer.

4. Show that the span of the three vectors in the previous problem is a plane through the origin in $\mathbb{R}^3$. Give an equation for this plane in the form $ax_1 + bx_2 + cx_3 = 0$ for some explicit constants $a$, $b$, and $c$.

5. Write the set of solutions to the equation $Ax = b$ in parametric vector form, where $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 4 \\ 3 & 7 & -11 \end{bmatrix}$ and $b = \begin{bmatrix} 8 \\ -2 \\ 28 \end{bmatrix}$.

6. Consider the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x) = Ax$, where

$$
A = \begin{bmatrix}
    1 & 0 & 0 \\
    0 & 1 & -1 \\
    0 & 1 & 1
\end{bmatrix}.
$$

What are the domain, codomain, and range of the linear transformation $T$? How would you describe this linear transformation geometrically, using the descriptive words we discussed in class (shear, rotation, dilation, contraction)? Be as precise as possible.
1. Write the solution set of the following homogeneous linear system of equations as the span of a set of vectors in \( \mathbb{R}^4 \):

\[
\begin{align*}
3x_1 + 5x_2 - 2x_3 + x_4 &= 0 \\
2x_1 - x_2 - 3x_3 + x_4 &= 0 \\
x_1 + 6x_2 + x_3 &= 0 \\
-x_1 + 7x_2 + 4x_3 - x_4 &= 0
\end{align*}
\]

2. Find an equation relating \( a \) and \( b \) such that the homogeneous linear system of equations represented by the following augmented matrix has infinitely many solutions if and only if your equation holds:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
2 & a & -7 & 0 \\
6 & 1 & b & 0
\end{bmatrix}
\]

3. Are the three vectors \( \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}, \text{ and } \begin{bmatrix} 0 \\ -1 \\ -5 \end{bmatrix} \) linearly dependent or linearly independent? Prove your answer.

4. Show that the span of the three vectors in the previous problem is a plane through the origin in \( \mathbb{R}^3 \). Give an equation for this plane in the form \( ax_1 + bx_2 + cx_3 = 0 \) for some explicit constants \( a, b, \) and \( c \).

5. Write the set of solutions to the equation \( Ax = b \) in parametric vector form, where

\[
A = \begin{bmatrix}
1 & -3 & 2 \\
0 & 2 & 1 \\
3 & -11 & 5
\end{bmatrix} \text{ and } b = \begin{bmatrix} -7 \\ -1 \\ -20 \end{bmatrix}.
\]

6. Consider the linear transformation \( T: \mathbb{R}^3 \to \mathbb{R}^3 \) defined by \( T(x) = Ax \), where

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
0 & \sqrt{3} & -2 \\
0 & 2 & \sqrt{3}
\end{bmatrix}
\]

What are the domain, codomain, and range of the linear transformation \( T \)? How would you describe this linear transformation geometrically, using the descriptive words we discussed in class (shear, rotation, dilation, contraction)? Be as precise as possible.
1. Write the solution set of the following homogeneous linear system of equations as the span of a set of vectors in \( \mathbb{R}^4 \):

\[
\begin{align*}
3x_1 + 2x_2 - 4x_3 + x_4 &= 0 \\
-x_1 - x_2 - 3x_3 + x_4 &= 0 \\
2x_1 + x_2 - 7x_3 + 2x_4 &= 0 \\
4x_1 + 3x_2 - x_3 &= 0
\end{align*}
\]

2. Find an equation relating \( a \) and \( b \) such that the homogeneous linear system of equations represented by the following augmented matrix has infinitely many solutions if and only if your equation holds:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
3 & a & -4 & 0 \\
-4 & 2 & b & 0
\end{bmatrix}
\]

3. Are the three vectors \( \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} \), \( \begin{bmatrix} 2 \\ -3 \\ -7 \end{bmatrix} \), and \( \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \) linearly dependent or linearly independent? Prove your answer.

4. Show that the span of the three vectors in the previous problem is a plane through the origin in \( \mathbb{R}^3 \). Give an equation for this plane in the form \( ax_1 + bx_2 + cx_3 = 0 \) for some explicit constants \( a, b, \) and \( c \).

5. Write the set of solutions to the equation \( Ax = b \) in parametric vector form, where

\[
A = \begin{bmatrix}
1 & 2 & 0 \\
0 & 5 & -5 \\
-3 & -1 & -5
\end{bmatrix}
\quad \text{and} \quad
b = \begin{bmatrix}
5 \\
15 \\
0
\end{bmatrix}
\]

6. Consider the linear transformation \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) defined by \( T(x) = Ax \), where

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & 2 \\
0 & -2 & 2
\end{bmatrix}
\]

What are the domain, codomain, and range of the linear transformation \( T \)? How would you describe this linear transformation geometrically, using the descriptive words we discussed in class (shear, rotation, dilation, contraction)? Be as precise as possible.
1. Write the solution set of the following homogeneous linear system of equations as the span of a set of vectors in $\mathbb{R}^4$:

\[
\begin{align*}
2x_1 + 2x_2 - 4x_3 + x_4 &= 0 \\
-x_1 - 3x_2 + 5x_3 - 2x_4 &= 0 \\
x_1 - x_2 + x_3 - x_4 &= 0 \\
5x_1 + 7x_2 - 13x_3 + 4x_4 &= 0
\end{align*}
\]

2. Find an equation relating $a$ and $b$ such that the homogeneous linear system of equations represented by the following augmented matrix has infinitely many solutions if and only if your equation holds:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
3 & a & -7 & 0 \\
-4 & 3 & b & 0
\end{bmatrix}.
\]

3. Are the three vectors \( \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix} \), \( \begin{bmatrix} 3 \\ -4 \\ -6 \end{bmatrix} \), and \( \begin{bmatrix} -1 \\ -6 \\ 10 \end{bmatrix} \) linearly dependent or linearly independent? Prove your answer.

4. Show that the span of the three vectors in the previous problem is a plane through the origin in $\mathbb{R}^3$. Give an equation for this plane in the form $ax_1 + bx_2 + cx_3 = 0$ for some explicit constants $a$, $b$, and $c$.

5. Write the set of solutions to the equation $Ax = b$ in parametric vector form, where

\[
A = \begin{bmatrix}
2 & 5 & -1 \\
0 & 3 & -3 \\
-1 & -1 & -1
\end{bmatrix}
\quad \text{and} \quad
b = \begin{bmatrix}
13 \\
9 \\
-2
\end{bmatrix}.
\]

6. Consider the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x) = Ax$, where

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & -\sqrt{3} \\
0 & \sqrt{3} & 2
\end{bmatrix}.
\]

What are the domain, codomain, and range of the linear transformation $T$? How would you describe this linear transformation geometrically, using the descriptive words we discussed in class (shear, rotation, dilation, contraction)? Be as precise as possible.