Toward A Unified Theory of Sandwiches: Sandwich Topology

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We present a unified picture of sandwich topologies, which considers the open-faced sandwich, dubbed $S_1$, to be the fundamental unit of sandwichhood. All other sandwich topologies are shown to be equivalent to convolutions of many $S_1$ units. This picture unites previous work in the field and is an important first step towards a Consistent Representation of a Unified Sandwich Theory (CRUST). The conditions on the nature of ingredients in a proper sandwich is left for future work.

INTRODUCTION

The sandwich is a staple of cuisine across continents, and can be found in the context of breakfast1, lunch, or dinner. It is said to have been invented by John Montagu fourth Earl of Sandwich; it is known that “The original sandwich was in fact a piece of salt beef between two slices of toasted bread” [2]. Today’s notion of a sandwich is somewhat more broad; the three defining features of the original sandwich, “toasted”, “beef”, “between two slices of bread” can each be relaxed. Uncontroversial sandwichhood is routinely granted to objects formed from untoasted ingredients; similarly, they may contain no meat at all, and they might even be formed from tortilla or pita rather than bread.

It is thus interesting to ask what constitutes sandwichhood in the first place. A complete classification will answer such controversial questions as: is a hot dog on a bun a sandwich? is a lettuce wrap a sandwich [3]? if I put two pizzas on top of each other, have I made a pizza sandwich (or is an ordinary pizza an open-faced sandwich [4])? among numerous others. Indeed, the consideration of questions like these represents an important motivation for this work. The ultimate goal is what we call a Consistent Representation of Unified Sandwich Theory (CRUST), which will answer these questions and related ones by providing a framework for unifying disparate areas of current sandwich research. In the end, a CRUST would provide the natural boundary for the field, inside which productive sandwich research can be conducted.

In lieu of a complete investigation, one might choose to blindly accept the proclamations of certain “governing bodies” on a case-by-case basis. We believe this approach is not acceptable, and is unlikely to end in a consistent classification. Take, for example, the 2015 announcement by the National Hot Dog and Sausage Council (NHDSC), that a hot dog is not a sandwich [5]. Does this settle the debate? Surely this council, by its name and charge,

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1 The most important meal [1]
To ground our work, we will use a certain set of “common-sense” notions on which to build our classification. For example, certain objects (e.g. a hammer, the Pacific Ocean, or a pile of grass clippings) cannot qualify as sandwiches, and a theory which grants such objects sandwichhood should be deemed a failure. Similarly, an edible object consisting of meat and cheese between two pieces of bread is clearly a sandwich; indeed, in a sense it is the exemplar of what a sandwich should be. It is valuable to check an abstract sandwich theory against such common-sense notions.

We will refer occasionally to this notion of an exemplar sandwich, one which cannot be excluded from the collection of sandwiches by a consistent theory; we call this a Sandwich of Unmistakeable Belonging (hereafter SUB); this will be a common-sense check of sandwichhood. The SUB consists of two pieces of bread, containing small slices or pieces of meat and cheese. One could imagine using a definitive SUB which included other optional ingredients, such as condiments (e.g. mayonnaise) or vegetables (e.g. lettuce and tomato), but this is a matter of taste. Crucially, we require that the SUB does not contain such controversial ingredients as, for example, egg, cream cheese, or additional slices of bread. Even more crucially, a SUB will also not contain hammers or the Pacific Ocean.

In the next section, we outline some basic terminology and assumptions of this work. We will then proceed to classify the topology of sandwiches, an important step towards the desired CRUST.

**FIRST STEPS**

As a starting point, it is crucial to distinguish orthogonal dimensions on which sandwichhood might depend. We identify two important broad classes of properties: (1) *Topology*, the topic of this work, is determined the organization of the ingredients in physical space; and (2) the *nature of the ingredients*, which we leave to a future work. There may indeed be further dimensions to specify for a full classification of sandwichhood, but we will not discuss this issue further here. It is important not to bite off more than one can chew.

Our acknowledgement of these two important dimensions of sandwichhood is somewhat subtle, and deserving of its own separate treatment. For example, is there a minimum number of ingredients necessary to constitute a sandwich? Need these ingredients fall into particular classes (meat, cheese, vegetables) or can they be anything at all? Need the outside consist of bread, or do other ingredients qualify? These questions are important, but we leave them for future work.

It will be adequate for our purposes here to accept only very basic assumptions to constrain the ingredients. We will assume that any sandwich must consist of an outer layer (which we will call *bread*) and some *toppings*. This terminology should not be taken too literally; clearly the outer layer of a sandwich might plausibly be made of something other than actual bread, as in the case of a wrap. In the original sandwich [2], the “bread” was literal bread, and the “toppings” consisted only of salted beef. For reasons that will become apparent shortly, we will not assume that toppings to be enclosed by bread, but rather assume that toppings live on a single side of a given bread unit.

We seek to classify all valid sandwich topologies. To avoid double-counting and to maintain consistent identification of equivalent topologies, a few rules are necessary. First, as we have already stated, we assume toppings to live on a single side of a given slice of bread; bread with toppings on both sides seems clearly to violate common-sense notions of sandwichhood, and they also make a mess in the kitchen [1]. Second, we impose the requirement that bread *not be torn*, so that we may count sandwich topologies consistently by number of bread slices; one slice may not be torn into two, as this would constitute an inequivalent topology. Third, the curving or deforming of a given piece of bread constitutes an equivalent topology (given that no new tears or holes are formed). In this sense, excluding ingrediential considerations, a tortilla wrap is equivalent to both a pita wrap and a bread-based open-faced sandwich.

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3 It has occasionally been suggested that a single piece of bread could be considered a sandwich; such inflammatory statements will not be commented on here, both due to the controversy surrounding this proposal and because of our focus on topology (rather than ingredients).

4 As we will make clear below, the distinction between an wrap and a bread-based open-face sandwich is ingrediential, not topological, but should be investigated further in the near future.
FIG. 2: Pictographic representations of the first few terms in the sandwich expansion.

a) The fundamental sandwich unit $S_1$;
b) A single $S_1$ flipped vertically, defined as equivalent to (a);
c) $S_2$, defined as a normal + an inverted $S_1$;
d) Two upright $S_1$ units, or $2S_1$;
e) An $S_2$ with an $S_1$ on top;
g) Two upright $S_1$ units and an inverted $S_1$, defining an $S_3$;
h) A vertical inversion of (g), which is equivalently an $S_3$.

TOPOLOGY OF A SANDWICH

Of crucial importance to the topological classification of sandwichhood is the distinction between “closed” and “open-faced” sandwiches. These are topologically distinct configurations, in the sense that an open-faced sandwich has a single bread boundary whereas a closed sandwich has two; one cannot deform an open-faced sandwich continuously into a closed one without tearing the bread manifold, an invalid procedure. We accept as intuitively obvious that both of these types are valid sandwich configurations, provided other relevant criteria (e.g. ingredients) are satisfied.

Once it is accepted that both closed and open-faced sandwiches are acceptable sandwich topologies, the natural next question is the relation between the two. We posit that a closed sandwich is fundamentally the conjunction of two open-faced sandwiches, making the open-faced sandwich the fundamental unit of sandwichhood. We refer to this fundamental unit as $S_1$. A typical closed sandwich (e.g. a SUB) is an $S_2$, which signifies both it’s closed-ness as well as the number of $S_1$ units it contains. On the other hand, two $S_1$ units on top of each other is not an $S_2$; we classify it instead as $2S_1$, two distinct sandwiches stacked together rather than a single closed sandwich.

It is also important to note certain symmetry considerations. Consider a single $S_1$. If we flip it upside-down, we clearly have not generated a new topology; similarly with a rotation in any direction by any angle. More generally, we suggest that the topology of a given sandwich should not be affected by its orientation compared to the external world; a sandwich in the vacuum of space is well-defined irrespective of any external influences. A sandwich can, however, change topology through the internal rotation of some of its parts, as illustrated by the qualitative difference between the open configuration $2S_1$ and the closed configuration $S_2$. This will have consequences when we count possible topologies. Without loss of generality, when we build higher-order sandwich contributions, we may regard the “bottom” $S_1$ unit as being “face-up” with respect to some fixed external observer. This naturally rules out also any sandwich candidate whose toppings are fully external, but we have already suggested that such objects cannot be considered sandwiches.

We may continue in this way, building up higher-order sandwiches in what we call the Sequential Approach to $N$-order Determination With Inequivalent Configuration-Having (SANDWICH). In Figure 2 we show all inequivalent configurations of $N \leq 3$ $S_1$ sandwich units. Clearly (a) and (b) are equivalent, as the rotation is defined only with respect to some outside observer. The difference between (c) and (d) has already been pointed out. Similarly, the $3S_1$ configuration (e) can be understood as three separate $S_1$ units stacked, compared with a single $S_1$ on top of an $S_2$ (f). Both of these open configurations should be contrasted with the closed $S_3$ (g) and (h), which are equivalent because one can be obtained by a complete vertical rotation of the other. An equivalent, but more compact, representation of the SANDWICH up to $N = 3$ is given in Figure 3, where an upright $S_1$ is given by an up-arrow, and an upside-down $S_1$ is given by a down-arrow.

We may represent an $N$-order SANDWICH configuration...
For $N > 3$, there is an additional ambiguity that must be acknowledged. Consider $N = 4$, as depicted in Figure 4. The labels on (j-p) indicate the ambiguity in how we define the sandwich decomposition.

Consider configuration (j) as an example. Starting at the bottom (left, in the Figure), the first two $S_1$ units form a united $S_2$; on top of this, is a disconnected (face-up) $S_1$, so we take this to be the natural splitting point. Indeed, we must not forget that the goal is to classify in a useful way how sandwiches will appear in the physical world. The fact that no toppings exist between the second and third $S_1$ units means that these should be regarded stacked, separated sandwiches. On the other hand, configuration (k) has the same set of possible decompositions, but under CLUB is unambiguously defined as an $S_1 S_3$; the top $S_1$ is disconnected from the lower $3$-sandwich. The final result for 4-sandwiches is

$$E_4 = \{4S_1, S_1 S_3, 2S_2, S_4\},$$

as expected.

We do not prove it rigorously here, but we believe the CLUB+SANDWICH is a well-defined and unambiguous procedure for sorting arbitrary $N$-sandwiches into equivalence classes, and that the resulting space of sandwich equivalence classes $E_N$ then spans all possibilities.

CONCLUSIONS

We have presented a rigorous analysis of the topology of sandwiches of arbitrary size. We have made very minimal assumptions, in particular that an open-faced sandwich is indeed a sandwich, and as a result we posit that the open-faced sandwich $S_1$ is indeed the fundamental sandwich unit. All other sandwiches are formed from permutations of different numbers of $S_1$ units. This classification only leaves out one possible collection of permutations, namely, ones in which all external $S_1$ constituents face outward. We have argued that such configurations do not qualify as valid sandwich topologies.

We have further constructed a procedure to classify any $N$-sized sandwich into its relevant equivalence class of sandwichhood, where $N$ is defined as the number of constituent $S_1$ units. The expansion into equivalence classes, dubbed SANDWICH, is a simple and direct procedure for any sandwich up to $N = 3$ (the Big Mac configuration). For $N > 3$, we supplement it by a CLUB selection rule which extends the validity of the procedure.
It is currently only a well-founded conjecture that the CLUB+SANDWICH analysis works at arbitrarily large \( N \), but certainly sandwiches beyond \( N = 6 \) or so are mostly of only theoretical interest anyway.

The question of ingredients is a complicated one, which we have mostly ignored here. We discussed in the introduction a few of the more pressing issues, but a full treatment will be delayed to a future work.

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[8] @matttomic, "The Sandwich Alignment Chart." https://twitter.com/matttomic/status/859117370455060481