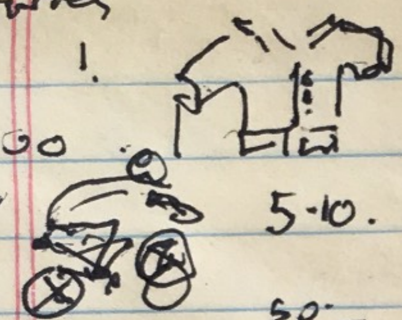
0. [Pyrennes anyone?] [Loose Ends, Questions, Open Questions]

stories

1. 

15. 2. Finish "Oscillatory."

25 3. " "Collisions + Minimality"

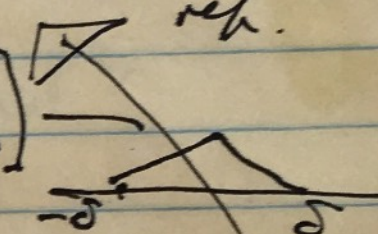
2300 — do π eight — p118-119

8 II equations 5-10. + Marchal. ref.

50.

$q_1 = q_1(t) + \epsilon \mu_1 s$

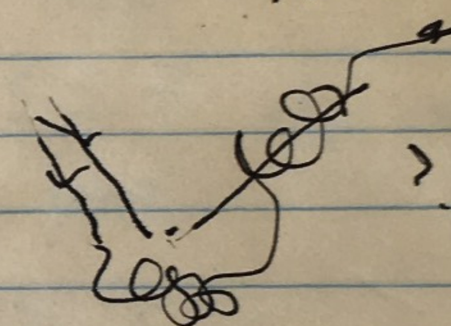
? $q_2(t) - \epsilon \mu_2 s$

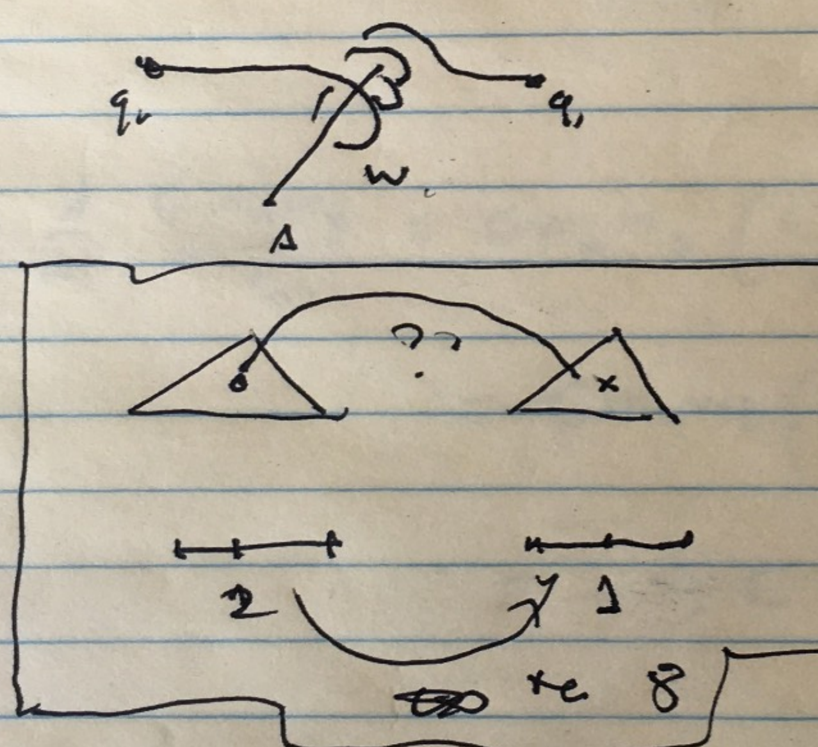


4. Quantum Questions?

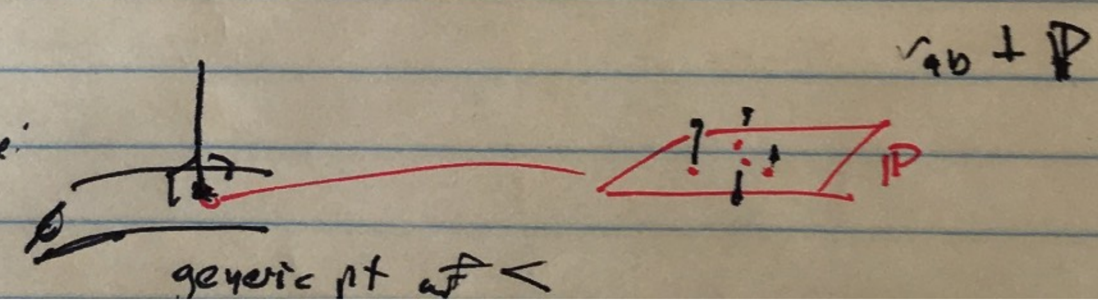
2030 5. Jacobi-Maupertuis metric approach

10 6. Open Qs.





2. I.



Theorem.

Consider any zero angular momentum solution to the standard attracive ($1/r$ potential) Newton's equations for $d+1$ bodies in d-dimensional. Suppose that along this solution the inter-body distances satisfy the bound

$$r_{ab} \leq c \tag{1}$$

Then, within every time interval of size $\frac{1}{\pi}(\frac{c^3}{GM})^{1/2}$, this solution has a degeneration instant.

Deriving the needed eqn. for S.

$$\sigma(t) = \pi(q(t)); \pi : M(d,d) \to Sh(d, d+1)$$

shape curve

sol'n to Newton's eqns having zero ang. mom. (J=0)

$$\nabla_{\dot{\sigma}} \dot{\sigma} = -\nabla \bar{V}(\sigma)$$

simple form of eq requires J = 0 along q(t)

$$\dot{S} = \langle \nabla S, \dot{\sigma} \rangle$$

$$\ddot{S} = \langle \nabla S, \ddot{\sigma} \rangle + \langle \nabla_v \nabla S, v \rangle$$

standard computation in Riem. geom.

$$\ddot{S} = \langle \nabla S(q), -\nabla V(q) \rangle + II_S(v, v)$$

$$= I + II$$

q solves Newt.

2nd f.f. of level sets of S = equidistants from deg. locus

PROP. I = -S g, g > 0 , and

$$g > \omega^2, \omega = GM/(\delta^3), M = \Sigma m_a, \text{ assuming bound } r_{ab}(t) \leq \delta$$

Pf I: Hamilton-Jacobi or `weak KAM' + $\|\nabla S\| = 1$

+ property of 2-body potential f(r) = -1/r

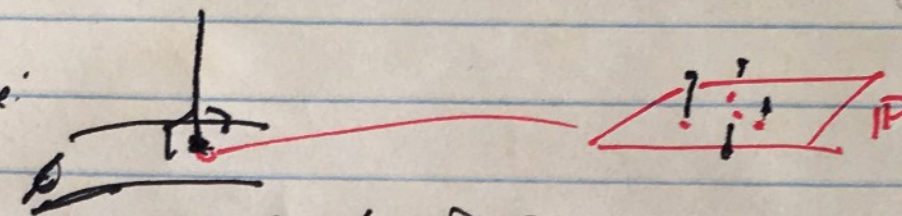
PROP. II = -S h, h > 0 .

Pf II: curv. shape space ≥ 0,

+ Σ is tot. good. + `Sign & The Meaning of Curvature.'

$r_{ab} \perp \mathbb{P}$

2. I.



generic pt of <

geod: $\pi(\gamma)$

$\gamma(t) = q_a + t v_a$ $\quad$ $v_a \perp$ plane $=$ all q_a's

$$\Rightarrow r_{ab}(t)^2 = \| q_a + t v_a - (q_b - t v_b) \|^2$$

$$= \| q_{ab} \|^2 + t^2 \| v_{ab} \|^2$$

No cross terms! $v_{ab} \perp q_{ab}$

$$\Rightarrow \frac{d}{dt} r_{ab} \quad -- \quad \frac{d}{dt} r_{ab}^2 = 2 r_{ab} \dot{r}_{ab}$$

$$= 2t \| v_{ab} \|^2$$

$$= \frac{d}{dt} \left(\| q_{ab} \|^2 + t^2 \| v_{ab} \|^2 \right)$$

$$\Rightarrow \dot{r}_{ab} = t \frac{\| v_{ab} \|^2}{r_{ab}} := s \frac{\| v_{ab} \|^2}{r_{ab}}$$

since $s = t$ alg non $\cap$ geod.

$$\Rightarrow \frac{d}{dt} \frac{G m_a m_b}{r_{ab}} = G m_a m_b \left(\frac{-1}{r_{ab}^2} \right) \dot{r}_{ab}$$

$$= G m_a m_b \left(\frac{-1}{r_{ab}^2} \right) s \frac{\| v_{ab} \|^2}{r_{ab}}$$

$$= -s G \frac{m_a m_b}{r_{ab}^3} \| v_{ab} \|^2$$

``Sturm comparison" with

$$\ddot{S} = -S\omega^2$$

S has a zero in any interval of time of size

$$\pi/\omega$$

implying theorem. For all d, N,
with N =d+1

Dynamical content of theorem:

the collinear states form a global Poincare section for the
negative energy
zero-angular momentum
3- body problem

Open Problems. For d=3, N=4.

¿Is `bounded' necessary?
In other words: ¿Is it true that every negative energy, zero angular momentum solution to the four-body problem either ends in a singularity (eg triple collision) or oscillates forever about the coplanar locus?

¿Is angular momentum zero even necessary?
In other words: ¿Is it possible that every bounded solution to the four-body problem oscillates forever about the coplanar locus?

¿Is there a `good' symbolic dynamics,
copying the planar 3 body case, where there are 3 symbols representing the three types of generic collinearity?

For N=4, d= 3 there are 7 symbols, representing 7 generic ways a tetrahedron can degenerate to a quadrilateral