

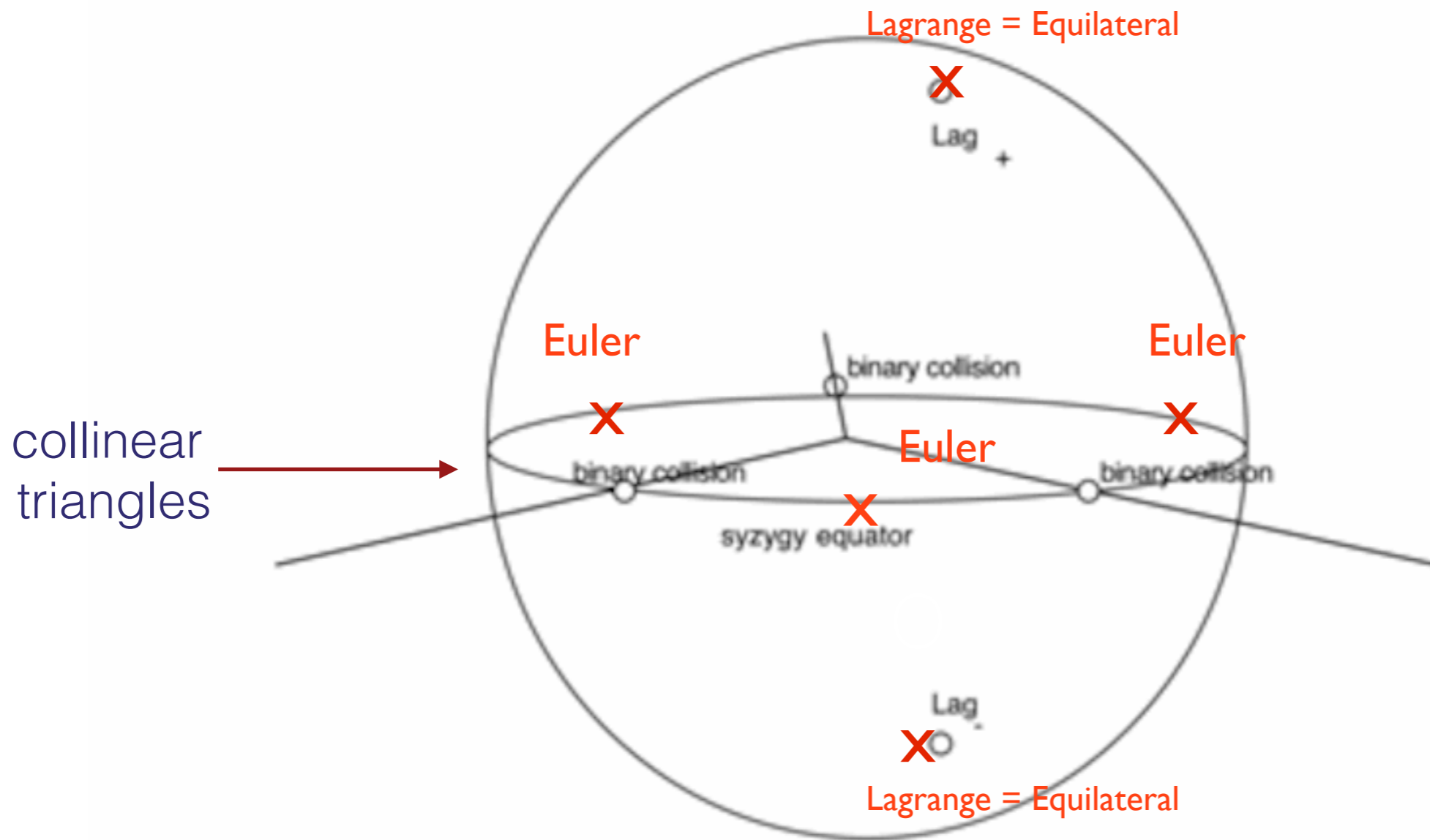
Recall:

Course Overview

1. Lagrangian systems. Natural Mechanical systems. Symmetry vs topological constraints. Works of: Gordon, Poincaré, me. Braids. Planarity. Strong vs weak forces.
- 2.** Shape space, shape sphere, reduction. The Eight
3. Hard core functional analysis. Two point problem. The eight again. time permitting: Marchall.
4. Infinitely many syzygies and coplanarities. Riem. geom methods.
5. Open ending. Open problems.

Lecture 2. Shape space, shape sphere. Reduction.
Application to the Eight

to Geogebra...



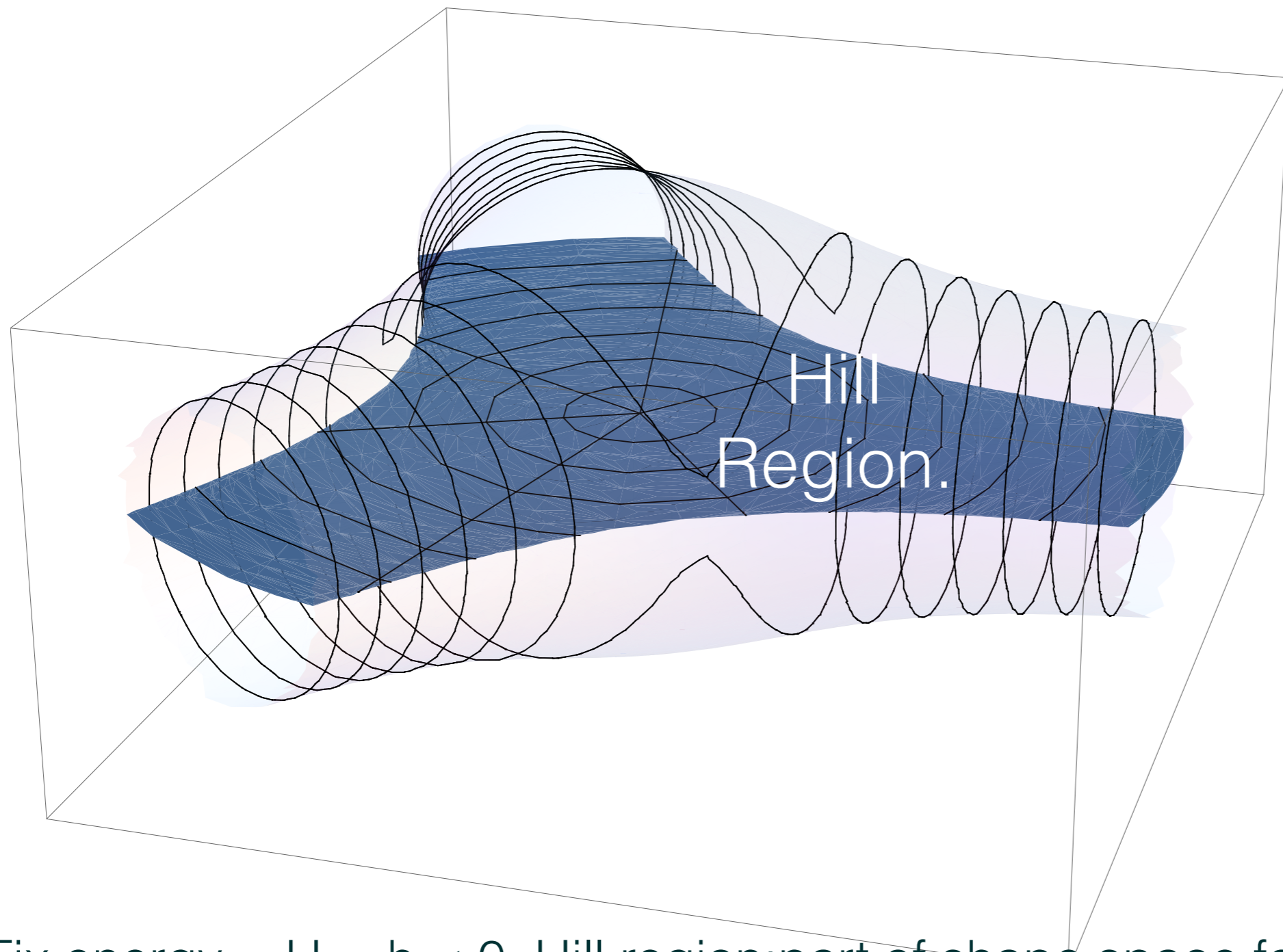
SHAPE SPHERE

Oriented similarity classes of triangles

\subset

SHAPE SPACE

Oriented congruence classes of triangles



Fix energy = $H = -h < 0$. Hill region: part of shape space for which there is a v and $H(q,v) = -h$. Domain where motion occurs. Identical to region with $U(q) > +h$

How does this picture arise out of the planar 3-body problem?



Galileo:

The laws of physics are invariant under the group G of isometries of space

1600

G : translations, rotations, reflections

(N -body) Eqns: dN coupled 2nd order ODEs on the ...

Configuration space $Q = N$ -fold product of space

$$G \curvearrowright Q$$

Galileo:

G is a subset of the Galilean group of isometries of space, or time, and of space-time 'boosts'

$q_a(t)$ solves

$q_a(t)$ solves	\iff	$q_a(t) + d$ solves	(space translations)
1600	\iff	$Rq_a(t), R \in O(d),$ solves	(space rotations)
	\iff	$q_a(t - t_0)$ solves	(time translations)
	\iff	$q_a(-t)$ solves	(time reflection)
	\iff	$q_a(t) + tv$ solves	(boosts)

`for each continuous symmetry there is a conserved quantity'

Emmy Noether



space translation \longleftrightarrow linear momentum

space rotation \longleftrightarrow angular momentum

time translation \longleftrightarrow energy

To get rid of translational degrees of freedom:

using a boost can make

linear momentum =0

then fix the center of mass of the system =0

using a translation

Lagrangian side

$$TQ \rightarrow (TQ)/G$$

or

Hamiltonian side

$$T^*Q \rightarrow (T^*Q)/G$$

Identify with..

$$T(Q/G) \oplus \text{something}$$

$$T^*(Q/G) \oplus \text{something}$$

$Q/G = \text{Shape space}$

‘something’ is foliated by level sets of Noether’s conserved momenta

in case of planar N-body problem: ‘something’ coordinatized by angular momentum J

Derivation of shape space metric
now. .. rough idea of reduction..

Derivation of shape space metric,
shape sphere .. now. ..

$$Q/G = \text{Shape space}$$

to blackboard !

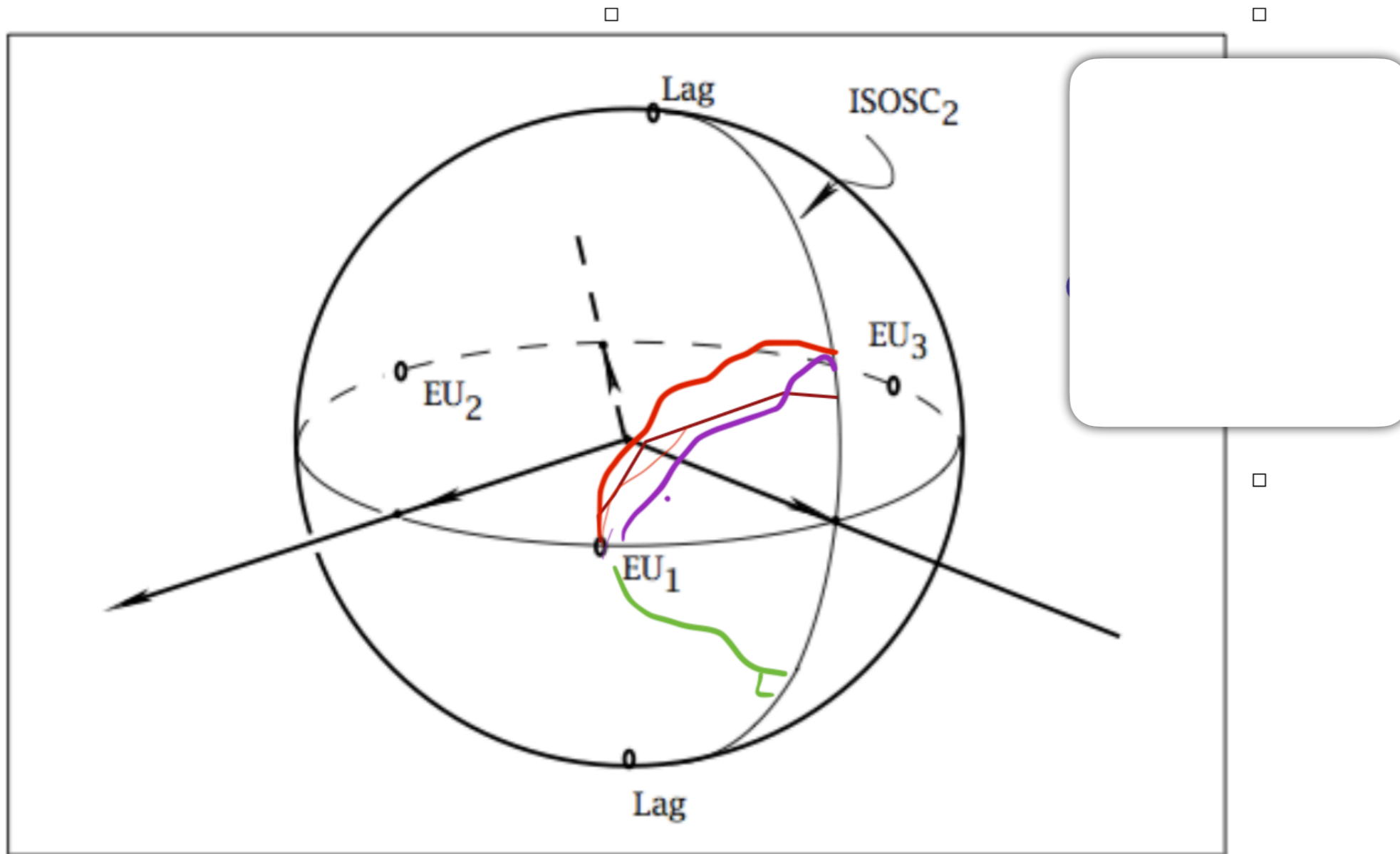
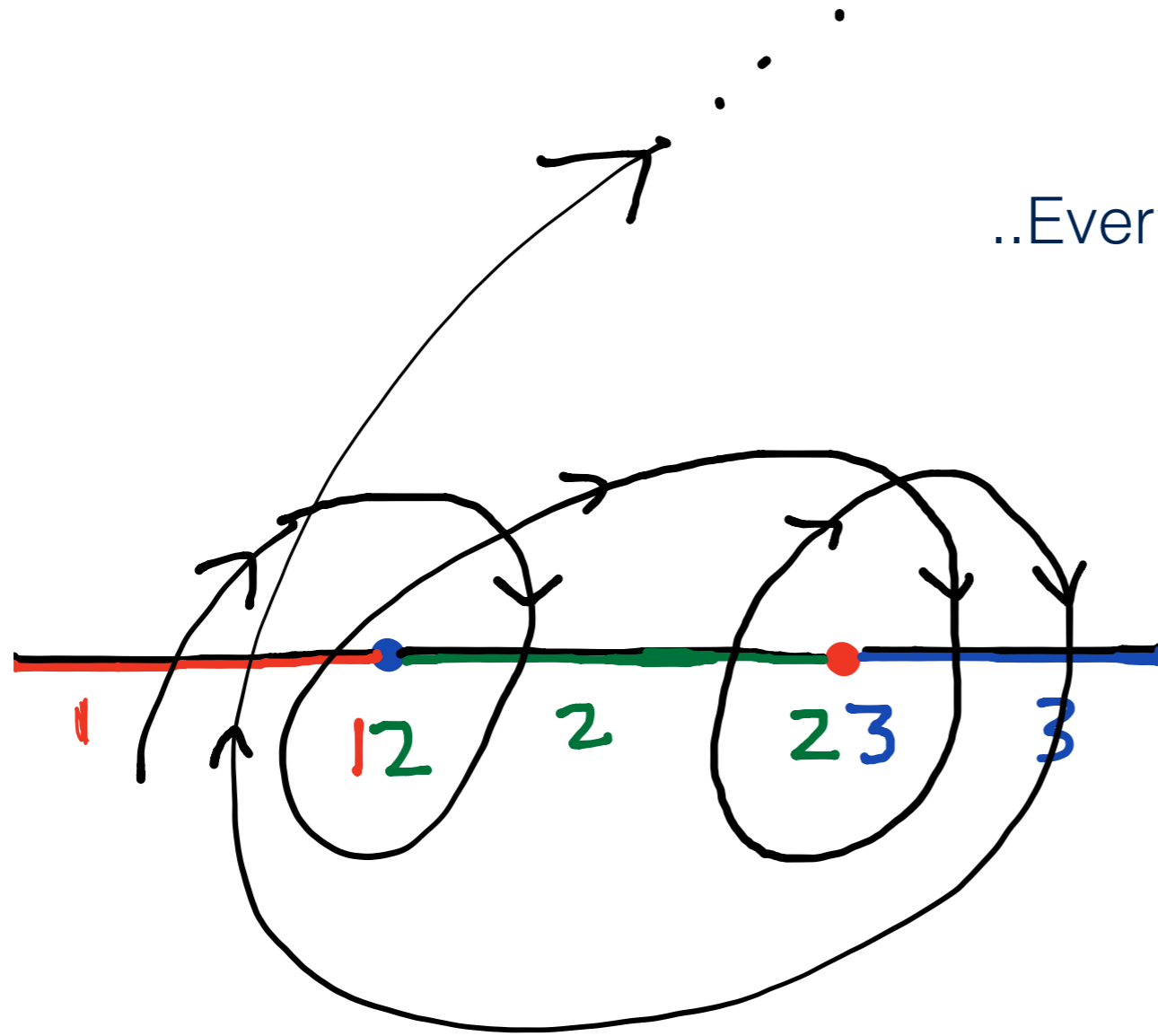


Figure 4. The shape sphere.

Min. Action $A(q)$
 among all paths
 q in shape space
 connecting the Euler Ray (Eu_1)
 to the Isosceles subspace
 with 2 as vertex (ISOSC_2)



..Every free homotopy class...

Figure 8: 123123

→ 1213231...

Blackboard pictures in shape
space.

FINI, lecture 2