1. Path Arguments.

Two edges are said to be "adjacent" or "consecutive" if they share a vertex.

A "path" in a graph is a list of consecutive edges. In a simple graph, we can list edges uniquely by specifying their vertices: $e = v_1 v_2$, $v_i \in V$, $e \in E$. Then a path in a simple graph can be written out as a list of vertices: $v_1 v_2 v_3 \dots v_k$ where v_i is adjacent to $v_{i\pm1}$. We insist that no edge is used more than once, so, in particular we do not allow backtracking, which means that we insist that $v_{i+2} \neq v_i$. A "maximal path" is then a path we cannot extend any edges to: we cannot move to another vertex without retracing our steps.

Let us now give an alternative proof that any finite tree has at least two leaves, meaning two vertices of degree 1.

Theorem. Any finite tree having more than one vertex has at least two distinct vertices of degree 1.

Proof. Let T be our finite tree. Pick any vertex $v \in T$. Choose a maximal path passing through v. This path must have two ends since the graph has no cycles. But the ends of a maximal path which is not a cycle must be vertices of degree 1 ! QED

Does this convince you? If not, here are some more details, details which involve the construction and existence of a 'maximal path".

Details. Write $v = v_1$. At least one edge leaves v since our graph is connected and has another vertex besides v. Choose one of these edges. Call it $v_1 v_2$. Now continue walking away from v_1, never retracing your steps, until you run out of places to go. More specifically, if v_2 has degree 1, stop. We have one end of our maximal path. Otherwise, $deg(v_2) > 1$ and there is another edge (maybe many) which leaves v_2 besides the edge we came in on ($v_2 v_1$). Pick one of these edges. Call it $v_2 v_3$. Now we have path $v_1 v_2 v_3$. If v_3 has degree 1, stop, we have one end of our maximal path. Otherwise, keep walking away from v_3. Continue in this manner until you run out of places to go. Specifically, there are only two ways this process can halt (i) we return to v_1, or (ii) we run out of edges by landing on a vertex of degree 1. The first possibility is impossible, since our graph is a tree and has no cycles. But the process has to stop because there are only finitely many vertices in T. Consequently, the process ends in a degree 1 vertex v_k, with associated "ray" $v_1 v_2 \dots v_k$.

If $deg(v_1) = 1$, we are done: we have a maximal path from v_1 to v_k and v_1, v_k are our two vertices of degree 1. Otherwise, there in another edge leaving v_1 besides the edge $v_1 v_2$ we chose above. Pick one of these other edges, call it $v_0 v_1$. Continue walking "backwards" away from v_0, never retracing your steps as before, until you run out of places to go. Again, the only way the process can stop is by ending on a vertex of degree 1. And it must stop because T is finite. When we are done, we have a maximal path $v_{-m} v_{-m+1} \dots v_0 v_1 \dots v_k$. with endpoints $v_{-m} \neq v_k$ both of degree 1. So we have our two degree 1 vertices.