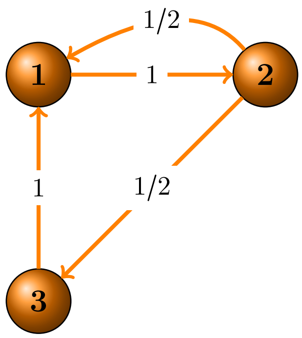
SOLUTIONS.

The columns and rows are labelled by the vertices as given.

EXER. 1. The matrix is

$$H = \begin{pmatrix} 0 & \frac{1}{2} & 1 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

EXER 2.



EXER 4. With

$$I = \begin{pmatrix} a \\ a \\ b \end{pmatrix}$$

and H as in exert. 1, we compute that

$$HI = \begin{pmatrix} \frac{1}{2}a + b \\ a \\ \frac{1}{2}a \end{pmatrix}$$

Setting $HI = I$ yields the equations $\frac{1}{2}a + b = a, a = a, \frac{1}{2}a = b$ with unique solution, up to scale

$$I = \begin{pmatrix} a \\ a \\ \frac{1}{2}a \end{pmatrix}$$

Normalizing by insisting that the entries sum to 1 yields $a = 2/5, b = 1/5$.

EXER 5. If we start off with 5 students in our Markov Maze after N iterations we ought to get the ratio $[2:2:1]$.

Did we?

How big was N?

COMPUTE: λ_2

The other two eigenvalues of H are $-\frac{1}{2} \pm \frac{1}{2}i$. The norm of either is $1/\sqrt{2}$

Say we want to get within 10 percent of the desired distribution. The error is estimated by $|\lambda_2|^N$. We are asking: what is the first N for which $|\lambda_2|^N < .1$ or $1 < (1/10)2^{N/2}$. Since $2^3 < 10 < 2^4$, $N = 7$ or 8 should work.

After 8 about 8 rounds we ought to settle down to $2:2:1$

[We simultaneously make 1 step from each vertex.]