# 1. Graph Theory, Lecture Tree Number.

Main source. Ch. 6, Biggs.

Beginning questions.

How many trees are there on n (labelled) vertices?

Do the cases : $n = 2, 3, 4$

Do $n = 5$.

The formula: $n^{n-2}$.

What is bigger: $2^{\binom{n}{2}}$ or $n^{n-2}$? By how much?

A tree on $n$ vertices has $n - 1$ edges.

Prove!

Find a counterexample to the assertion: "A simple graph on n vertices is a tree if and only if it has $n - 1$ edges".

Prove: if a graph on n vertices has $n - 1$ edges AND is connected, then it is a tree.

Tree number, Laplacian, etc.

The tree number of a graph $\Gamma$ is the number of spanning trees for the graph.

Example: Let $K_n$ be the complete graph on n vertices. Then EVERY tree on these n vertices is a spanning tree. Hence the tree number of $K_n$ is $n^{\binom{n}{2}}$.