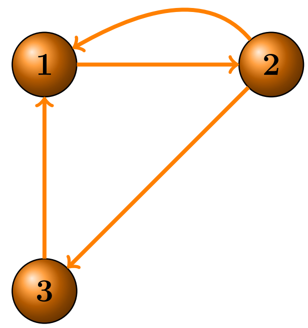
Reference: How Google Finds Your Needle in the Web's Haystack; Featured Column, AMS.

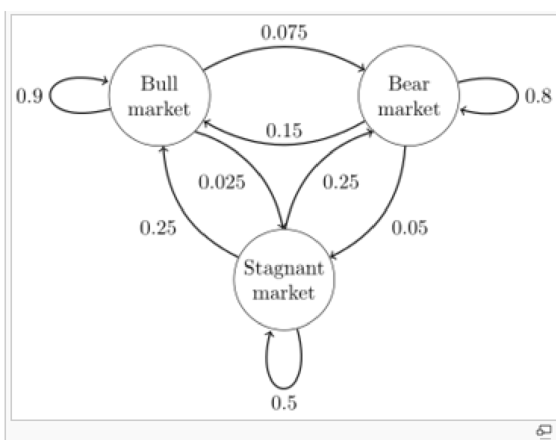
Read the description of the first method for attaching a positive square matrix to a directed graph.



EXERCISE 1 : Find the matrix H for this graph:

ANSWERS are in the Answers supplement to this note "PageRankAnswers.pdf'.

Here, copied from wiki, is an example of a good, quick and dirty way to encode the transition matrix associated to a "Markov Chain" on a graph by labeling or "weighting" the edges of directed graph: Let '1' = Bull, '2' = Bear, '3'= Stagnant.



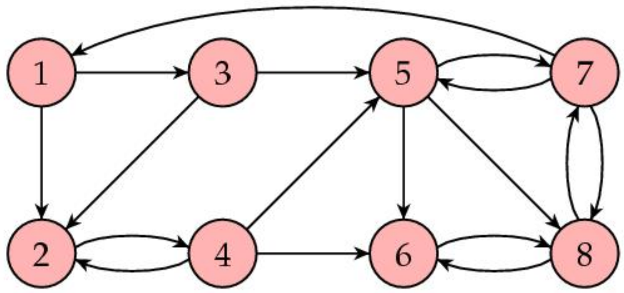
Write $H_{ij} = Prob(j \mapsto i)$. Then we read off the transition matrix for this stock market model:

$$H = \begin{pmatrix} 0.9 & 0.15 & 0.025 \\ 0.75 & 0.8 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{pmatrix}$$

WARNING: wiki uses the transpose of this matrix for the transition matrix.

EXERCISE 2: Weight the edges of first graph appropriately so the matrix you get is the correct matrix.

Here is the Markov chain matrix from the beginning of that article:



EXERCISE 3.
A) Label its edges according to the Google page rank description.
B) Verify that its associated transition matrix is the 8 by 8 matrix H the author claims.

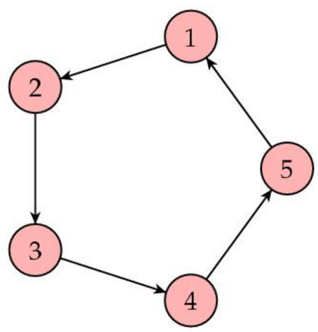


FIGURE 1. Left:

EXERCISE 4. Let H be the matrix you found in *exercise 1.* Find the eigenvector I for the matrix H which Find I by the method of symmetry plus guessing. A bit of staring at the picture suggests a *symmetry* which interchanges vertices 1 and 2, keeping 3 fixed. This suggests the *guess* that the eigenvector has the form

$$I = \begin{pmatrix} a \\ a \\ b \end{pmatrix}$$

Plug this I into the eigenvector equation $HI = I$. Solve for b in terms of a. Normalize, thus finding the probability eigenvector

COMMENTARY. The end result of a PageRank search is an eigenvector I for some matrix M built out of linking data for web pages. Then I_j is the ranking of page j. The I_j are all positive numbers. The j whose I_j is biggest is the first page listed from the search. Our first pass at the matrix M will be the H just described. We will make two successive approximations to M, called S and then G in this paper. I is always the eigenvector for the eigenvalue 1. 1 is always the largest eigenvalue of M. It is often of multiplicity 1, which means that I is unique up to scale. We normalize I so that it becomes a probability: the sum of its entries are 1. This eigenvector I is also called the "stable vector" for M because, in the good cases, if we start off with any nonzero vector v and proceed by the mapping $v \mapsto Mv \mapsto MMv \mapsto v_{k+1} = Mv_k$ the vector v_k limits, or stabilized to (a multiple) of I. (IN dynamical systems terms, I is a "stable attractor" of the mapping M.) We will understand this better later. Our first pass at the PageRank algorithm is to use the matrix H just described and look for its eigenvector I with eigenvalue 1.

EXERCISE 5. Read the Probability section of the Feature. Set up 3 chairs labelled 1, 2, 3. Assign a coin-flipper to one or more of the chairs. Use chair 1 as a source. Implement the Markov chain with students 'entering' in through the source chair 1.

Let's see if it works!!