

Lecture schedule. Every Thursday there will be a student talk. Sign up starting today. HW is handed out each Tuesday, due the next Tuesday. Hand back something on at least one problem each week.

1. Start with ODEs on the circle. $\dot{\theta} = -\sin(\theta)$ versus $\ddot{\theta} = -\sin(\theta)$. Their phase portraits. Qualitative long time theory. Gradient vs Hamiltonian flows. First orderizing to rewrite 2nd eqn as a vector field on TS^1 . Flows from vector fields. Finding the fixed points. Linearizing at the fixed points. Character of linearizations.

Generalizing the two examples to n . To manifolds.

Sign ups for talks. Straightening lemma or flow box lemma

Autonomous linear systems: $\dot{X} = AX$ and e^{tA}

HW hand out.

2. A bit on the gradient flows. Morse theory, overview and basic idea. Perturbing gradient flows. Ideas from structural stability.

General culture: the oldest problem. Stability in the 3-body problem. Time-dependent perturbation of the pendulum: $\ddot{\theta} = -\sin(\theta) + \epsilon \cos(t)$. Poincare section and map. IFT to get nearby hyperbolic / elliptic orbits for the perturbed pendulum. $W^s \neq W^u$ and consequences thereof.

Student Talk.

3. Well-posedness of ODEs on manifolds. Homoclinic Tangles ct'd. Horseshoes. Semi-conjugacy to Bernoulli shifts.

4. The extremes in behaviour: Integrable vs Anosov. Examples of Integrable: Harmonic Oscillator. Geodesic flow on sphere. Kepler's problem. Geodesic flows on surfaces of Revolution. Examples of Anosov behaviour. The cat map. Geodesic flow on a negatively curved surface.

Student talk.

5. Perturbations of Anosov systems: (topological stability!. Gromov quote. Notion of quasi-geodesic. Sketch proof of rigidity, geod flow case.

6. Perturbations of integrable systems. History. Small denominators. KAM. What happens near the elliptic fixed point.

Return to pendulum. Hartman-Grobman. Stable and unstable manifolds theorem.. Smale's theorem.

7. Mixes of integrable and 'Anosov'. The standard map as a model.

8. Measures of "chaos". Lyapunov exponents. Devaney's definitions. Entropy I (topological).

9. Ergodic theory basics. Measurable maps preserving a measure. Koopmanism. Spectral theory. Baker's map. Doubling map.

10. Entropy, II (measure theoretic). As distinguishing Bernoulli shifts. Ornstein's theorem. (Statement) Rokhlin's theorem (statement).

11. Diffeos and homeos of S^1 . Rotation number. Irrational vs irrational.

12. Arnol'd tongues (1). Standard circle map. Appearance of devils staircase. [See wiki]

other topics: Axiom A. N-body. Periodic orbit counting. Lefschetz formula. Poincare-Hopf formula. Logistic map. From topological dynamics. omega-limit set. alpha-limit set. Nonwandering set. Recurrence. Chain recurrence. Fundamental theorem of Conley. Poincaré-Dulac theory, the bare idea [Lie transform method] . Birkhoff normal form. Averaging. Laplace and Gauss's work. The Denjoy example (ref: Brin.) Bifurcation theory. Aubry-Mather theory. Cantori. Arnold conjectures. Seifert conjec and Kuperberg's example.