into the eye in exactly the same manner as it would have come into the eye if the object were at B', then the eye-brain system interprets that, assuming it does not know too much, as *being* an object at B'. So the illusion that there is an object behind the mirror is merely due to the fact that the light which is entering the eye is entering in exactly the same manner, physically, as it would have entered had there *been* an object back there (except for the dirt on the mirror, and our knowledge of the existence of the mirror, and so on, which is corrected in the brain).

Now let us demonstrate that the principle of least time will give Snell's law of refraction. We must, however, make an assumption about the speed of light in water. We shall assume that the speed of light in water is lower than the speed of light in air by a certain factor, n.

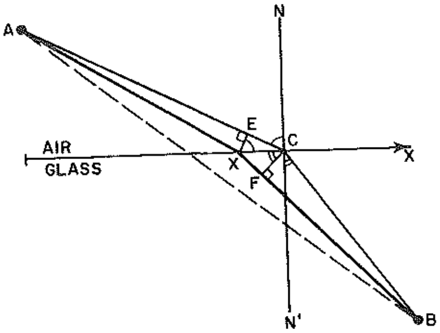

Fig. 26–4. Illustration of Fermat's principle for refraction.

In Fig. 26–4, our problem is again to go from A to B in *the shortest time.* To illustrate that the best thing to do is not just to go in a straight line, let us imagine that a beautiful girl has fallen out of a boat, and she is screaming for help in the water at point B. The line marked X is the shoreline. We are at point A on land, and we see the accident, and we can run and can also swim. But we can run faster than we can swim. What do we do? Do we go in a straight line? (Yes, no doubt!) However, by using a little more intelligence we would realize that it would be advantageous to travel a little greater distance on land in order to decrease the distance in the water, because we go so much slower in the water. (Following this line of reasoning out, we would say the right thing to do is to *compute* very carefully what should be done!) At any rate, let us try to show that the final solution to the problem is the path ACB, and that this path takes the shortest time of all possible ones. If it is the shortest path, that means that if we take any other, it will be longer. So, if we were to plot the time it takes against the position of point X, we would get a curve something like that shown in Fig. 26–5, where point C corresponds to the shortest of all possible times. This means that if we move the point X to points *near* C, in the first approximation there is essentially *no change* in time because the slope is zero at the bottom of the curve. So our way of finding the law will be to consider that we move the place by a very small amount, and to demand that there be essentially no change in time. (Of course there is an infinitesimal change of a *second* order; we ought to have a positive increase for displacements in either direction from C.) So we consider a nearby point X and we calculate how long it would take to go from A to B by the two paths, and compare the new path with the old path. It is very easy to do. We want the difference, of course, to be nearly zero if the distance XC is short. First, look at the path on land. If we draw a perpendicular XE, we see that this path is shortened by the amount EC. Let us say we gain by not having to go that extra distance. On the other hand, in the water, by drawing a corresponding perpendicular, CF, we find that we have to go the extra distance XF, and that is what we lose. Or, in *time*, we gain the time it would have taken to go the distance EC, but we lose the time it would have taken to go the distance XF. Those times must be equal since, in the first approximation, there is to be no change in time. But supposing that in the water the speed is $1/n$ times as fast as in air, then we must have

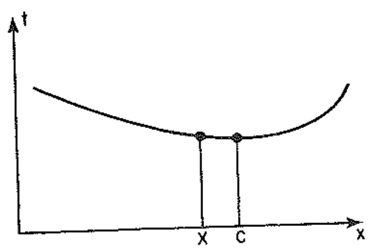

Fig. 26–5. The minimum time corresponds to point C, but nearby points correspond to nearly the same time.

$$EC = n \cdot XF. \tag{26.3}$$

Therefore we see that when we have the right point, $XC \sin EXC = n \cdot XC \sin XCF$ or, cancelling the common hypotenuse length XC and noting that

$$EXC = ECN = \theta_i \quad \text{and} \quad XCF = BCN' = \theta_r,$$

we have

$$\sin\theta_i = n \sin\theta_r. \tag{26.4}$$

So we see that to get from one point to another in the least time when the ratio of speeds is n, the light should enter at such an angle that the ratio of the sines of the angles θ_i and θ_r is the ratio of the speeds in the two media.