1. You are the life guard on a beach whose shore line is straight and runs directly North-South You spot someone drowning. You are a meters from the shoreline. They are out a meters away from the shoreline but also $2a$ meters north of you.

Draw a geometric picture of the situation.

Attach!

You run exactly twice as fast as you swim. Consider two possible routes to swimmer:

Route 1: draw a straight line from your location to the swimmer. Follow that.

Route 2: aim to spot on the shoreline as close to the swimmer as possible: so: on the shore, but $2a$ meters north, and then swimming directly west from there.

Verify that Route 2 is quicker than Route 1.

Attach picture of two routes.

In route 1 we travel the diaganal of two congruent squared, each of side a, one in the land on in water. So each has length $\sqrt{2}a$. Since (speed) = (length)/(time) we have (time) = length/(speed). Thus the time of travel for route 1

$$t_1 = \sqrt{2}(a/v_l) + \sqrt{2}(a/v_w)$$

where v_l is our speed on land and v_w our speed in the water. Now $v_L = 2v_q$ so that

$$t_1 = [(1/2)\sqrt{2} + \sqrt{2}](a/v_w)$$

In route 2, the path traveled on land is the hypotenuse of a right triangle with sides $a, 2a$ so has length $\sqrt{5}a$, while the path in the water is straight out so has length a. By the same reasoning the time for route 2 is

$$t_2 = [(1/2)\sqrt{5} + 1](a/v_w)$$

The question then: $t_2 < t_1$? becomes the question: $[(1/2)\sqrt{5} + 1] < [(1/2)\sqrt{2} + \sqrt{2}]$?

Use a calculator now. $\sqrt{2} \cong 1.414$, while $\sqrt{5} \cong 2.236$. Thus we have the first number is about 2.118 while the second is about 2.121.

Yes: the 2nd route is faster. (Not by much.)

Ratio of times: $2.118/2.121 = .998$

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Problem 2: Basically the same, same routes, same situation, except now we run 10 times as faster than we swim : $v_L = 10v_w$. The ratio 1/2 which occurs in the time formulae multiplying the land distances now becomes 1/10

We get:

$$t_1 = [(1/10)\sqrt{2} + \sqrt{2}](a/v_W)$$

$$t_2 = [(1/10)\sqrt{5} + 1](a/v_W)$$

These numbers become 1.555 and 1.236 times a/v_w so that now the ratio is about .795.

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Reality check and Aside: Some people got this ratio to be around 2 instead of .8. As a reality check imagine that $v_L = +\infty$ then the time of flight on the land is zero. In that case $t_2/t_1 = 1/\sqrt{2}$ which is approximately .704. The factor of 1/10 appearing in both t_1 and t_2 is v_W/v_L and tends to zero as $v_L \to \infty$. As long as $v_W < v_L < \infty$ we have $t_2/t_1 > .704$..

Problem 3.

We take the hint.

Then FIGURE HERE!!

The length of the route in medium 1 is $\sqrt{a_1^2 + y^2}$ so the time-of-flight in medium 1 is $\sqrt{a_1^2 + y^2}(/1v_1)$. The length of the route in medium 2 is $\sqrt{(y - b_2)^2 + a_2^2}$, so its time-of-flight is $\sqrt{(y - b_2)^2 + a_2^2}(1/v_2)$

The total time of flight then is

$$T = T_1 + T_2 = (1/v_1)\sqrt{a_1^2 + y^2} + (1/v_2)\sqrt{(y - b_2)^2 + a_2^2}$$

Now, use the chain rule to differentiate the square-roots arising in pythagoras' theorem, we have that

$$dT_1/dy = (1/v_1)y/\sqrt{a_1^2 + y^2}$$

while

$$dT_2/dy = (1/v_2)(y - b_2)/\sqrt{(y - b_2)^2 + a_2^2}$$

Now, from the figure, we recognize the fractions arising as (opposite/hypotenuse) in each situation!

Thus:

$$dT_1/dy = (1/v_1)\sin(\theta_1)$$

$$dT_2/dy = -(1/v_2)\sin(\theta_2)$$

*** carefuly for minus sign – check out pic.! ** so that

$$dT/dy = (1/v_1)\sin(\theta_1) - (1/v_2)\sin(\theta_2)$$

Setting equal to zero yields Snell's law in the form

$$\sin(\theta_1)/\sin(\theta_2) = v_1/v_2$$

and since $v_1/v_2 = n_2/n_1$ we get the original form of Snell also.