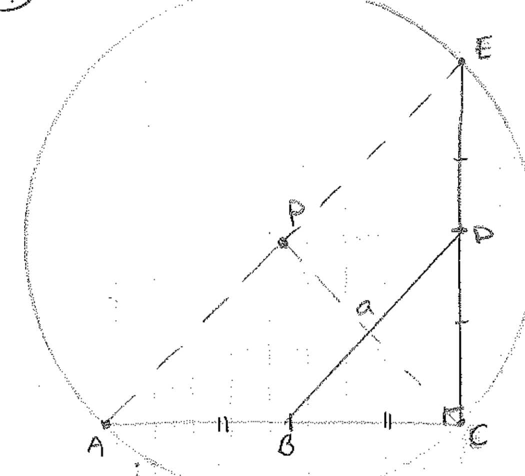
HW #4 Daele Mattner

↳ Group 8 problem connected.

(19) Two ⊥ cords are drawn from a point on the circle. The distance between the midpoints of the cords is a. Find the diameter of the circle!



10/10 total
[see ~~back of~~ back p.]

Claim #1: AE is the diameter.
Proof: Let AC and CE be cords on circle C, centered at P, where AC and CE are perpendicular to each other at C so $\measuredangle ACE = 90°$. Let B be the midpoint of AC and D be the midpoint of CE. Let the measure of BD equal a. By properties of inscribed angles on a circle $\measuredangle ACE = \frac{1}{2}\measuredangle APE$ so $\frac{1}{2}\measuredangle APE = 90°$ which implies that $\measuredangle APE = 180°$. Thus the line connecting AE goes through the centers so is the diameter.

Claim #2: AE = 2a.
Proof: By pythagorean theorem we know $(AC)^2 + (CE)^2 = (AE)^2$ and $(BC)^2 + (DC)^2 = (a)^2$. Since B and D are midpoints then $AC = 2BC$ and $CE = 2(CD)$. Thus,

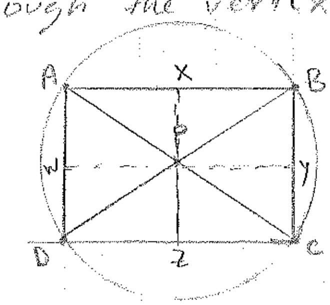
$(AE)^2 = (AC)^2 + (CE)^2$ by pythagorean thm
$= (2BC)^2 + (2CD)^2$ by substitution $AC = 2BC, CE = 2CD$
$= 4[(BC)^2 + (CD)^2]$
$= 4(a)^2$ by substitution $a^2 = (BC)^2 + (CD)^2$

This is cool.

and so $AE = \sqrt{4(a)^2} = 2a$.

QED.

(33) Prove that any rectangle with a symmetry axis not passing through the vertex admits a circumscribed circle.



Proof: Let ABCD be a rectangle. Then AB = DC and AD = BC by definition of a rectangle and $\measuredangle DAB = \measuredangle ABC = \measuredangle BCD = \measuredangle CDA = 90°$. So $\triangle ABC = \triangle ACD = \triangle DCB = \triangle ADB$ by SAS.
By homework #2 there exists a circle such that the vertices of a triangle all lie on the circle, and by previous problem a triangle constructed with two cords perpendicular with each other at a single point on the circle then the opposite ends of the cords when connected goes through the center ~~of~~ of the circle.

Proof: By ...

Thus $\triangle ABC$ admits a circumscribed circle and AC lies on the diameter. Since $\triangle ABC = \triangle ACD$ then $\triangle ADC$ also admits the same circumscribe circle with AC on the diameter thus D and B are equidistance from the center of circle as are A and C so ABCD admits a circumscribed circle.
QED