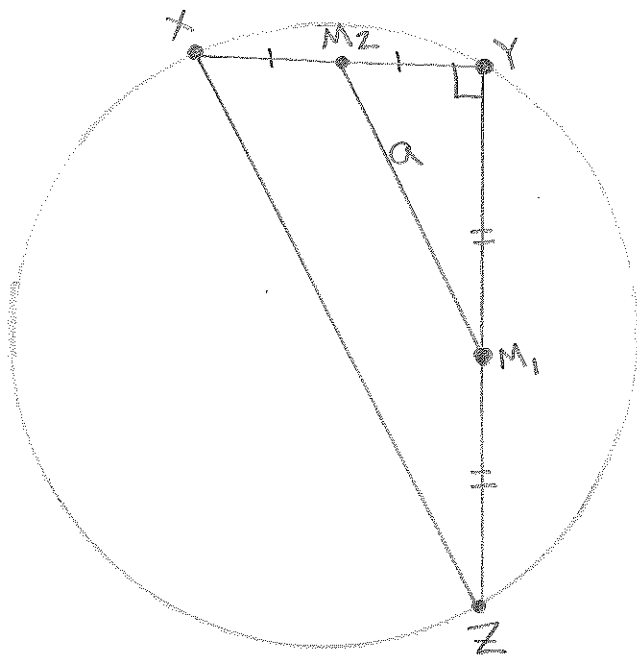


19.



By Thales's theorem if you have a right triangle inscribed in a circle then two points must lie on the diameter.

By construction  $\angle XYZ$  is a right angle and  $X, Y, Z$  lie on the circle. Thus  $\triangle XYZ$  has two points that lie on the diameter. Since  $XY$  and  $YZ$  are arbitrarily chosen chords,  $XZ$  must be the diameter of the circle.

Now since  $\overline{XY} = 2\overline{M_2Y}$  and  $\overline{YZ} = 2\overline{M_1Y}$ ,

$\triangle XYZ$  is similar to  $\triangle M_2YM_1$  and

$\triangle XYZ$  differs from  $\triangle M_2YM_1$  by a

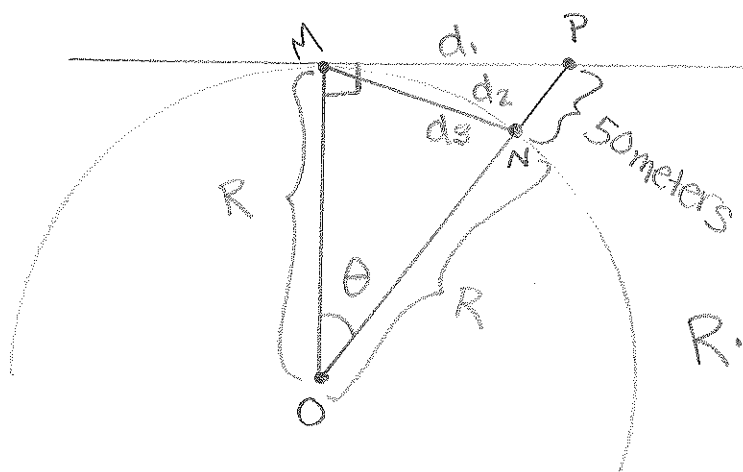
$2:1$  ratio. Therefore  $a = \frac{1}{2} \cdot \text{diameter}$

$\Rightarrow a = \text{radius}$ .

good!

4.2 Find the radius of the planet:

4.2 | 5  
19 | 5



$$R \cdot \theta = 2,000 \text{ meters} = d_2$$

$\angle OMP = 90^\circ$  since the line through  $MP$  is tangent to the circle at pt.  $M$ .

Also  $d_1 \approx d_2 \approx d_3 \approx 2,000$  meters by the small  $\angle$  approximation on  $\theta$ . If  $\theta = \pi/2$  then the mass height would be  $\infty$ . Hence  $\theta$  is small.

Now by the Pythagorean theorem,

$$R^2 + 2000^2 = (R + 50)^2$$

$$\Rightarrow R^2 + 4,000,000 = R^2 + 100R + 2500$$

$$\Rightarrow 100R = 3,997,500 \quad \text{pretty slick}$$

$$\Rightarrow R = 39,975$$

Hence the radius of the planet is approximately 39,975 meters.