Homework 1

Natalie Nowicki
128 due 10/2/13
Worked with Carolina

1. 10/10

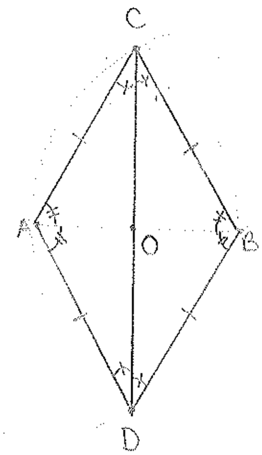
1) construct the perpendicular bisector of a given line segment AB

Construction:

1) measure the given line segment AB with a compass, for the remainder of part 1, do not change the distance set on the compass.
2) create a circle center A, radius AB.
3) create a circle center B, radius AB.
4) Label the intersections of the circle with center A and the circle with center B, points C and D.
5) Using a straightedge, connect points C and D.
6) call the intersection of line segment AB and CD, point O.
7) Again, using a straight edge, connect points: A and C, B and C, A and D, and points B and D creating line segments: AC, BC, AD and BD respectively.

Claim: CD perpendicularly bisects AB at O.

Recall AB is the radius of both the circle centered at A and the circle centered at B.




Pf: So AC = AD = AB since they are radii of the circle centered at A, and BC = AB = BD since they are the radii of the circle centered at B. So AC = AD = BC = BD = AB. Same radii for both circles.

$\triangle CAD$ and $\triangle CBD$ share a common side, line segment CD and since AC = AD = BC = BD, $\triangle CAD$ and $\triangle CBD$ are congruent by SSS. ✓

Since $\triangle CAD$ and $\triangle CBD$ are congruent triangles, we know corresponding angles are equal. So $m\angle ADC = m\angle ACD = m\angle BDC = m\angle BCD$ ✓

Now observe that $\triangle ACB$ shares line segment AB with $\triangle ADB$.

So again by SSS, $\triangle ACB$ is congruent to $\triangle ADB$.

Thus $m\angle CAB = m\angle CBA = m\angle BAD = m\angle ABD$. ✓

Observe $\triangle CAO$ shares line segment CO with $\triangle CBO$. So by SAS $\triangle CAO$ is congruent $\triangle CBO$. ✓

Since these are congruent triangles, AO = OB. ✓

Since $m\angle COA = m\angle COB$ and they are supplementary, we can conclude that they are right angles. ✓

Thus CD perpendicularly bisects AB. ∎

perfect

2) Given points A, B, prove that the locus (= set) of points equidistant from A and B is equal to the perpendicular bisector of line segment AB.

From part 1, Recall CD perpendicularly bisects AB.

We want to show that CD contains a set $S = \{P : |PA| = |PB|\}$.

Pf: Let $P \in CD$.

Notice that $\triangle AOP$ and $\triangle BOP$ share the line segment OP.

Since CD is a perpendicular bisector of AB and $P \in CD$.

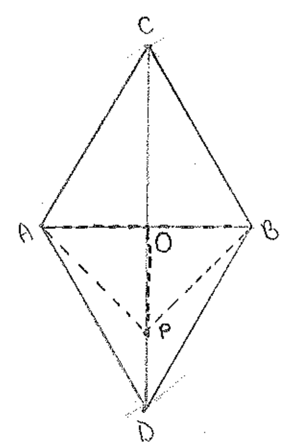
$m\angle AOP = m\angle BOP$ and AO = OB, by SAS $\triangle AOP$ is congruent to $\triangle BOP$.

So we observe AP = BP so $|AP| = |BP|$.

This is a biconditional statement, so now Assume $|AP| = |BP|$.

Then $m\angle PAO = m\angle PBO$. by construction we know AO = OB

Thus by SAS and the congruency of $\triangle POA$ and $\triangle POB$, $P \in CD$. ∎



Gabriella Pimentel
HW # 1
9·30·13

10/10.

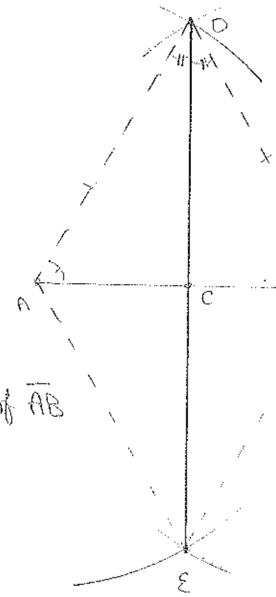
1. Construct the $\perp$ bisector of $\overline{AB}$

Steps: a) Draw 2 semi circles of radius $\overline{AB}$ at centers A & B.

b) label the intersections of the semi circles D, E and connect the two points

c) Label the intersection of $\overline{DE}$ and AB C.

Claim $\overline{DE}$ is the $\perp$ bisector of $\overline{AB}$




Pf: Consider $\triangle DBE$ & $\triangle EAD$

| | |
|---|---|
| $\overline{AD} = \overline{DB} = \overline{BE} = \overline{AE}$ | (radii) |
| $\triangle DBE \cong \triangle EAD$ | (SSS) |
| $\measuredangle ADC = \measuredangle BDC$ | (CPCTC) |
| $\triangle ADC \cong \triangle BDC$ | (SAS) |
| $\measuredangle DCB = \measuredangle DCA$ | (CPCTC) |
| $\measuredangle DCB = \measuredangle DCA = 90°$ | (def of right angle) ✓ |
| $\overline{AC} = \overline{CB}$ | (CPCTC) |
| DE is the $\perp$ bisector of AB | $\square$ |

perfect.

2. Given points A, B prove the locus of points equidistant from A, B is equal to $\perp$ bisector of line segment $\overline{AB}$

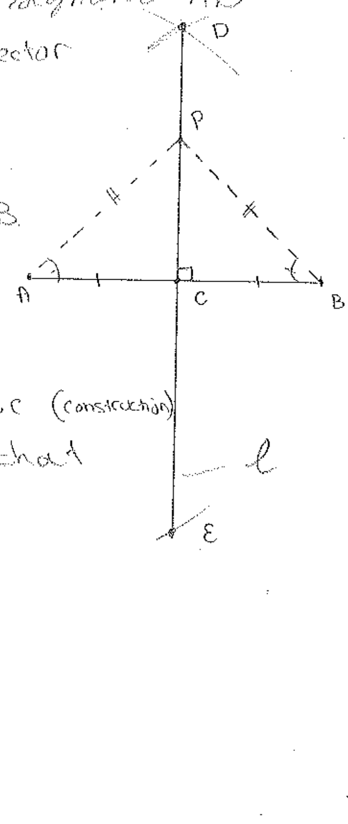
From problem 1, $\overline{DE}$ is the $\perp$ bisector of AB, thus $\overline{AC} = \overline{CB}$ and $\measuredangle PCB = \measuredangle PCA = 90°$

Pick any point, P on DE and draw lines from P to A & B.

Consider $\triangle APC$ & $\triangle BPC$.

by SAS, $\triangle APC \cong BPC$

PA = PB (CPCTC)




If PA = PB, then $\measuredangle PAC = \measuredangle PBC$ & AC = BC (construction)

$\Rightarrow \triangle PCA = \triangle PCB$, which implies that $P \in \ell$.

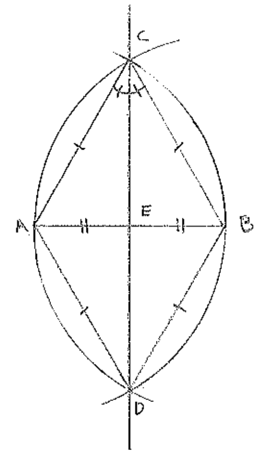
Wesley Price
Prof. Montgomery
Math 128 A

1. 10/10.

HW 1

1. Construct the perpendicular bisector of a given line segment $\overline{AB}$.

- Construct circle of radius AB centered at A. Construct circle of radius BA & centered at B.
- Denote intersection of circle A & circle B C & D as indicated in diagram.
- Draw line through point C & D & denote intersection of $\overline{AB}$ & $\overline{CD}$ as E.
- Draw lines AC, BC, BD, & AD




$\overline{AC}$, $\overline{AB}$, $\overline{AD}$ are the radius of circle A and so $\overline{AC} \cong \overline{AB} \cong \overline{AD}$. $\overline{BC}$, $\overline{BA}$, & $\overline{BD}$ are radius of circle B and so $\overline{BC} \cong \overline{BA} \cong \overline{BD}$. Also since $\overline{AB} \cong \overline{BA}$, $\overline{AC} \cong \overline{BC} \cong \overline{AD} \cong \overline{BD}$. $\triangle ACD$ & $\triangle BCD$ share line segment $\overline{CD}$ & $\overline{AC} \cong \overline{BC}$ & $\overline{AD} \cong \overline{BD}$. By I.8 (SSS) $\triangle ACD \cong \triangle BCD$ & implies that $\angle ACD \cong \angle BCD$. Since $\triangle ACE$ & $\triangle BCE$ share $\overline{EC}$ and $\overline{AC} \cong \overline{BC}$ & $\angle ACE \cong \angle BCE$, therefore by I.4 (SAS) $\overline{AE} \cong \overline{BE}$. Thus, $\overline{CD}$ bisects $\overline{AB}$.

yes!

Since $\triangle ACE \cong \triangle BCE$, it follows that $\angle AEC \cong \angle BEC$ and are adjacent angles which by definition is a right angle. Therefore $\overline{CD}$ is a perpendicular bisector of the given line $\overline{AB}$.

yes! perfecto!