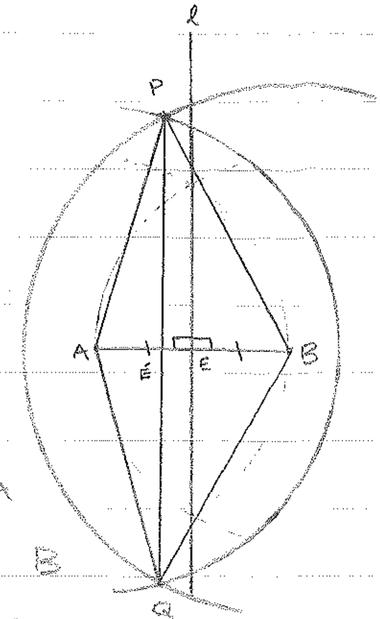
Wesley Price

2.

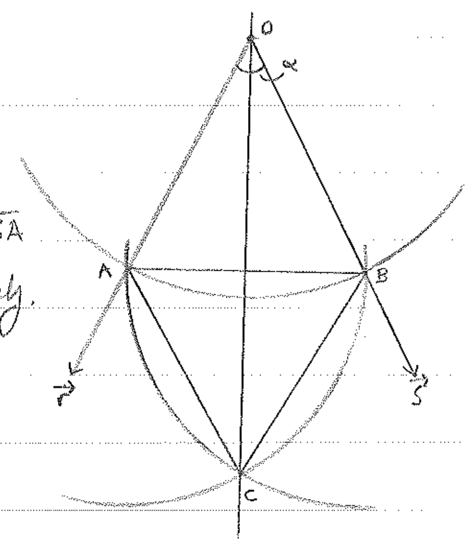


- Given segment AB construct a perpendicular bisector l using the construction from problem 1.
- Denote intersection as E.
- Construct a point $P \notin l$
- Construct line $\overline{AP}$ & $\overline{BP}$
- Construct circle with radius AP centered at A and circle with radius BP centered at B
- Label second point of intersection of circle Q and construct lines $\overline{AQ}$ & $\overline{BQ}$
- Construct line $\overline{PQ}$ & denote intersection of $\overline{PQ}$ & $\overline{AB}$ as E'

Proof by contradiction. Assume there exist a point P not on perpendicular bisector of $\overline{AB}$, or l, that is equidistance from A & B. Then $\overline{AP} \cong \overline{BP}$ and since $\overline{AP}$ & $\overline{AQ}$ are radius of circle A & $\overline{BP}$ and $\overline{BQ}$ are radius of circle B, it implies that $\overline{AQ} \cong \overline{BQ}$. $\triangle APQ$ & $\triangle BPQ$ share $\overline{PQ}$ & by I, 8 (SSS) $\triangle APQ \cong \triangle BPQ$ & by proposition I.5 $\angle APQ \cong \angle AQP \cong \angle BPQ \cong \angle BQP$. $\triangle APE'$ & $\triangle BPE'$ share $\overline{PE'}$ and so by I.4 (SAS) $\overline{AE'} = \overline{BE'}$. However $E' \neq E$ and $\overline{AE} = \overline{BE}$ and thus a contradiction. Therefore all points on l are equidistant from A & B.

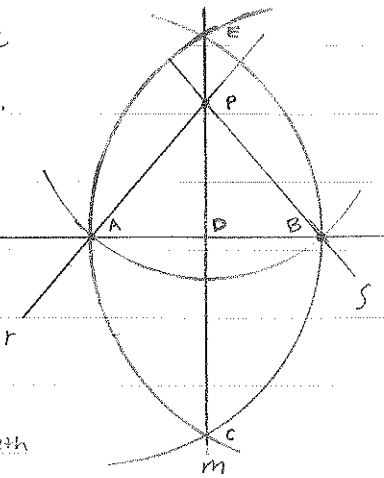
3. Construct the angle bisector of given angle $\alpha = \angle_0(r, s)$

- Given angle α, set compass on O and draw circle centered at O.
- Denote intersection of Circle O with ray $\vec{r}$ & $\vec{s}$ as A & B respectively
- Draw line through A & B and construct circle centered at A with radius AB & circle centered at B with radius BA.
- Denote lower intersection of circle A & circle B as C & draw line through C & O, and line $\overline{AC}$ & $\overline{BC}$



Segment $\overline{AO} \cong \overline{BO}$ since both are radius of Circle O. $\overline{AC}$ is radius of circle A & $\overline{BC}$ is radius of circle B, however $\overline{AB} \cong \overline{BA}$ which are radii of circle A & B respectively. Thus $\overline{AC} \cong \overline{BC}$. $\triangle ACO$ & $\triangle BCO$ Share segment $\overline{CO}$ and so by I.8 (sss) $\triangle ACO \cong \triangle BCO$. This implies $\measuredangle AOC \cong \measuredangle BOC$ as required. Thus, line $\overline{CO}$ bisects angle α.

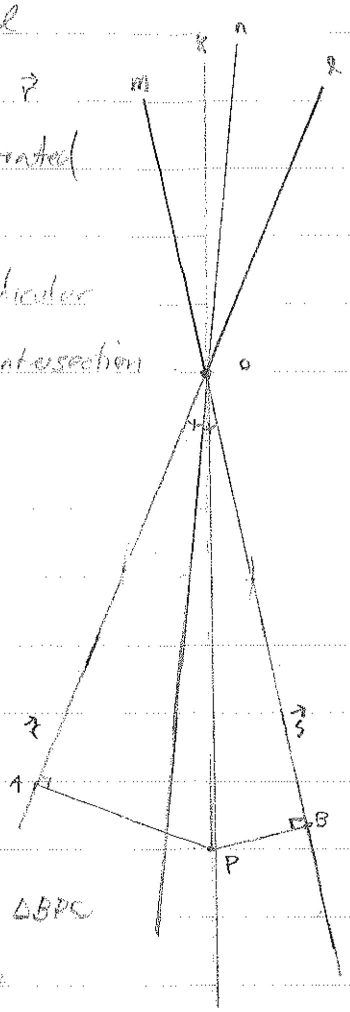
4.



- Given line l & point $P \notin l$, construct line r through point P and intersecting l at A.
- Construct circle radius $\overline{AP}$ centered at P & mark intersection of circle P & l as B.
- Construct perpendicular bisector m through $\overline{AB}$ by creating circle A with radius $\overline{AB}$ & circle B with radius $\overline{BA}$ as proved in problem 1.
- Label points of intersection C & E on the circles' A & B and draw a line m through C & E and label D as the intersection of l & m.

m is the perpendicular to l since m is perpendicular to $\overline{AB}$ of which $\overline{AB} \subset l$. $\overline{AP}$ & $\overline{BP}$ are congruent since both are radii of circle P & thus P is equidistance from A & B. Since P is equidistance from A & B therefore $P \in m$ as proven in problem 2. Thus $\overline{PD} \perp l$.

5. • Given angle $\angle_O(r,s)$ and lines m & l from which have been extended from rays $\vec{s}$ & $\vec{r}$ respectively, construct angle bisector n as demonstrated in problem 3.



• Construct point $P \notin n$ and drop a perpendicular from P onto m and P onto l and mark intersection points B & A respectively.

• Construct line k through points P and O.

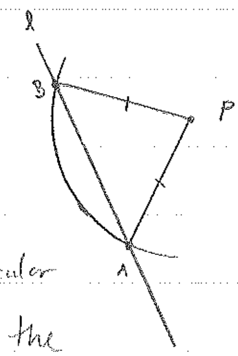
Proof by contradiction. Suppose $\exists P \notin n$ such that it is equidistant from both lines l & m. Then $\overline{AP} \cong \overline{BP}$ since $d(P,m) = \overline{BP}$ & $d(P,l) = \overline{AP}$. $\triangle APO$ and $\triangle BPO$ share segment $\overline{PO}$. Let's superpose $\triangle APO$ onto $\triangle BPO$ by superposing P onto P & $\overline{AP}$ onto $\overline{BP}$. Since $\overline{AP} \cong \overline{BP}$ then A superposes B. This implies that $\overline{AO}$ is on top of $\overline{BO}$ since all right angle equal one another. However, since both triangles share $\overline{PO}$, which is congruent to itself, and point P therefore point P superpose P on the second triangle and thus $\triangle APO \cong \triangle BPO$. However, this implies $\measuredangle BOP \cong \measuredangle AOP$ and that P must be in line n: which is a contradiction

Extra Credit A

$d(\ell, P)$ is the considered the distance from line ℓ to point P because it is the shortest (smallest) constructable line segment between the two. Thus a definition of distance $d(\ell, P)$ would be the shortest line segment from ℓ to P.

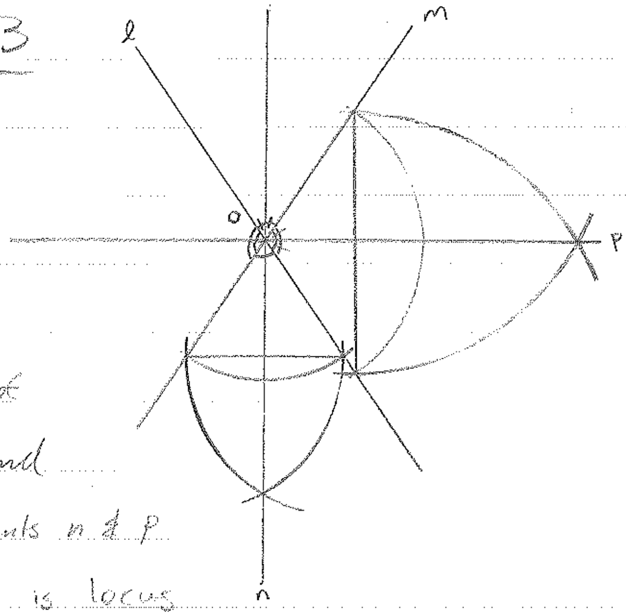
~~Proof~~ construction: • Given line ℓ & point $P \notin \ell$, mark a point A on line ℓ.

• Join line segment $\overline{AP}$ and create circle P of radius $\overline{AP}$ and mark second intersection on ℓ as B and join line segment $\overline{BP}$



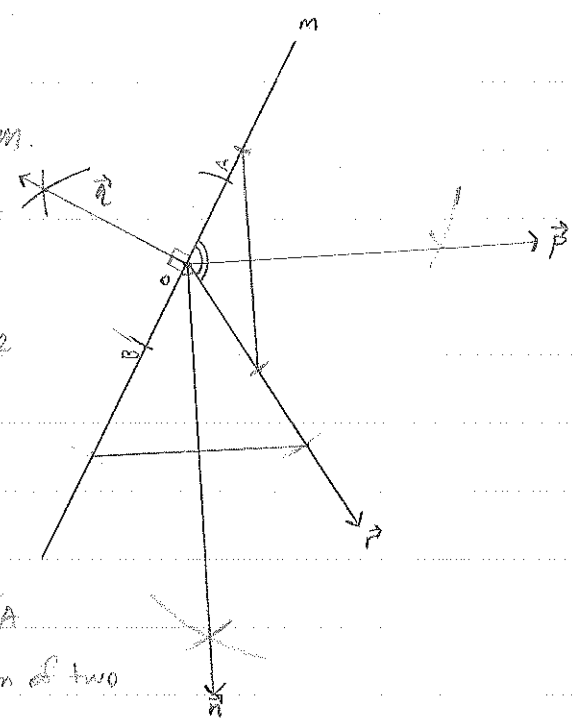
Proof: Suppose $\overline{AP} = d(\ell, P)$. Then $\overline{BP} = d(\ell, P)$ since $\overline{AP} = \overline{BP}$ of radius of circle P. From problem 4 the perpendicular line segment from P to ℓ is said to be the line segment whose length is $d(\ell, P)$. Thus $\overline{AP} \perp \ell$ & $\measuredangle PAB$ is a right angle and $\overline{BP} \perp \ell$ & $\measuredangle PBA$ is a right angle. However adjacent angles of right angles are also right. The parallel postulate imply the sum of the angles of a triangle is two right angles. Thus, $\measuredangle BPA = 0$ & $\overline{BP}$ is the same line as $\overline{AP}$ and is unique.

Extra Credit B



Given l & m two angles are create ∡ & ∡. Construct angle bisector of ∡ & ∡ and extend line segments n & p respectively. n is locus of l & m through ∡ & P is the locus of l & m through ∡.

Extra C



Given m & ray $\vec{r}$ starting on $O \in m$. Construct angle bisector of ∡ & ∡ and extend ray $\vec{n}$ & $\vec{p}$ respectively. Construct perpendicular ray from O on side opposite $\vec{r}$ by selecting point $A \in m$, $A \neq O$ & construct circle O radius $\overline{AO}$ and denote second intersection as B. Construct circle A & B with radius $\overline{AB}$ & $\overline{BA}$ respectively and denote C as intersection of two circles. $\overline{CO}$ is a perpendicular line segment to m, extend $\overline{CO}$ to $\vec{q}$. $\vec{q}$, $\vec{p}$ & $\vec{n}$ are locus of $\vec{r}$ & m.