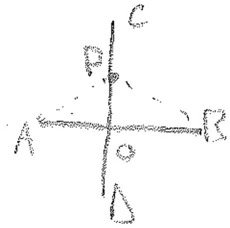
Paul Sinkules

2| If there is a point P on the line, then one can use it distance from A, |AP| as the radius used in 1. to construct a perpindicular bisector.

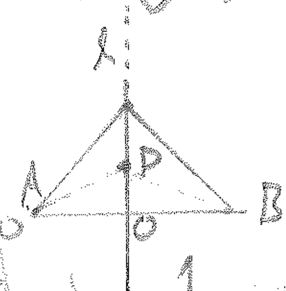


By pythagoras, |AP| = |BP| since |AO| = |BO|.

If a point P is known to be equidistant to A and B, then one can use |AP| to construct the perpindicular bisector. There can only be one perpindicular bisector, so this corresponds to CD from 1, and hence P will lie on CD.

Therefore, P is on the line CD if and only if it is equidistant from A and B, and therefore, since P was arbitrary, the desired result is achieved.

2| In the previous construction, it will be shown that any point along the extended line l, formed by extending CD by any amount in either direction, is equidistant from A and B. It will also be shown that if a point is equidistant from A and B, it must lie on l. Thus the set of points equidistant from AB is equal to the line l, since a line is a set of points.



If a point P lies on l, then we can use the distance $|AP|$ as a radius to construct the perpendicular bisector as in 1. Call this m. m must be parallel to l, for if it weren't, it would violate Euclid's parallel postulate. Since $|AO| = |OB|$, the perpendicular bisector must intersect P since it is parallel and has one point common, namely O. Therefore m coincides with l; they are the same line.

If a point P is equidistant from A and B, one can use $|AP|$ as the radius

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Paul Sutelex
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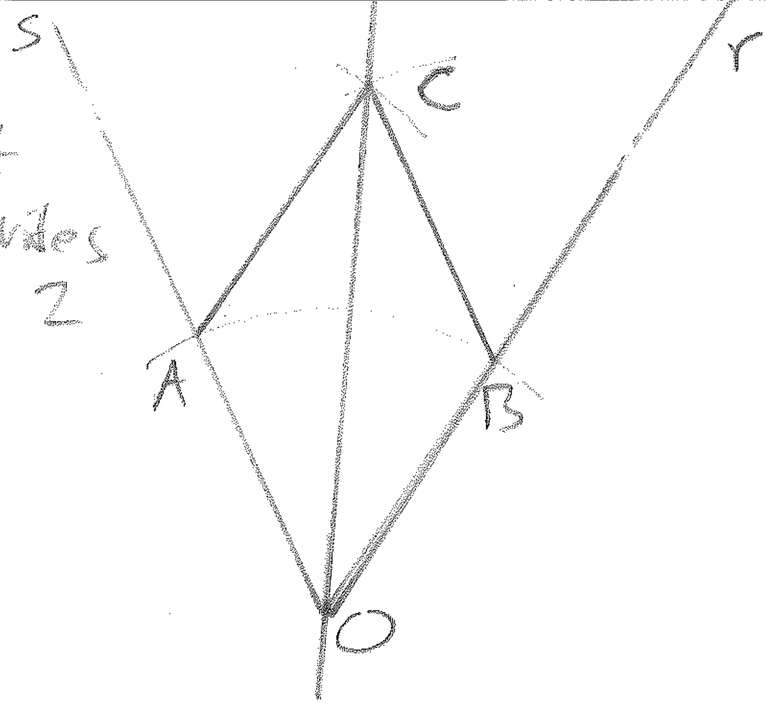
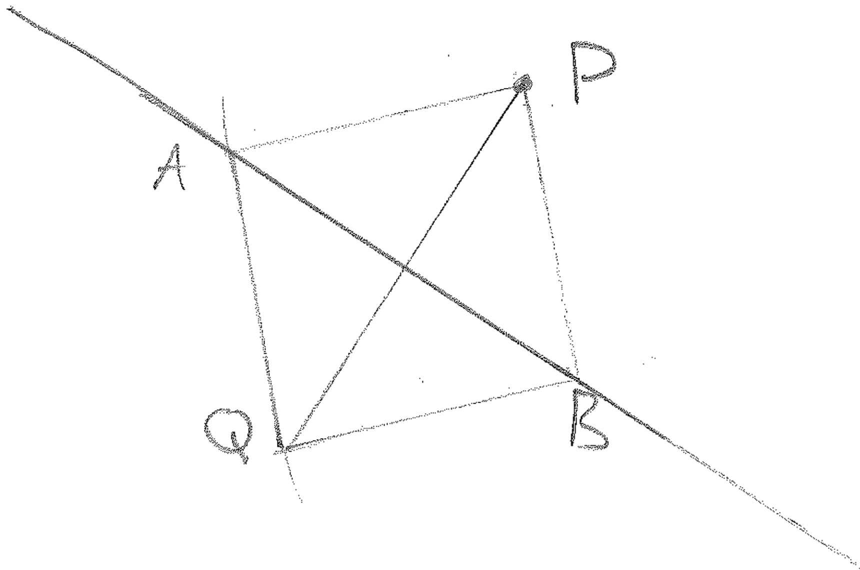
3. To construct a line that divides an angle into 2 equal parts.



Set compass to some length and draw a circle with center O. Label the intersection with s A and the intersection with r B.
Next draw circles with radius $|AB|$ and with centers A and B. Label the intersection C which lies opposite O from the segment AB.
Lastly, join O and C to form the angle bisector.

Pf: $AO \cong BO$ by construction, likewise, $AC \cong BC$. Since $\triangle AOC$ and $\triangle BOC$ have the side OC in common, and therefore are congruent by SSS.
Thus, $\angle AOC \cong \angle BOC$ by corresponding parts of congruent triangles.

4. To 'drop' a perpendicular from a point $P \notin \ell$ to a line ℓ.



Set the compass to some length such that the circle drawn with that radius and centered at P will intersect ℓ twice. Draw that circle and label the intersections A and B. Now draw circles with that same radius, with centers A and B, and mark their intersection as Q (which is distinct from P). By a parallel argument as given in 1, PQ is a perpendicular bisector of AB, therefore perpendicular.

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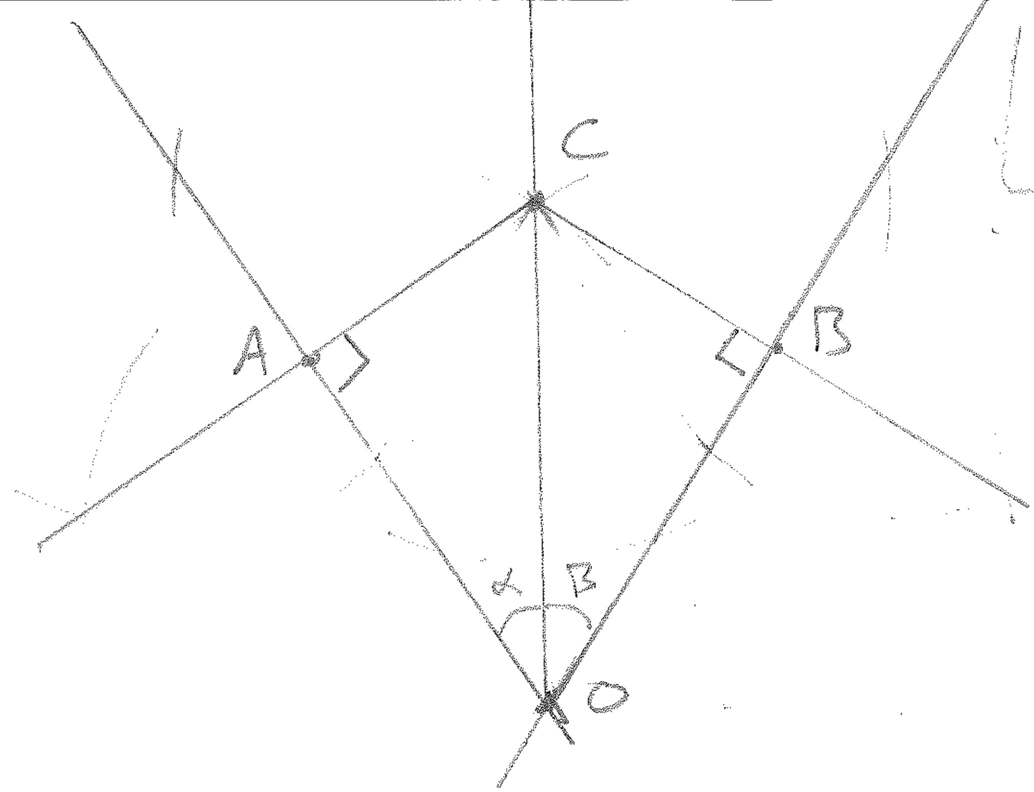

Using 3 and 4, we have $|OA| = |OB|$, and $\angle OAC \cong \angle OBC$

$\triangle OAC$ and $\triangle OBC$ share OC.

Since $\angle AOC = \alpha \cong \angle BOC = \beta$, $\angle OCA$ must be congruent to $\angle OCB$ by the sum of angles axiom. Since all three angles of $\triangle OAC$ and $\triangle OBC$ are congruent, they must be similar, and since they share a side, they are in fact congruent. Therefore, $|AC| = |CB|$. ∎

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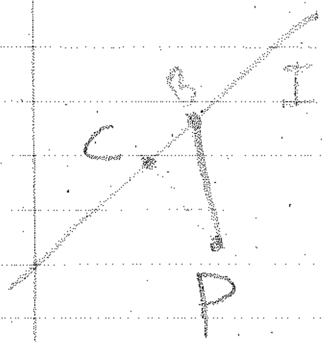
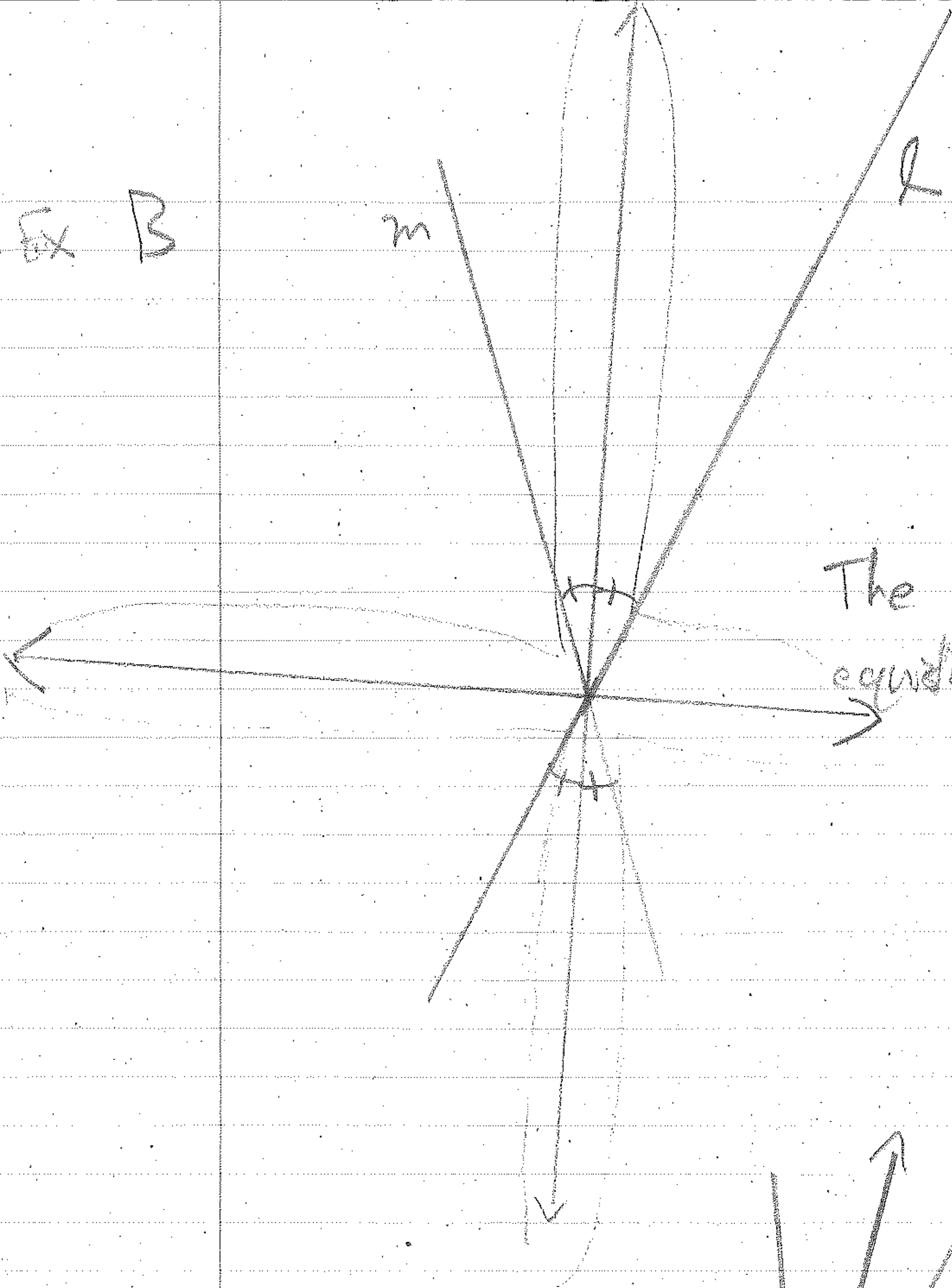
Ex. A. Define $d(\ell, P)$ the distance between a line ℓ and a point P to be $\inf\{|AP| \text{ such that } A \in \ell\}$



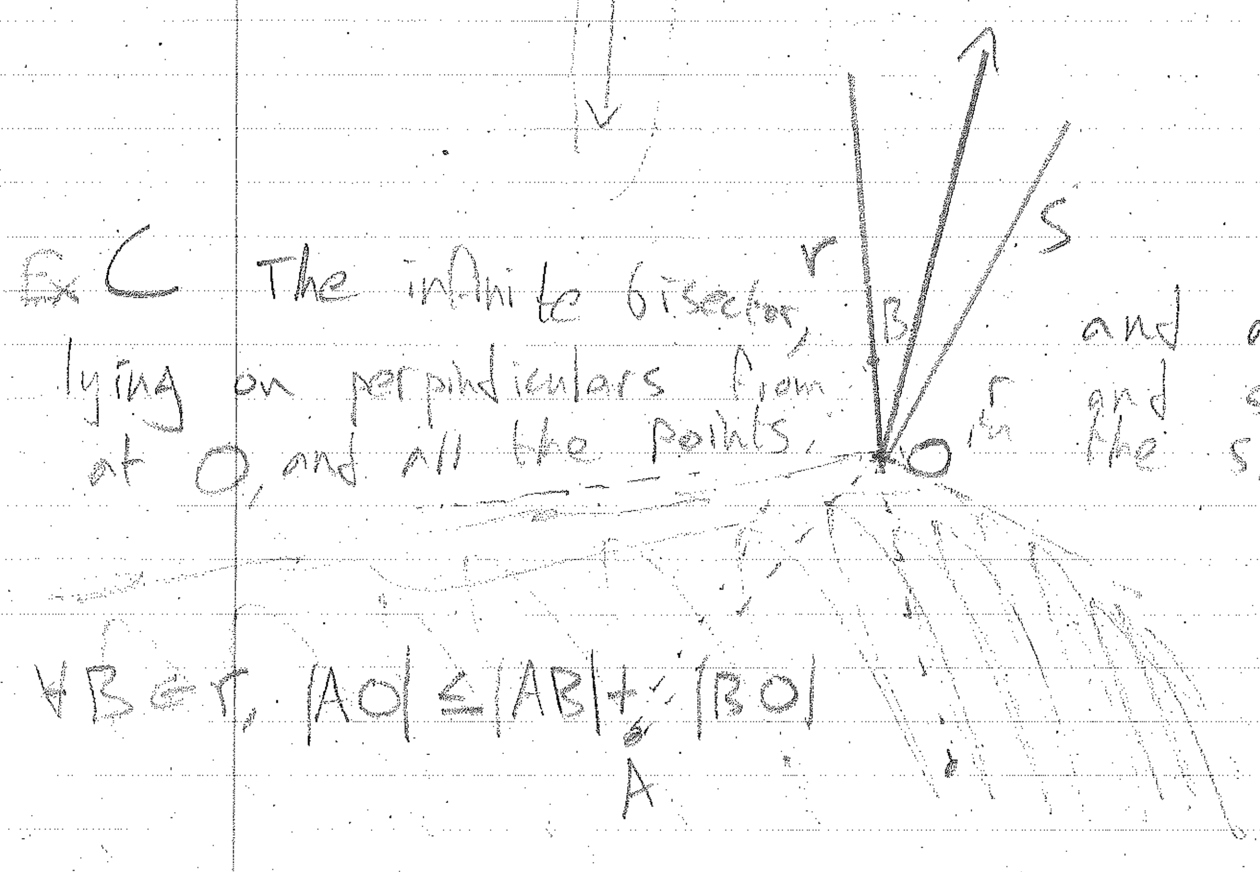
I say $d(\ell, P) = |CP|$, where C is the point of intersection between ℓ and the perpindicular constructed in 4. If there is some point B distinct from C, then pythagoras tells us $|CP|^2 + |CB|^2 = |BP|^2$. Since $|CB|$ is positive, $BP \geq CP$, with equality only in the degenerate case in which $|CB| = 0$.

Ex B



The locus of all points equidistant from l and m ('circled')

Ex C The infinite bisector, and all the points lying on perpendiculars from r and s which inters at O, and all the points in the shaded region



$\forall B \in r, |AO| \le |AB| + |BO|$