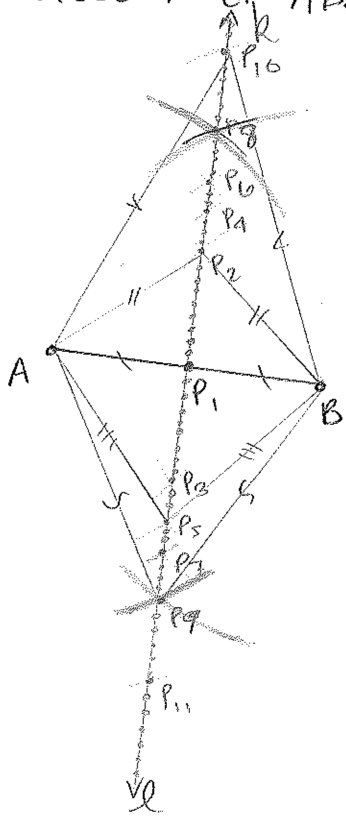
2 - Given points A & B, prove that the locus of points equidistant from A & B is equal to the perpendicular bisector of AB.



Proof: (informal)

Given AB, use a compass & straightedge to find the bisection of AB (as in #1), label it P_1, as well as the bisector, k. By construction, $AP_1 = BP_1$.

Setting the compass to a radius greater than AP_1 (or BP_1), mark arcs with centers at A & B, resepectively. Label the arcs' points of intersection P_2 & P_3.
By construction, $AP_2 = BP_2$ & $AP_3 = BP_3$.

Repeating this step m times with any & all radii greater than AP_1 (or BP_1) yields a collection of points P_4 to P_{2m-1} where each point is equidistant from A & B.
That is, $AP_{2m-1} = BP_{2m-1}$ by construction.
* Repeating this step with <u>any</u> radius smaller than AP_1 (or BP_1) will <u>not</u> result in any point because the arcs cannot intersect.

These points are situated so that only one line, ℓ, lies through all of them.
Since ℓ & k lie on top of one another, $\ell = k$.

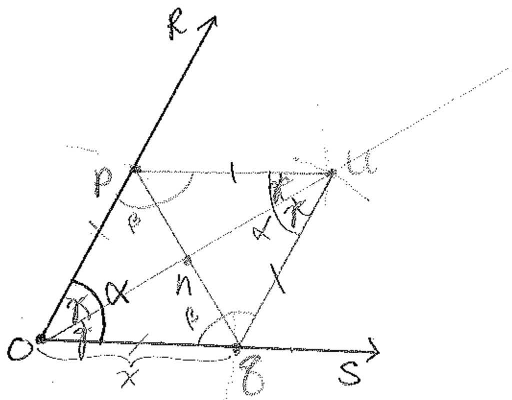
$\therefore$ The locus of all points equidistant from A & B, ℓ, is equal to the perpendicular bisector of A & B, k.

$\square$

* possible to use induction with geometry?

Marie Wittenkamp

3. Construct the angle bisector of a given angle α.



Steps:

1. Given an angle $\alpha = \angle_O(R,S)$, set the compass to an arbitrary radius x.
2. Mark an arc with radius x & vertex O through R & S.
3. Mark points where the arc intersects R & S p & q.
4. With same radius x, mark an arc with vertex P.
5. With same radius x, mark an arc with vertex q.
6. Label point where arcs intersect u.
7. Connect O & u.

Claim: Ou bisects α.

Proof:

Letting $\alpha = \angle_O(R,S)$, draw Pu, qu, & Pq. Label point where Ou & Pq intersect n.

By construction (with radius x), $OP = Oq = Pu = qu$.

Note Pq & Ou are diagonals of the rhombus OPuq.

Also note $Pq = Pq$.

Since $OP = uP$ & $Oq = uq$, ΔOPu & ΔOqu are isosceles.

So $\angle POu = \angle PuO$ & $\angle qOu = \angle quO$.

By SSS, ΔOPu & ΔOqu are congruent.

By CPCTC, $\angle POu = \angle PuO = \angle qOu = \angle quO = \gamma$ & $\angle OPu = \angle Oqu = \beta$.

Since $\angle POu + \angle qOu = \alpha$, $\angle PuO + \angle quO = \alpha$.

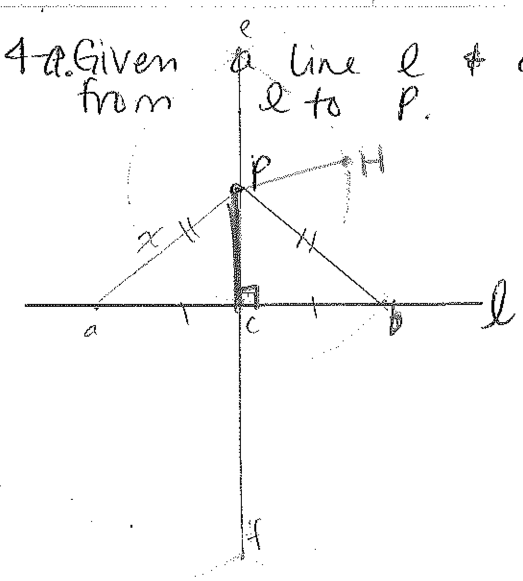
But more importantly, $\gamma + \gamma = \alpha$.

So $2\gamma = \alpha$ & $\gamma = 1/2\,\alpha$

$\therefore$ Ou bisects α.

$\square$.

4 a. Given a line ℓ & a point P, define the distance from ℓ to P.



[• The distance from ℓ to P is the radius of a circle with center P & tangent line ℓ.] alt.

• The distance from ℓ to P is the line segment $\perp$ to ℓ which goes through P.

b. - construct a line segment with one endpoint P whose length is $d(\ell, P)$. (see above diagram)

Steps:

1. Given a point P & line ℓ, set the compass to a radius, x, obviously larger than the shortest distance between P & ℓ.
2. With P as the vertex, mark an arc with this radius x through ℓ.
3. Label the points where the arc intersects ℓ a & b.
4. Construct the perpendicular bisector, ef, of segment ab.
5. Label the point of intersection between bisector ef & ℓ C.
6. Set the compass to Pc.
7. Draw a circle with center P & radius Pc.
8. Connect P to any point on the circle & label the point H.

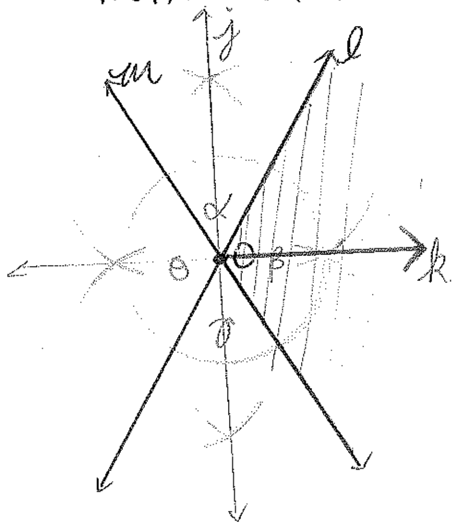
Claim: $PH = Pc = d(\ell, P)$

Proof:
Let ℓ be a line, P a point not on ℓ, & ef $\perp \ell$ through P.
By construction $Pa = Pb = x$, & $ac = bc = \frac{1}{2} ab$, & $\angle Pca = \angle Pcb = 90°$;
So $\triangle Pac$ & $\triangle Pab$ are right $\triangle$s (& congruent by SSS)
So $Pa (= Pb = x)$ will always be the hypotenuse of the $\triangle$ because the hypotenuse is always the side opposite the right angle.
So $Pa (= Pb = x)$ will always be larger than Pc.

$\therefore$ $Pc = d(\ell, P) :=$ the shortest distance between P & ℓ.

$\square$

4-c. Given 2 lines l & m intersecting at point O, sketch the locus of points P equidistant from l & m.



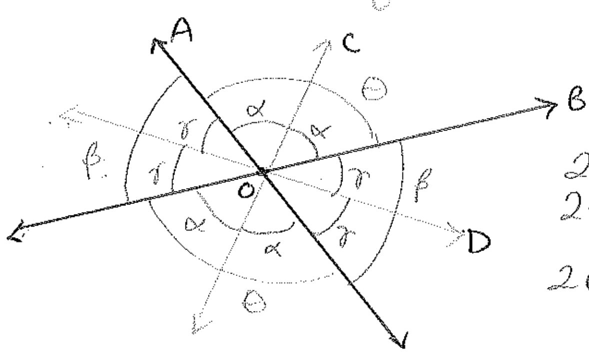
Steps:
1. Label angles $\alpha, \beta, \gamma, \theta$ as shown.
2. Bisect α as in #3.
3. Repeat with β.
4. Repeat with γ.
5. Repeat with θ.
6. Label angle bisectors j & k.

Claim: The angle bisectors of the l-m intersection are the locus of points equidistant from l & m.

Proof:
The proof for #2 can easily be adapted to the straight angle AB & then to any angle.

d. The angle & angle bisector of angle β above.

e. What is the locus of points equidistant from the 2 rays of an angle?
The angle bisector. See "Proof" above.
Sketch the ~~this~~ equidistant locus.



$2\alpha = \theta$
$2\gamma = \beta$
$2\theta + 2\beta = 360°$