Homework #1

1- 10/10.

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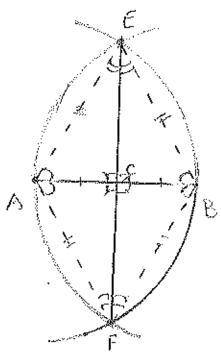
Math 128A
Montgomery

1. Construct the perpendicular bisector of a given line segment AB.



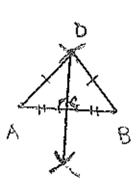
Construction
1. Set compass at radius AB
2. Draw a circle at center A of radius AB
3. Draw a circle at center B of radius AB
4. Mark the points of intersection E and F ✓
5. Draw line EF and mark the intersection of EF and AB C.
6. Claim: EF bisects AB at C.

Pf: 1. Lines AE, AF, BE, and BF are congruent since they were drawn with the same compass length AB.
2. AB = AB by Reflexive Property
3. △ABE ≅ △ABF by SSS
4. ∡EAC, ∡EBC, ∡FAC, and ∡FBC are congruent by CPCTC. ✓
5. EF = EF by Reflexive Property ✓
6. △AEF ≅ △BEF by SSS ✓
7. ∡AEC, ∡BEC, ∡AFC, and ∡BFC are congruent by CPCTC. ✓
8. △ACE, △BCE, △ACF, and △BCF are congruent by ASA. ✓
9. ∡ACE, ∡BCE, ∡ACF, and ∡BCF are congruent by CPCTC. ✓
10. The four angles add up to 360° and are equal ⇒ each angle is 90° ⇒ EF ⊥ AB
11. AC = CB by CPCTC
12. Therefore, EF bisects AB at C

QED

ok
good.

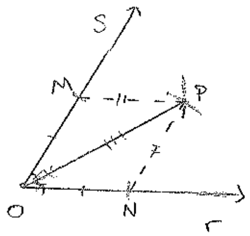
2. Given points A, B prove that the set of points equidistant from A and B is equal to the perpendicular bisector of line segment AB.



Pf: 1. Given a line segment AB, let point D be equidistant from the endpoints A and B ⇒ AD = BD.
2. Let C be the midpoint of AB using problem #1 ⇒ AC = BC
3. CD = CD by Reflexive Property ⇒ △ACD ≅ △BCD by SSS
4. ∡ACD = ∡BCD by CPCTC
5. Since ∡ACD + ∡BCD = 180° ⇒ ∡ACD = ∡BCD = 90° ⇒ CD ⊥ AB
6. Thus, D lies on the perpendicular bisector of AB

QED

3. Construct the angle bisector of a given angle $\alpha = \angle_O(r, s)$.



Construction

1. Set the compass to any width with the point at vertex O.
2. Draw an arc on each leg S and r. Mark points M, N
3. Place the compass point on point M and make an arc in the angle's interior, any width
4. Repeat on point N without changing compass width.
5. Mark where the arcs cross point P
6. Draw line OP.
7. Claim : OP bisects angle α

Pf: 1. $OM = ON$ since they were drawn with the same compass width.

2. $MP = NP$ since they were drawn with the same compass width

3. $OP = OP$ by Reflexive Property $\Rightarrow \triangle OMP = \triangle ONP$ by SSS

4. $\measuredangle MOP = \measuredangle NOP$ by CPCTC

5. Therefore, OP bisects angle α. QED

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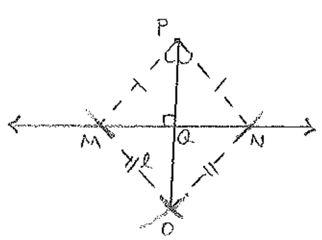
4. Given a line ℓ and a point $P \notin \ell$ construct the line segment through P and perpendicular to ℓ. In other words, show how to drop a perpendicular from P to ℓ. The length of this line segment is this distance $d(\ell, P)$ from P to ℓ.



Construction

1. Place the compass point on point P and make the width beyond line ℓ
2. Draw two arcs on line ℓ and label points M and N.
3. From point M, draw an arc below line ℓ. Repeat for point N so the arcs intersect. Mark point O.
4. Draw line OP. Mark intersection Q.
5. Claim: PQ is perpendicular to line ℓ.

Pf: 1. $MP = NP$ and $MO = NO$ since they were drawn with the same compass width.

2. $OP = OP$ by Reflexive Property $\Rightarrow \triangle PMO = \triangle PNO$ by SSS

3. $\measuredangle QPM = \measuredangle QPN$ by CPCTC

4. $PQ = PQ$ by Reflexive $\Rightarrow \triangle QPM = \triangle QPN$ by SAS

5. $\measuredangle PQM = \measuredangle PQN$ by CPCTC

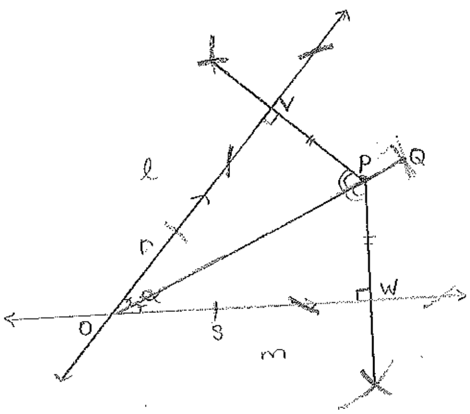
6. Since $\measuredangle PQM + \measuredangle PQN = 180° \Rightarrow \measuredangle PQM = \measuredangle PQN = 90°$

7. Therefore, QP is perpendicular to line ℓ QED

Extra Credit A. Why is this the distance? Define the distance $d(\ell, P)$ from ℓ to P. Prove that this distance is indeed the length of the segment constructed

Pf. By definition, the distance from a point to a line is the shortest distance between them, which is the length of a perpendicular line segment from the point to the line. Since PQ is perpendicular to line ℓ, it is the distance $d(\ell, P)$.

5. Extend the rays r and s of problem 3 to lines ℓ and m. Thus $r \subset \ell$, $s \subset m$, and $\ell \cap m = O$. Prove that each point P of the angle bisector you constructed in problem 3 is equidistant from ℓ and m: $d(\ell, P) = d(m, P)$



Pf: 1. Let P be a point on the angle bisector of $\measuredangle \alpha$.

2. Let VP be a line segment perpendicular to line ℓ and WP a line segment perpendicular to line m.

3. Then $\triangle POV$ and $\triangle POW$ are right triangles.

4. $\measuredangle POV = \measuredangle POW$ since PO bisects $\measuredangle \alpha$.

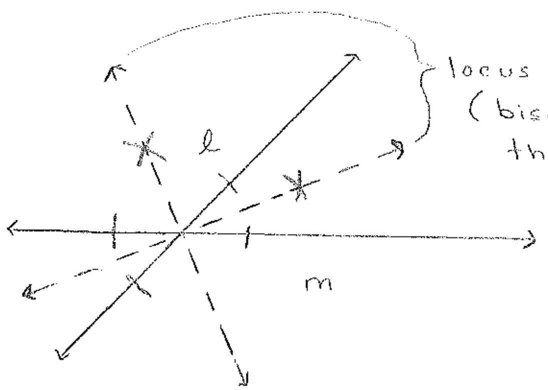
5. Thus, $\measuredangle OPV = \measuredangle OPW$ since they are non-adjacent complementary angles to $\measuredangle POV$ and $\measuredangle POW$ in $\triangle POV$ and $\triangle POW$.

6. $OP = OP$ by Reflexive Prop. $\Rightarrow \triangle POV = \triangle POW$ by ASA

7. $PV = PW$ by CPCTC

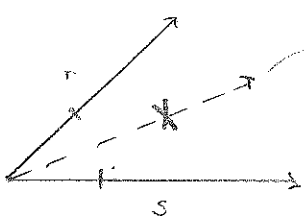
8. Therefore, $d(\ell, P) = d(m, P)$ QED

Extra Credit B. Draw the locus of all points equidistant from ℓ and m



locus of all points equidistant from ℓ and m (bisectors that bisect the angles formed by the intersection of ℓ and m)

Extra Credit C. Draw the locus of all points equidistant from rays r and s.



locus of all points equidistant from rays r and s (angle bisector of the vertex of r and s)