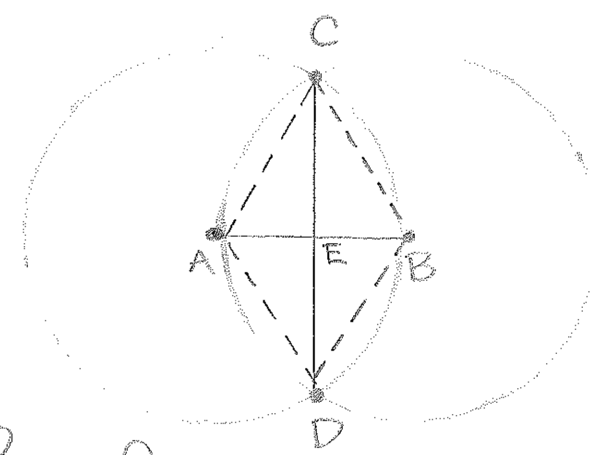
Anne Maher,

(2.) Given points A,B prove the locus of points equidistant from A and B is equal to the perpendicular bisector of AB.



Proof:

Given points A and B choose any point C equidistant from A and B. Set a compass to distance AC = BC and with this set distance draw a circle centered at point A and a circle centered at B. C is one point where the circles intersect. Label the other point of intersection as D. By construction AC = AD and BC = BD, therefore point D is also equidistant from point A and point B. By drawing line segments AB, CD, AC, AD, BC, and BD we can now apply the proof from exercise #1 and see CD is the perpendicular bisector of AB. Therefore any point equidistant from two points A,B lies on the perpendicular bisector of line segment AB. Since C represented any point equidistant from AB, we can

Observe that the set of all points equidistant from A,B is equal to the perpendicular bisector.

QED

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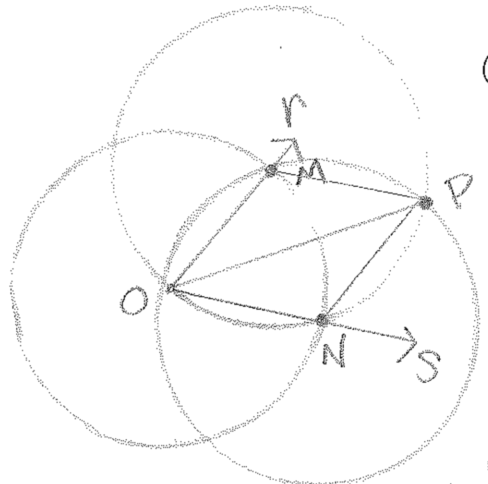
③ Construct the angle bisector of a given angle $\alpha = \angle_O(r,s)$.



<u>Construction:</u>

Set the compass to any width such that when centered at O a circle would intersect $\vec{r}$ and $\vec{s}$. With this set width draw a circle centered at O. Mark the points of intersection as M and N. With out changing the compass draw a circle centered at M and a circle centered at N. Mark point of intersection as P. Draw line OP. Draw line segments MP and NP

<u>Claim:</u>

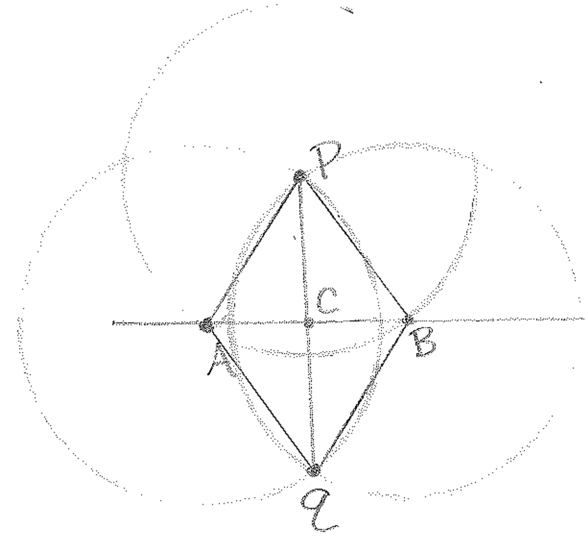
The line segment OP is the angle bisector of given angle $\alpha = \angle_O(r,s)$.

<u>Proof:</u>

By construction line segment OM = ON. Also by construction MP = NP. Since OP is shared, by side, side, side $\triangle OMP$ is congruent to $\triangle ONP$. Therefore, $\measuredangle MOP = \measuredangle PON$. Hence the line segment OP is the angle bisector of given angle α.

Q.E.D.

(4.) Given a line ℓ and point $P \notin \ell$ construct the line segment through P and perpindicular to ℓ.

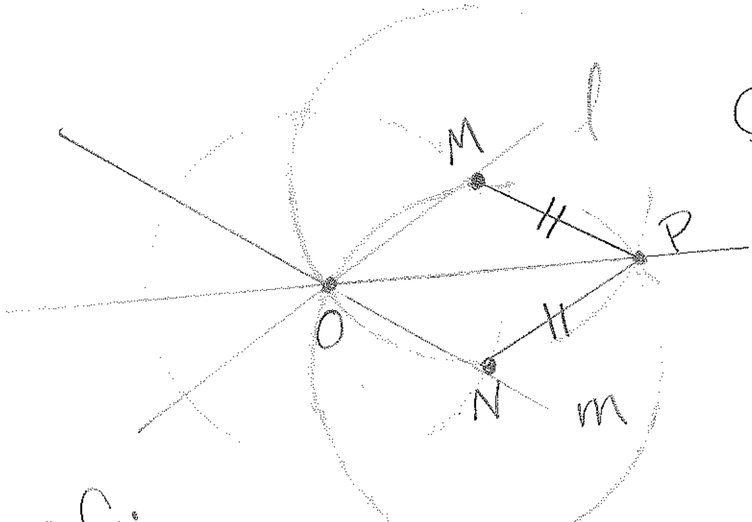


Construct: Given a line ℓ and a point P set a compass to some width such that a circle centered at P would intersect line ℓ at two places. Draw this circle centered at P. Mark the points of intersection as A and B. Now without changing the width of the compass draw a circle centered at A and a circle centered at B. Mark where these two circles intersect and label as Q. Draw line segments PQ, AP, PB, BQ, AQ. Label the intersection of PQ and AB as C.

Claim: Given a line ℓ and a point $P \notin \ell$, PQ is the perpindicular line segment through P.

Proof: By construction $AP = AQ = AB$ and $BP = AB = BQ$, thus $AP = AQ = AB = BP = BQ$. Hence $\triangle APQ$ is congruent to $\triangle BPQ$ by side, side, side. Also by properties of isosceles triangles $\measuredangle APC = \measuredangle AQC = \measuredangle BQC = \measuredangle BPC$. Now $\triangle APC$ is congruent to $\triangle BPC$ by side, angle, side. Thus $\measuredangle PCA = \measuredangle PCB$. Therefore PC is perp. to AB. It follows PQ is perp. to AC. QED.

(5.) Extend the rays $\vec{r}$ and $\vec{s}$ of Problem 3 to lines l and m. Thus $r \subset l$, $s \subset m$ and $l \cap m = O$. Prove that each point P of the angle bisector is equidistant from l and m: $d(l,P) = d(m,P)$.



Construction:
Same as problem 3 with rays extended into lines.

Proof:
By construction $ON = OM = MP = NP$. Thus we see that at point P, $d(l,P) = d(m,P)$. Since we set the width of the compass arbitrarily, we can reajust it and repeat this process infinitly many times until we have the line that bisects the angle created by l and m. By construction of an angle bisector each point on the angle bisector will be equidistant from l and m so $d(l,P) = d(M,P)$.

QED.