<u>Claim</u>: $\square CJKI$ has the same area as $\triangle ABC$.

<u>Pf</u>: By construction, $\triangle ABC$ has area $= \frac{1}{2}(\overline{AC})(\overline{BD})$ since $\overline{BD}$ is the height of $\triangle ABC$ and $\overline{AC}$ is the base.

Also, by construction, $\overline{BD} = \overline{EO} = \overline{FC}$ and $\overline{OC} = \overline{EF}$.

Since $\overline{OC} = \overline{EF} = \frac{1}{2}\overline{AC}$ by construction, we can conclude that

$$\text{Area } \square EOCF = \text{Area } \triangle ABC = \frac{1}{2}(\overline{AC})(\overline{BD}).$$

Now, observe that $\overline{FC} = \overline{CG} = \overline{EO}$ by construction.

Let $\overline{OC} = a$ and $\overline{FC} = \overline{CG} = \overline{EO} = b$ so that the area $\square EOCF = ab$. Next, notice that $\overline{IC}$ is the geometric mean of $\triangle OCI$ and $\triangle ICG$ by construction.

Therefore since $\overline{OC} = a$ and $\overline{CG} = b$, we know that $\overline{IC} = \sqrt{ab}$.

By construction, $\overline{CI} = \overline{CJ} = \overline{JK} = \overline{IK} = \sqrt{ab}$.

Hence, the area $\square CJKI = (\sqrt{ab})^2 = ab$.

Translating the area of $\triangle ABC$ into terms of a and b, we get

$$\text{area } \triangle ABC = \frac{1}{2}(\overline{AC})(\overline{BD}) = ab.$$

$\therefore$ Area $\triangle ABC$ = Area $\square EOCF$ = Area $\square CJKI = ab$. ∎

17) A triangle's sides are 3 and 4 inches. What can its area be?

Great !!!

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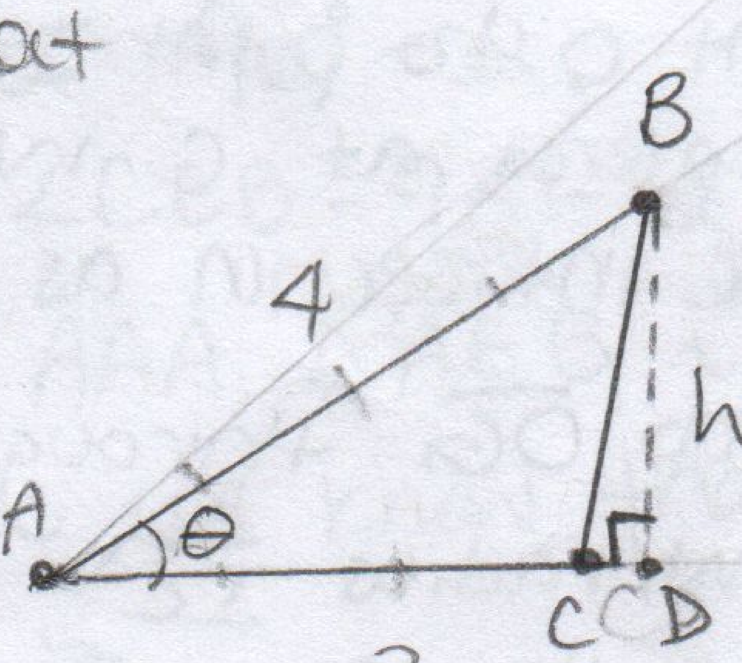
Let $\overline{AC} = 3$, $\overline{AB} = 4$, and $\overline{BD}$ = height $= h$.

Through the properties of trig, we see that

$\overline{BD}$ $h = 4\sin\theta$. ✓



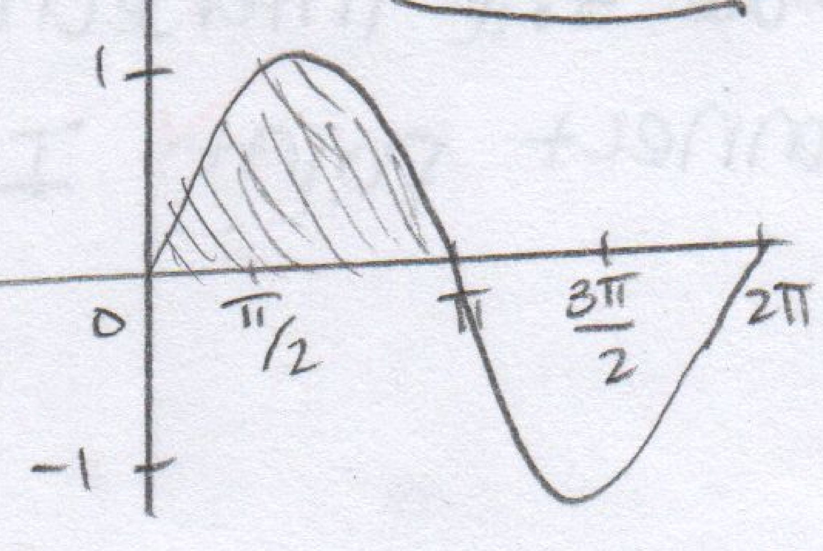
$\Rightarrow$ area $\triangle ABC = \frac{1}{2}bh$ $\frac{1}{2}(3)(4\sin\theta)$

$= \frac{1}{2}(3)(4\sin\theta)$

$= 6\sin\theta$. ✓

For a positive area, we restrict $\theta \in (0, \pi)$.

<u>Sine curve</u>



when $\theta = 0$: area $\triangle ABC = 6\sin 0° = 0$ ✓

when $\theta = \pi/2$: area $\triangle ABC = 6\sin\frac{\pi}{2} = 6(1) = 6$

$\therefore$ Area $\triangle ABC \in (0, 6]$. ∎ ✓