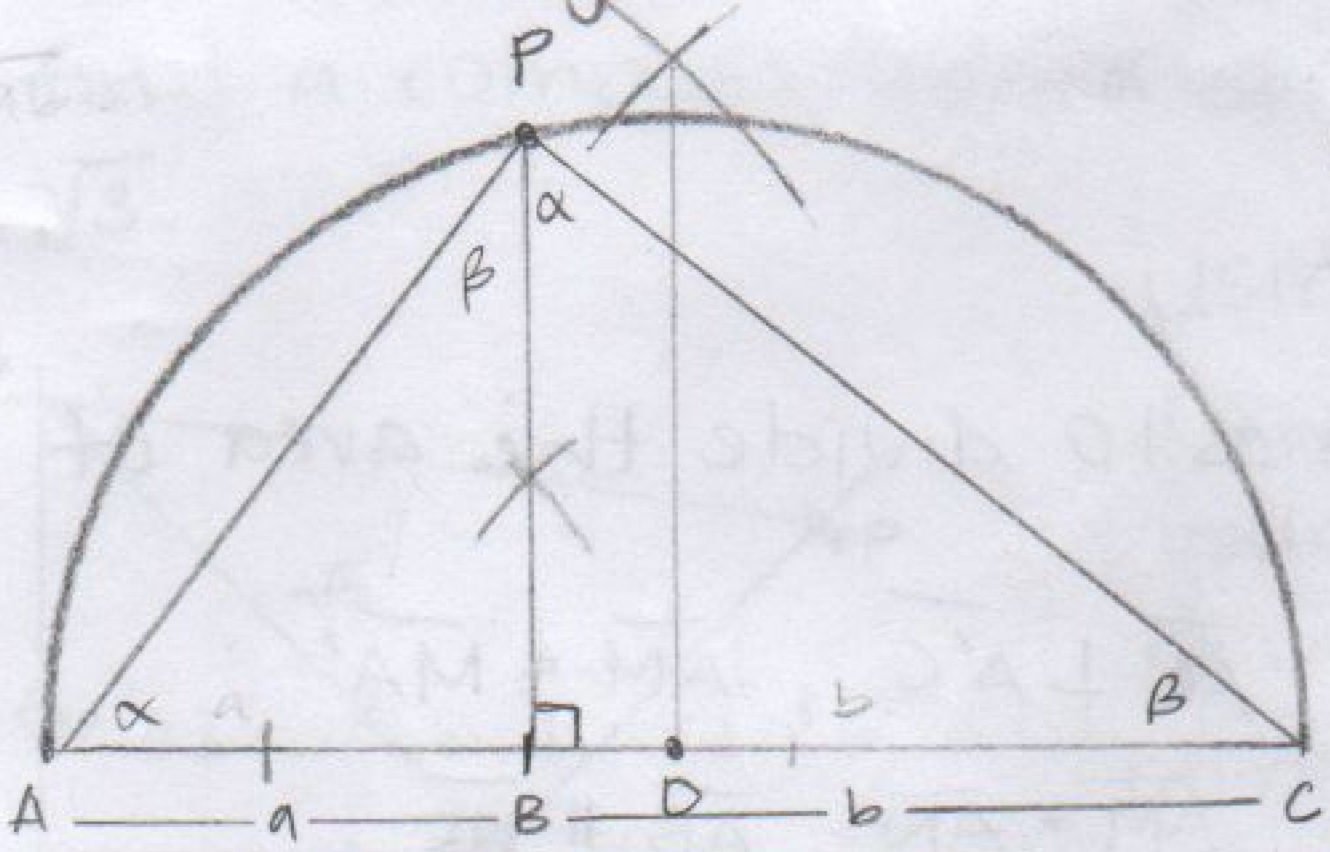
8. Given two segments a and b. Construct the segment $\sqrt{ab}$



Start by placing the two line segments end to end in a straight line. Let the end points of the line segment of length a be A and B, let the end points of the other line segment be B, C.

Using methods from HW1 find the perpendicular bisector of $\overline{AC}$. Let that point be D. Draw a semicircle centered at D with radius $\overline{AD}$, with endpoints at A and C. Using methods from HW1, find the perpendicular to $\overline{AC}$ at B. Let the intersection of that line and the semicircle be point P. Connect A and P, B and P with line segments

Claim: $\overline{PB}$ has length $\sqrt{ab}$.

Consider $\triangle PBC$. Let $\measuredangle BPC$ be α, and let $\measuredangle PCB$ be β. Since the sum of the angles of a triangle are 180° and $\measuredangle PBC$ is 90° by construction this implies $\alpha+\beta=90°$.

Now consider $\triangle ABP$. By Thales theorem, we know that $\measuredangle APC$ is 90°, therefore $\measuredangle APB = 90-\alpha$, but above we have that $\beta = 90-\alpha$ therefore $\measuredangle APB = \beta$. Finally $\measuredangle PAB = 90-\beta = \alpha$. By AAA, $\triangle PBC \sim \triangle ABP$. Since these two triangles are similar we know the ratio of two similar sides are equal. Therefore

$$\left(\frac{\overline{AB}}{\overline{PB}}=\right)\frac{a}{\overline{PB}} = \frac{\overline{PB}}{b}\left(=\frac{\overline{PB}}{\overline{BC}}\right)$$

Now cross multiply

$$ab = \overline{PB}^2$$

Which implies $\overline{PB} = \sqrt{ab}$.

Great!!

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