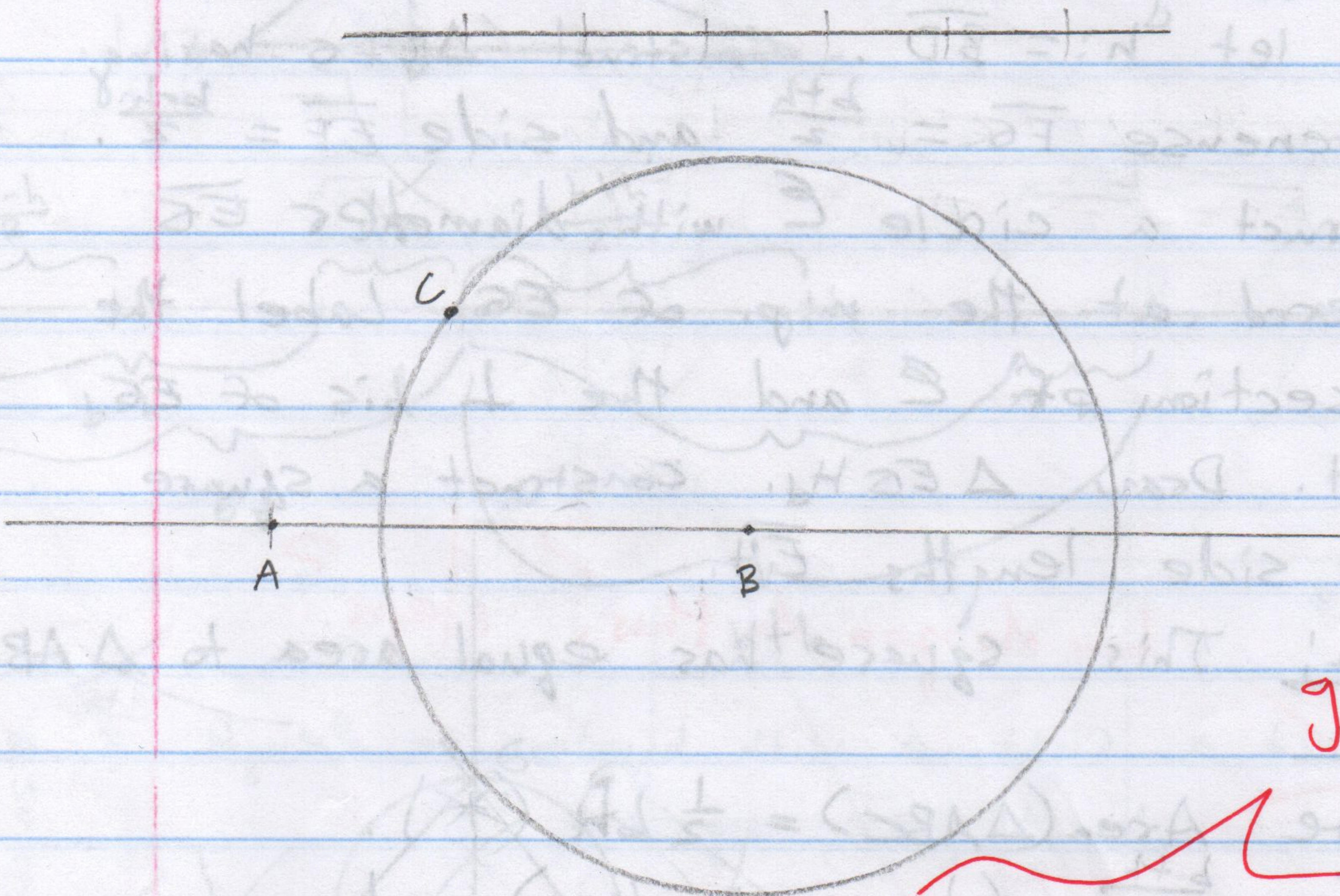Lemma: The $\Delta$ of maximum area with side lengths 3 and 4 units is the 3-4-5 $\Delta$.

pf:



good!

Consider the above diagram. $\overline{AB} = 4$, and the distance from B to C is the radius of the circle is 3 by construction. As C moves around the circle, $\overline{BC}$ remains equal to 3 and $\Delta ABC$ remains a 3-4-X $\Delta$. C moving along the upper half of the circle exhausts all possible 3-4-X $\Delta$s. Letting the base of all these $\Delta$s equal AB, the Area of these $\Delta$s depends only on the height of these $\Delta$s which takes on its maximum value at $\angle ABC = 90°$. Thus the 3-4-X $\Delta$ of maximum area is indeed a 3-4-5 $\Delta$. $\square$