1. You are the life guard on a beach whose shore line is straight and runs directly North-South You spot someone drowning. You are a meters from the shoreline. They are out a meters away from the shoreline but also $2a$ meters north of you.

Draw a geometric picture of the situation.

You run exactly twice as fast as you swim. Consider two possible routes to swimmer:

Route 1: draw a straight line from your location to the swimmer. Follow that.

Route 2: aim to spot on the shoreline as close to the swimmer as possible: so: on the shore, but $2a$ meters north, and then swimming directly west from there.

a)Verify that Route 2 is *quicker* than Route 1.

b) What is the ratio: (time for route 2)/time for route 1?

2. Repeat problem 1a and 1b, except now you run 10 times as fast as you swim. Use the same two routes.

3. Light travels through water slower than it travels through air. The ratio of these speeds: (speed in air)/(speed in water) is called the "index of refraction" and written n (and later as n_{water}/n_{air} so you may think of n_{air} as having been normalized to 1.)

a) Look up Snell's law of refraction for the angle which light rays bend upon hitting water.

b) Look up *and state* "Fermat's principle" for predicting the shape of light rays moving through variable media.

c) Using Fermat's principle, and 1st quarter calculus, derive Snells law in its standard form

$$\sin(\theta_1)/sin(\theta_2) = n_2/n_1 = v_1/v_2.$$

The situation is of light travelling from medium 1 (say 'air') to medium 2 (say 'water'). The interface separating the media is a line. The speeds within each medium are assumed constant, v_1 in medium 1 and v_2 in medium 2. The light rays are assumed to travel on straight lines *within* each medium. The direction of the line bends at the interface and that bending can be described by the two angles θ_1, θ_2 which are defined so that the angle θ_i is angle that the light ray in medium i makes with the NORMAL to the interface line. (That normal and the vector of the light ray are aligned as per standard pictures, eg: so both point INTO the medium.)

Hint: Set up your coordinates so that the interface is the y-axis, with medium 1 to be the left-half plane $x < 0$ and medium 2 to be the right half plane $x > 0$. Imagine a light ray leaving from a (fixed) point $(x, y) = (-a_1, 0)$ in medium 1, hitting the interface at some (variable) point $(0, y)$ and then continuing on to the (fixed) point (a_2, b_2) in medium 2. Compute the time T of flight of the light in terms of the lengths of the two segments and the two given speeds. Done properly you get an expression in y alone (with a_1, a_2, b_2 occuring as parameters). How do you minimize a function of y?

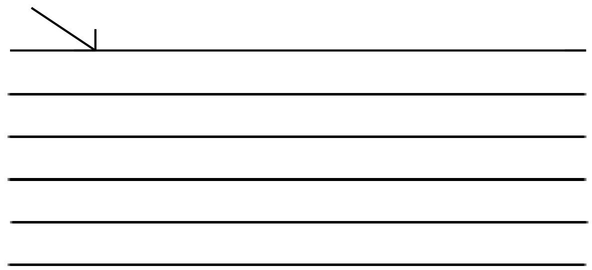

FIGURE 1. PROBLEM 4 A. Assume the index of refraction increases as we descend from each layer to the next, so the speed is slower. How will the incoming path from the top deflect?.

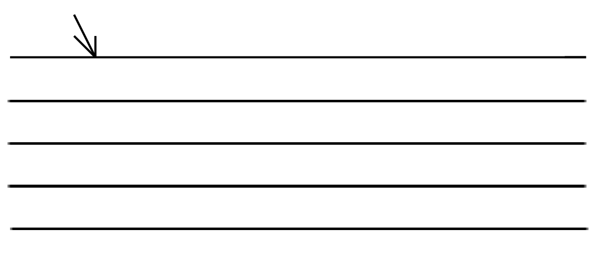

FIGURE 2. PROBLEM 4b): Assume the index of refraction *decreases* as we descend from each layer to the next, so the speed is faster as we drop from one level to the next. How will the incoming path from the top deflect?.

6. Now: Imagine that the index of refraction varies continuously and is of the form $n(y)$. Thus $dt = n(y)ds$ or $ds/dt = 1/n(y)$ describes how the light speed varies with y. By considering a continuum limit of the previous examples, how do ligt rays bend if $n(y)$ is a decreasing function of y? An increasing function? Sketch pictures.

(On the margins of Feynman, vol 1. 28-5, 28-7 are 5 drawings. Find the one explaining mirages. Compare with problem 6.

7. Go back to Feynman v.1, 28-5 and 26-5. You will find 7 pictures in the margins explaining various optical phenomenon. Take any one of them. Explain it in your own words, in detail, and be ready to explain it to the class.

8. Elementary calculus of variations.

We write $n(y)ds = n(y)\sqrt{dx^2 + dy^2}$ to represent the "$dt = nds$ of the previous problem. Switch notation: $n = f$. By the Cauchy-Schwartz argument minimizing the integral of dt over all paths joining A to B is equivalent to minimizing the integral of

$$L = (1/2)f(y)^2(\dot{x}^2 + \dot{y}^2)$$

over all such paths.

Look up the Euler-Lagrange [EL] equations associated to a Lagrangian $L = L(x, y, \dot{x}, \dot{y})$.

Read about the [EL] equations, the principle of least action, and the calculus of variations.

Find the EL equations for our L!

Show that the "energy" $(1/2)f(y)^2(\dot{x}^2 + \dot{y}^2)$ is constant along any solutions to the Euler-Lagrange equations. Without loss of generality, set the value of energy equal to $(1/2)$.

Under the above assumption on energy, show that there is an angle such that $(f\dot{x}, f\dot{y}) = (\cos(\theta), \sin(\theta))$. Interpret θ in terms of the angle a path makes with the x or y axis.

Show that $f^2\dot{x}$ is constant along any solution to the Euler-Lagrange equations.

(The EL equations associated to x are special. The variable x is a "cyclic variables" for our Lagrangian.)

Show that $f\cos(\theta) = const.$ along solutions to the EL equations.

Use this constancy to verify the continuous version of Snell's version you derived from pictures in 6.