HW 4. DUE Oct 14. In class! To be presented then.

My problem 4.1 [Ct'd fractions] Show, two ways, that $\sqrt{2}-1 = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\dots}}}$

Way 1: Verify that $0 < \sqrt{2}-1 < 1$ and that $\frac{1}{\sqrt{2}-1} = 2+r$ with $r = \sqrt{2}-1$. Now describe what happens when you apply the Euclidean algorithm to the ratio $\sqrt{2}-1; 1$.

Way 2. Set $x = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\dots}}}$
and show that $x = \frac{1}{2+x}$ thus deriving a quadratic equation for x.

EC1: Find a 'picture proof' of this expansion a la the Reader's picture proof, copied from Hsiang, that the golden mean is irrational.

EC2

Follow way 2 so as to find the value of the continued fraction

$$x = \cfrac{1}{n+\cfrac{1}{n+\cfrac{1}{n+\dots}}} \tag{1}$$

where $n > 0$ is a positive integer.

(*) My problem 4.2. To compute the earth's radius from the mast of a sailing ship.

You are on a strange planet on an islad with your crew and your sailing ship. Its mast is 50 meters above sea level. You have some of the crew sail the ship directly towards the setting sun. Its mast completely disappears over the horizon when it is 2 kilometers away. What is the radius of the planet?

Assume the planet is spherical.

Let h be the mast's height , d the distance sailed and R the planet's radius.

Draw a picture of the situation, marking R, h, and d on your diagram.

Let h be the mast's height , d the distance sailed and R the planet's radius.

You will want to use small angle approximations valid since $h/d << 1$. In other words: if trig functions arise, just keep the first non-vanishing term in their Taylor expansion.

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From Levi [back pages of our Reader]

18, (*) 19, 20, 21, 24, 25, 28, 32, (*) 33: Prove that any rectangle admits a circumscribing circle. EC: tell me what the heck Levi needs the 'symmetry axis not passing through its vertex" restriction for!