

HW 1. (Classical Geometries.) Due Monday, Oct 1, 2012, i.e.: Next class.

1. [10] *Construct* the perpendicular bisector of a given line segment AB .

Here: You need to use propositions from Euclid Book 1 to get a working proof. Namely you will need some or all of SAS (Euclid I.4), the propositions on isosceles triangles (1.5 and or 1.6) and SSS (Euclid I.8). And to do this correctly you best ask yourself how do I define ‘perpendicular?’, i.e. “right angle?” Remember: no protractor!

2. [5] Given points A, B *prove* that the locus (=set) of points equidistant from A and B is equal to the perpendicular bisector of line segment AB. (Use your work in Exer 1.)

3. [10] *Construct* the angle bisector of a given angle $\alpha = \angle_O(r, s)$. (Notation: r and s denote rays issuing forth from a common base point O which is vertex of the angle.)

4. [5] a. *Given* a line ℓ and point $P \notin \ell$ *construct* the line segment through P and perpendicular to ℓ . In other words, show how to “drop a perpendicular” from P to ℓ .

The length of this line segment is this distance $d(\ell, P)$ from P to ℓ .

EXTRA CREDIT A Why is this the distance? *Define* the distance $d(\ell, P)$ from ℓ to P . Prove that this distance is indeed the length of the segment you constructed.

5. Extend the rays r and s of problem 3 to lines ℓ and m . Thus $r \subset \ell, s \subset m$ and $\ell \cap m = O$. *Prove* that each point P of the angle bisector you constructed in problem 3 is equidistant from ℓ and m : $d(\ell, P) = d(m, P)$.

A moral: There is a tight analogy between the construction of a perpendicular bisector of a line segment and the bisector of an angle.

EXTRA B Draw the locus of all points equidistant from ℓ and m . Hint: it is larger than the line spanned by the angle bisector.

EXTRA C Draw the locus of all points equidistant from rays r and m . Hint: this set is strictly larger than the set you drew in EXTRA B.

Instructions and WARNING. In Euclidean geometry “construct” means to do three things:

A. Draw the object using ruler and straightedge (or nowadays, using Geogebra or Geometer’s Sketchpad or the like),

B. Describe and explain each step of the drawing – each line and point constructed – Label lines and points that will be needed for the proof.

C. Prove that what you say you drew is actually what you drew.

In mathematics generally, when we say “given” (as in “given a line and a point”) we mean that the objects (line, point) are to be imagined handed to you in a random or “general” position. However, for clarity of exposition it is often best to locate objects on your page in a way that makes drawings simple. For example, one side might be parallel to the page’s edge.