Chaos in the forced pendulum—an easy case of Lagrangian Coherent Structures

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Control & Dynamical Systems

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Invariant Manifolds: Standard View

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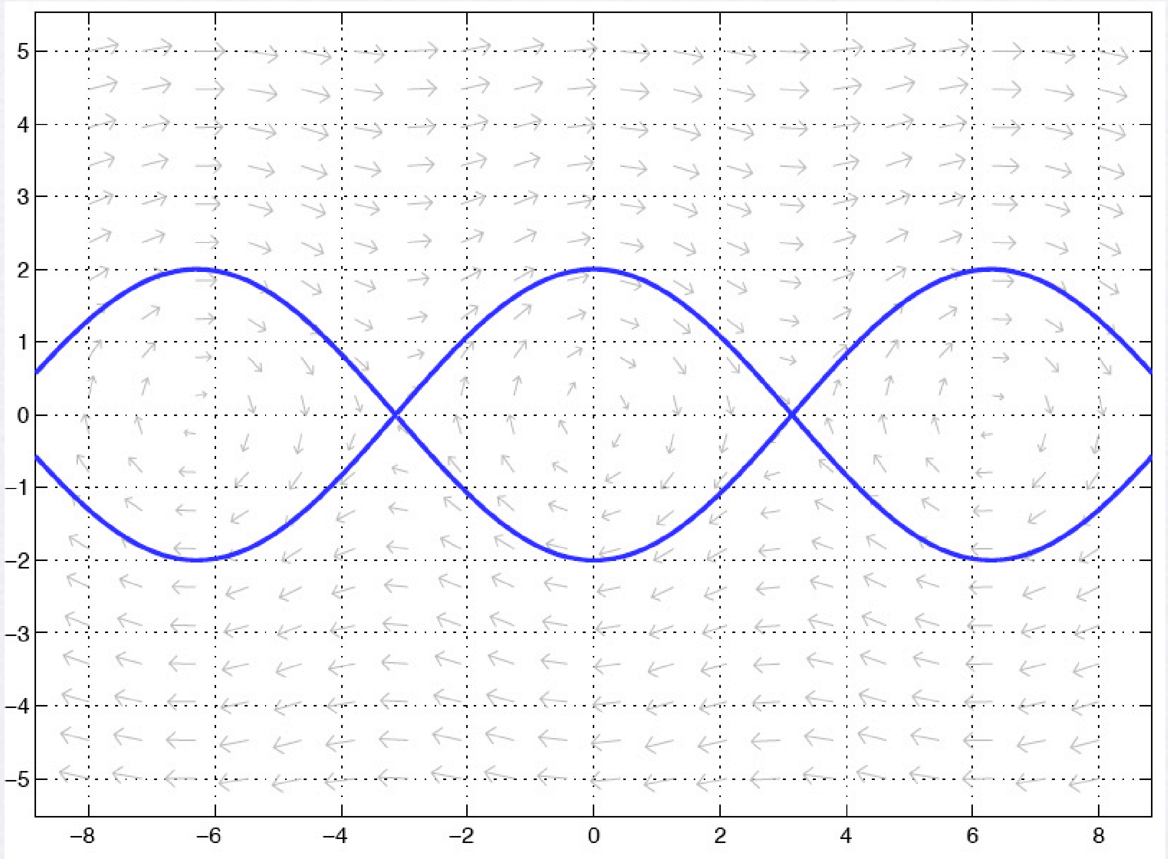

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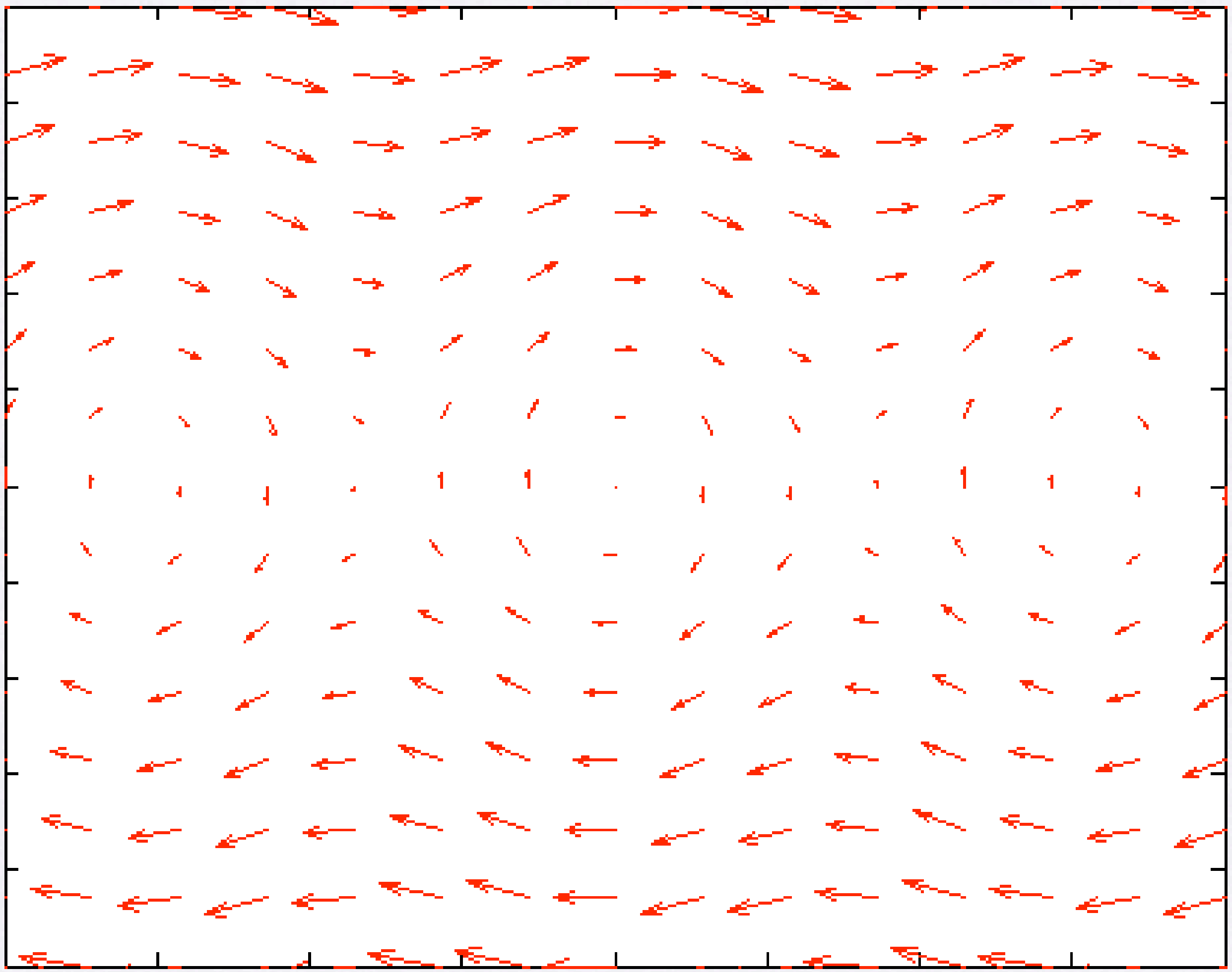

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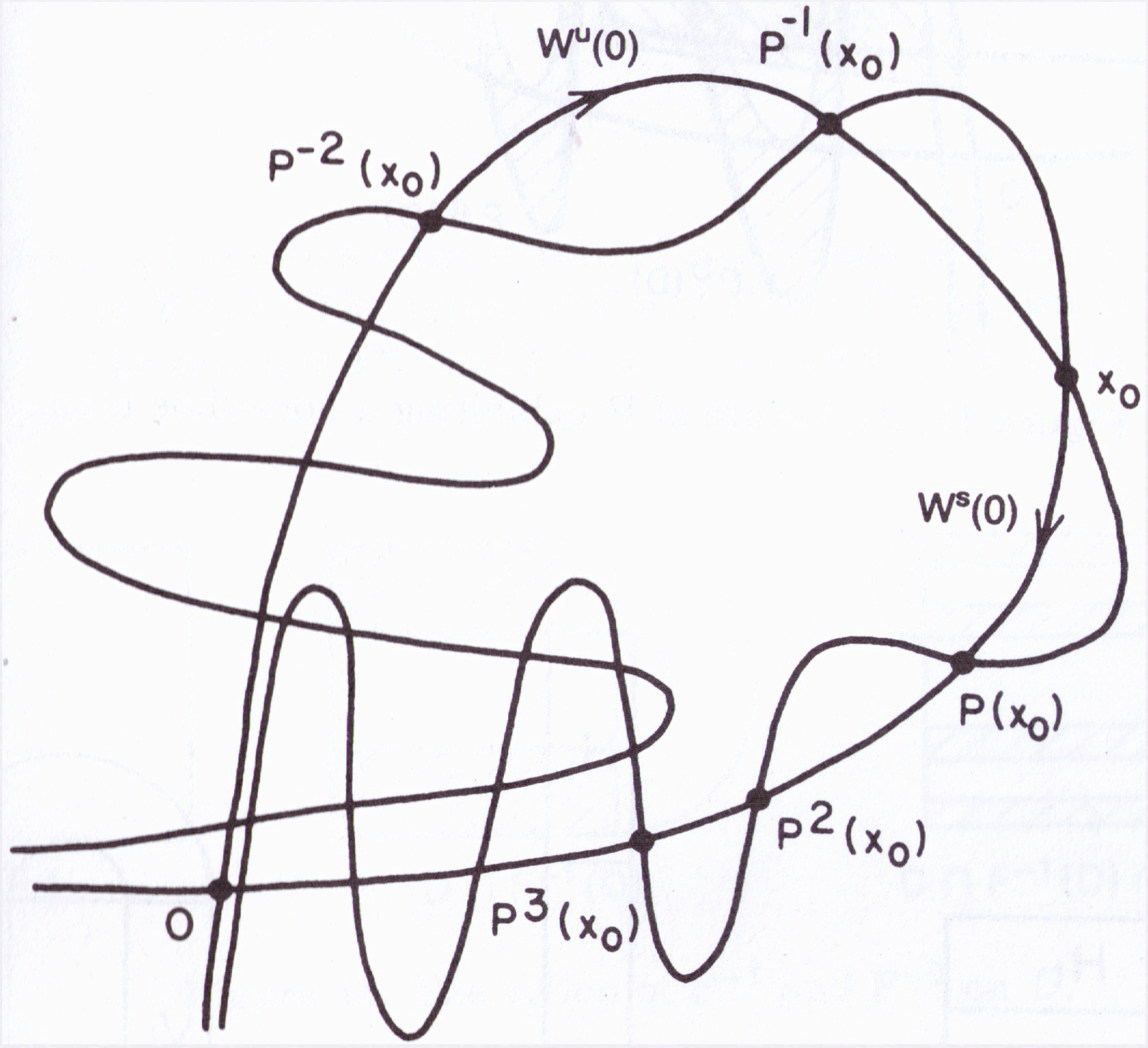

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- First, a bit more about the tangle

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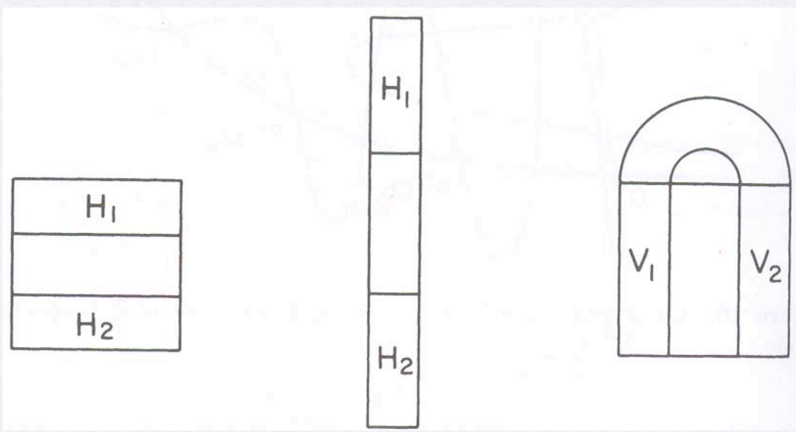

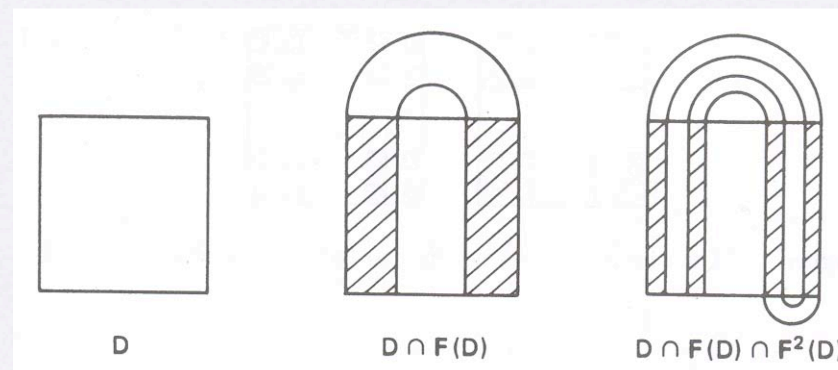

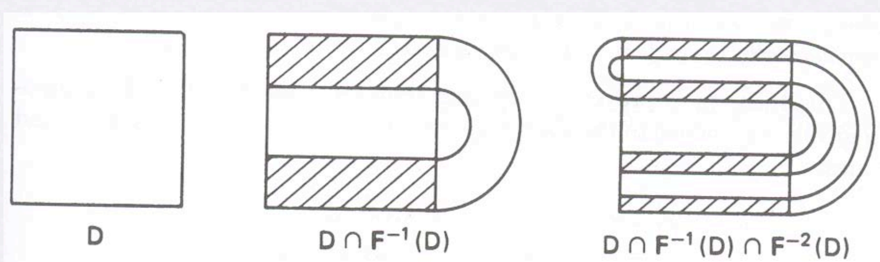

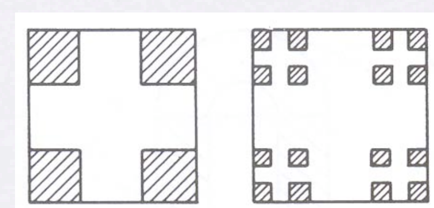

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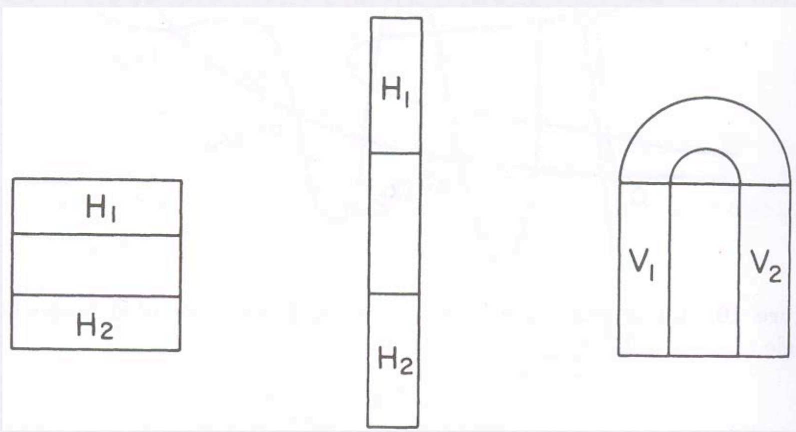

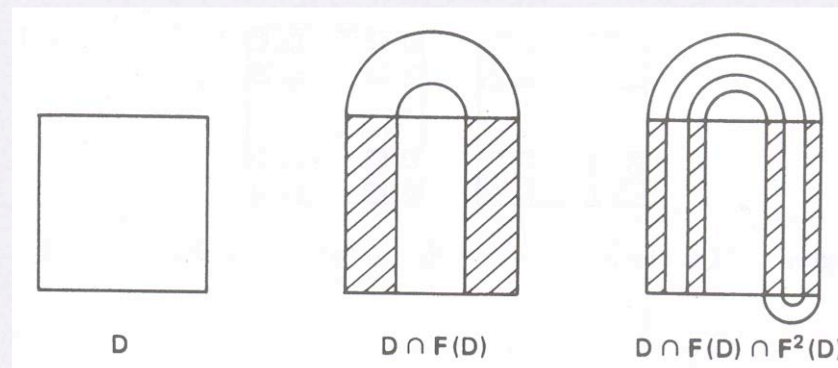

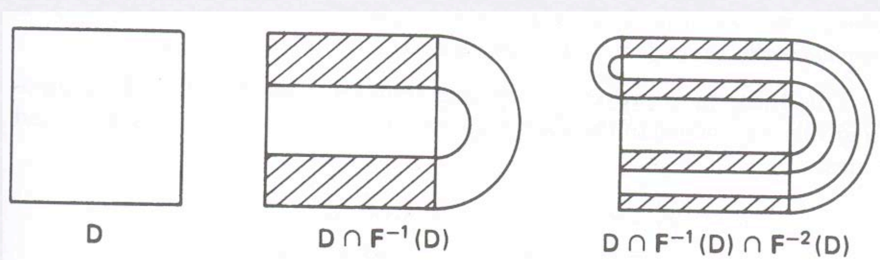

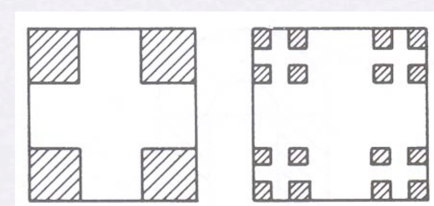

- Proved lots of nice things—eg, an invariant Cantor set.

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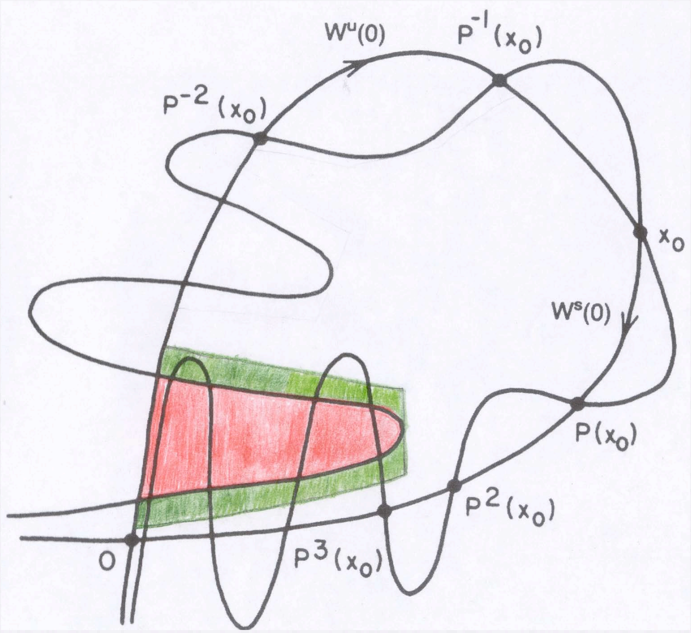

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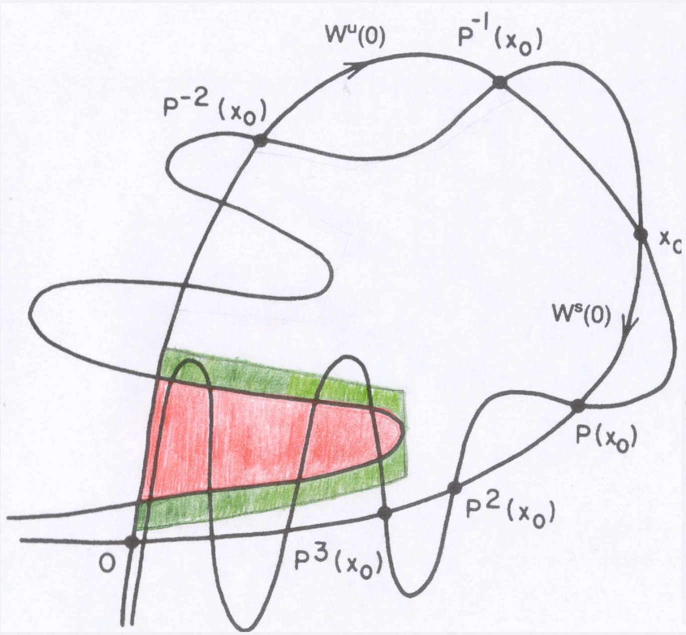

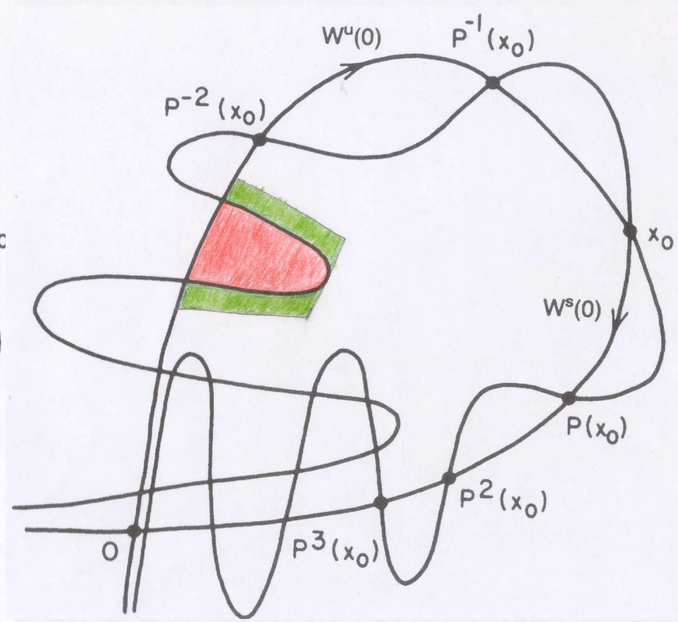

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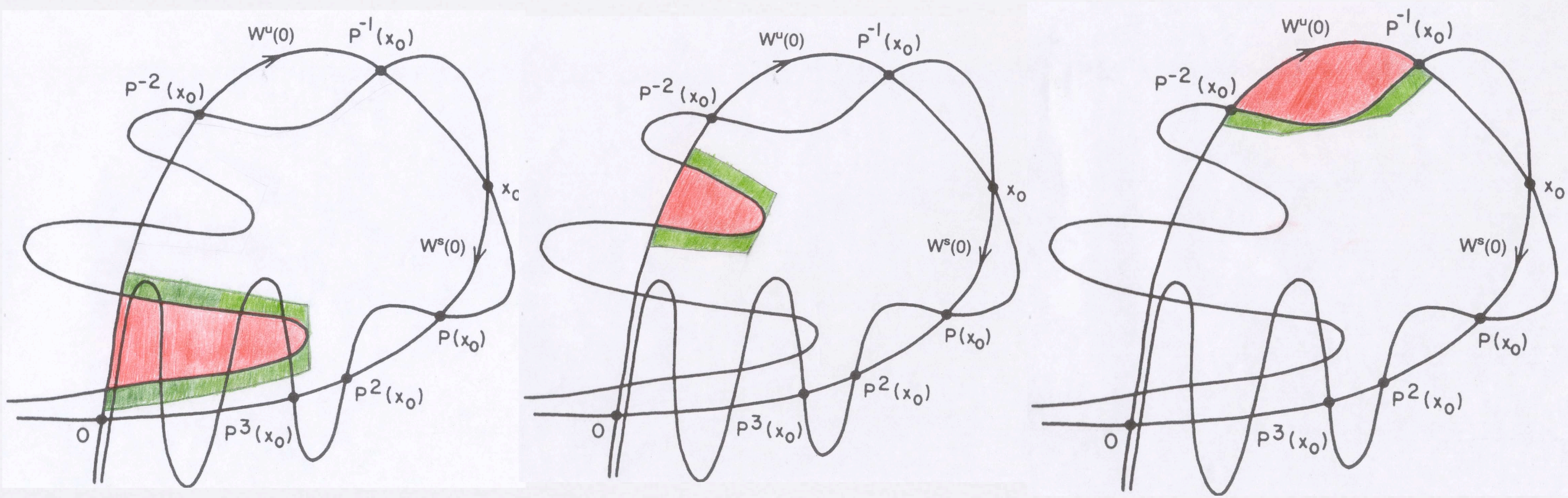

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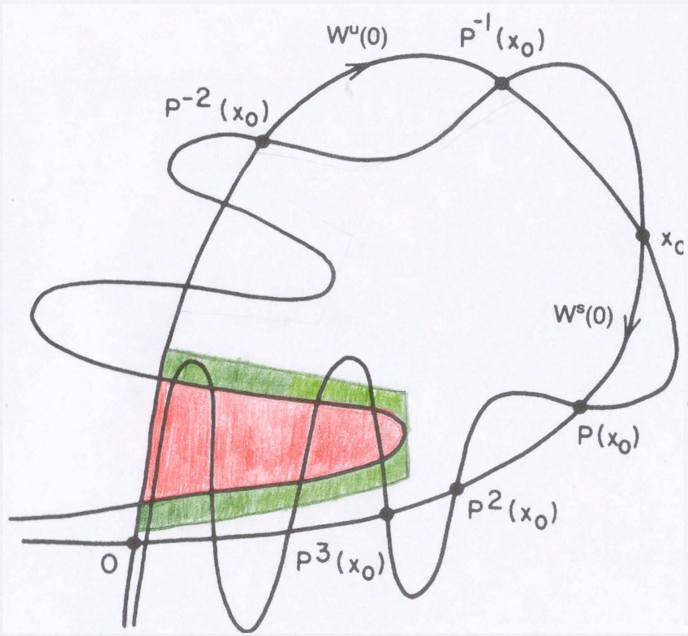

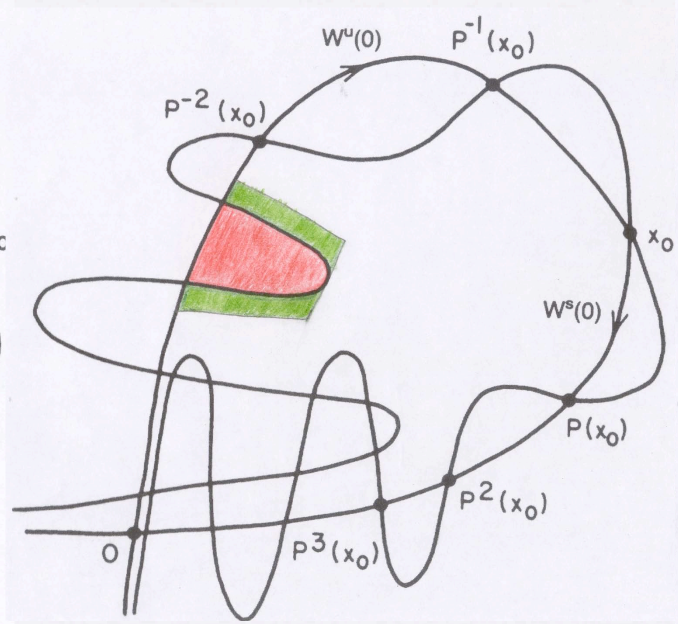

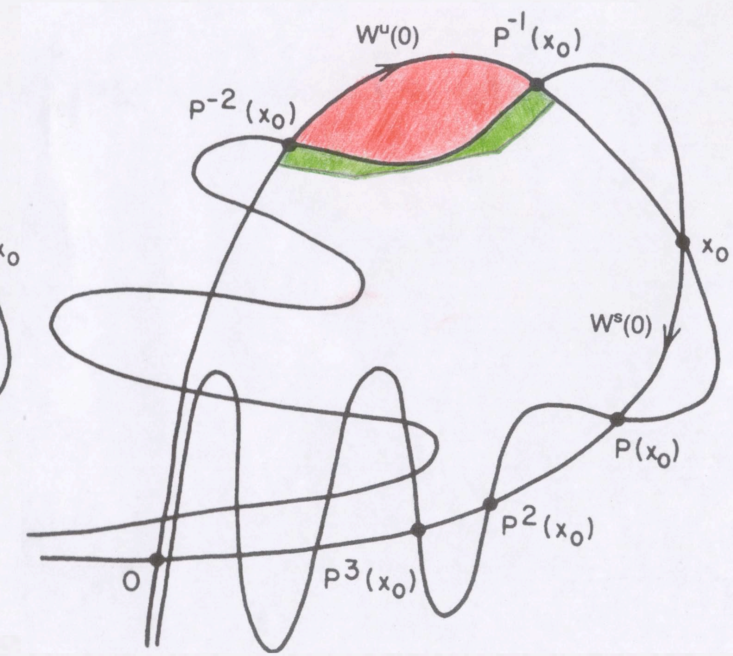

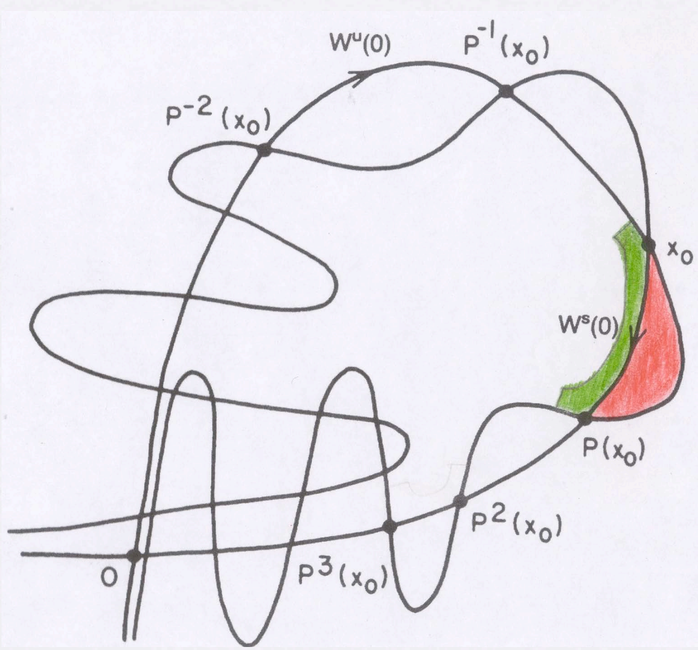

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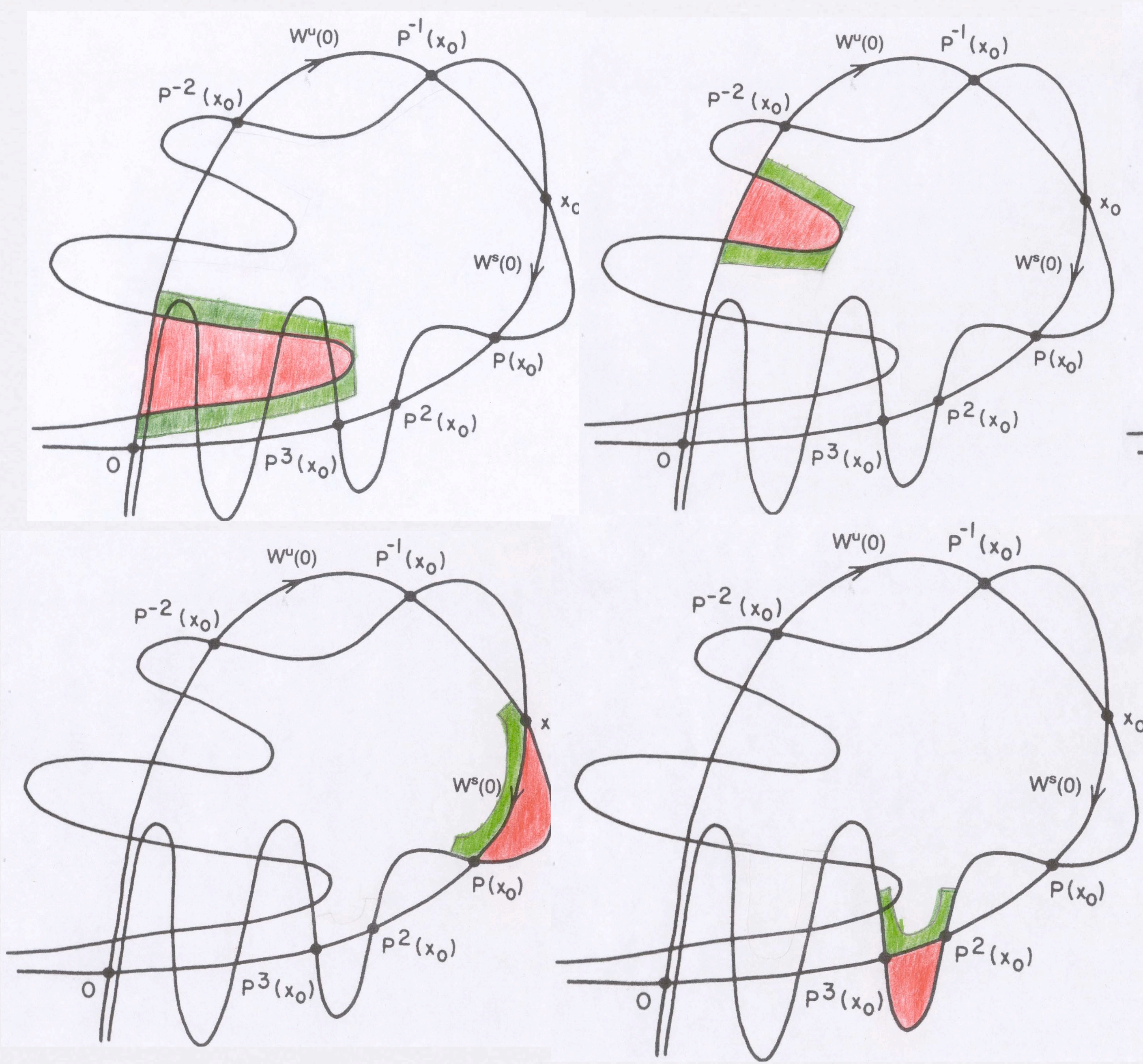

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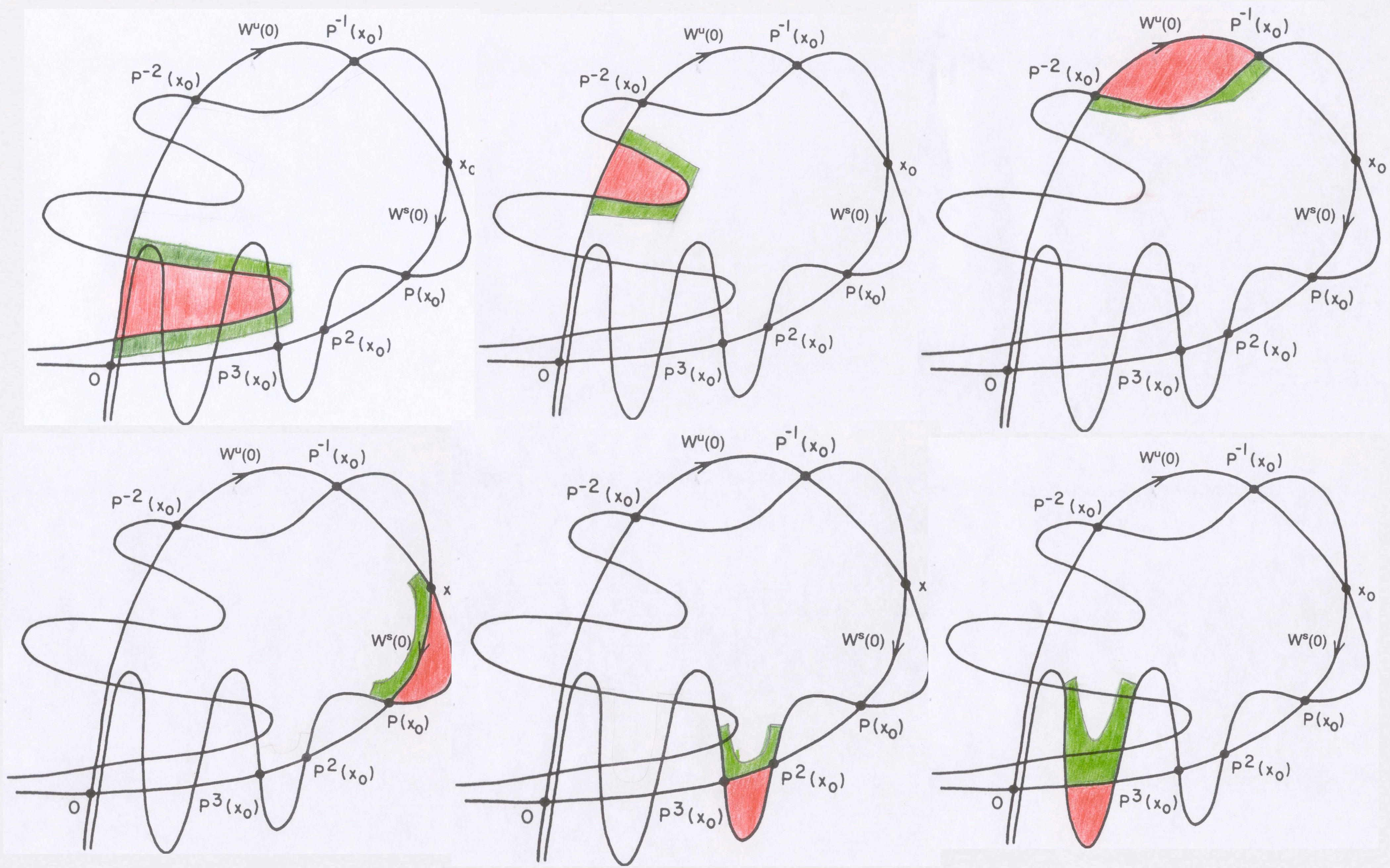

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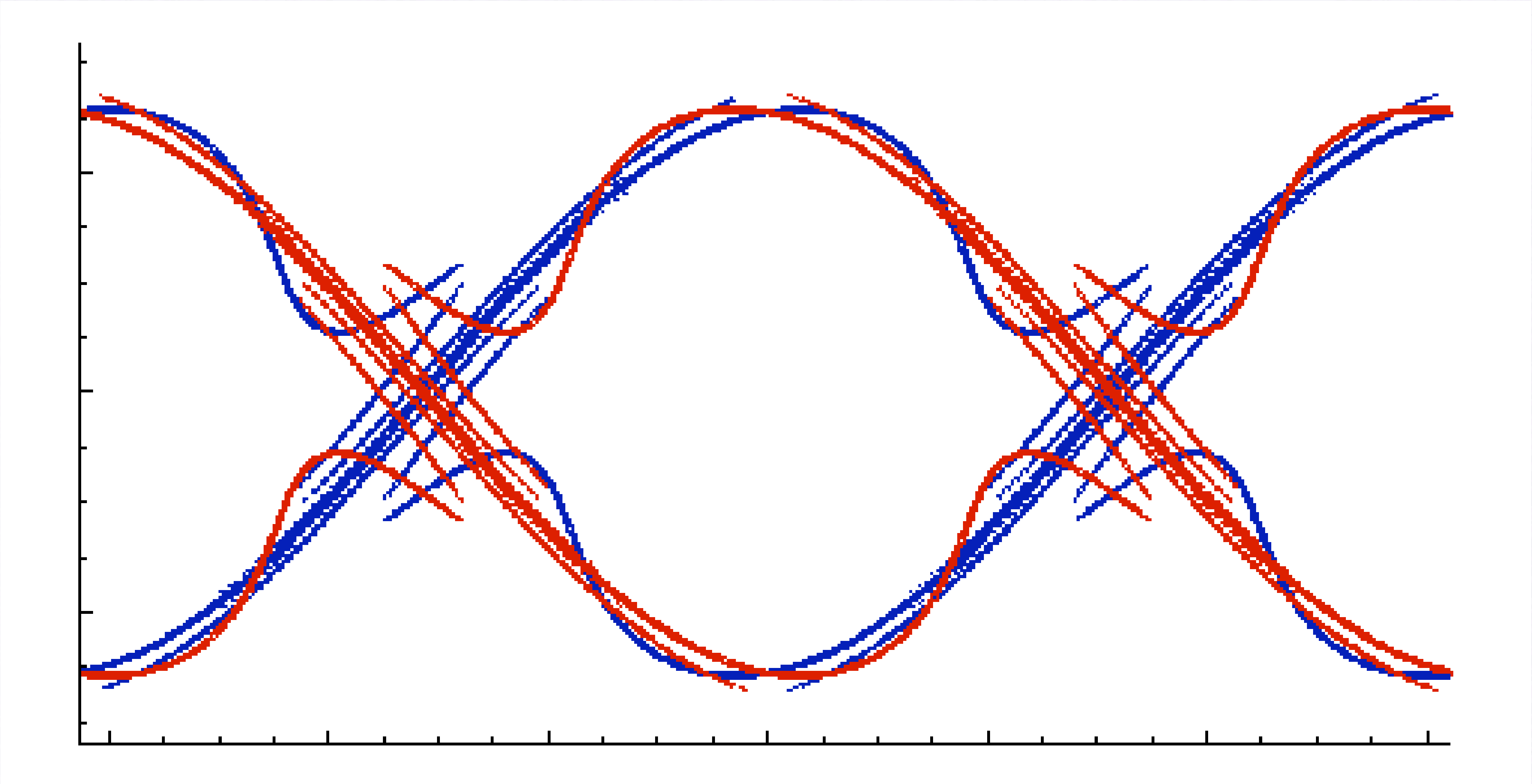

Look at lobes, mixing, dynamically