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MATH 145: HOMEWORK 2

DUE 1/22/13

(A) GIVEN $q(x) = Ax^2 + Bx + C$, SHOW $q(x)$ IS CONJUG. TO $x^2 + c$, $c \in \mathbb{R}$, using $\phi(x) = ax + b$.
POSSIBLE q CONJUG. to $x^2 + c$?
RELATION BETWEEN A, B, C AND c.

LET $f(x) = x^2 + c$.

WTS: $$q(\phi(x)) = \phi(f(x))$$

$$A(ax+b)^2 + B(ax+b) + C' = q(\phi(x))$$
$$= Aa^2x^2 + 2Aaxb + Ab^2 + Bax + Bb + C'$$

and $$\phi(f(x)) = a(x^2+c) + b$$
$$= ax^2 + ac + b$$

$$Aa^2 = a \qquad a = \frac{1}{A}$$

$$2Aab + Ba = 0 \qquad 2Ab + B = 0 \qquad b = -\frac{B}{2A}$$

$$Ab^2 + Bb + C' = ac + b \qquad \frac{Ab^2 + Bb + C' - b}{a} = c$$

THUS, $q \underset{\phi}{\sim} f$, SINCE a, b, c CAN BE WRITTEN IN TERMS OF A, B, C'.

A relation between A, B, C' and c is

$$A\left(A\left(\frac{-B}{2A}\right)^2 + B\left(\frac{-B}{2A}\right) + C' + \frac{B}{2A}\right) = c$$

$$\frac{B^2}{4} - \frac{B^2}{2} + C' + \frac{B}{2} = c$$

$$-\frac{B^2}{4} + \frac{B}{2} + C' = c$$

(B) WTS: $kx(1-x) \underset{\phi}{\sim} x^2+c$

Let $l(x) = kx(1-x)$, $f(x) = x^2+c$, $\phi(x) = ax+b$.

(WTS: $l(\phi(x)) = \phi(f(x))$)

$$\begin{aligned} l(\phi(x)) &= k(ax+b)(1-(ax+b)) \\ &= kax+bk - k(ax+b)^2 \\ &= kax+bk - ka^2x^2 - 2kaxb - kb^2 \\ &= -ka^2x^2 - 2kaxb + kax - kb^2 + bk \end{aligned}$$

$$\begin{aligned} \phi(f(x)) &= a(x^2+c)+b \\ &= ax^2+ca+b \end{aligned}$$

$\Rightarrow -ka^2 = a \qquad -2kab+ka = 0 \qquad kb - kb^2 = ca+b$

$a = -\frac{1}{k} \qquad -2b+1 = 0 \qquad kb(1-b) = ca+b$

$b = \frac{1}{2} \qquad \frac{kb(1-b)-b}{a} = c$

THUS, $l \underset{\phi}{\sim} f$.

AND $c = \left[\frac{k}{2}\left(\frac{1}{2}\right) - \frac{1}{2}\right] \cdot (-k) = -\frac{k^2}{4} + \frac{k}{2}$.

let $\psi(k) = -\frac{k^2}{4} + \frac{k}{2}$.

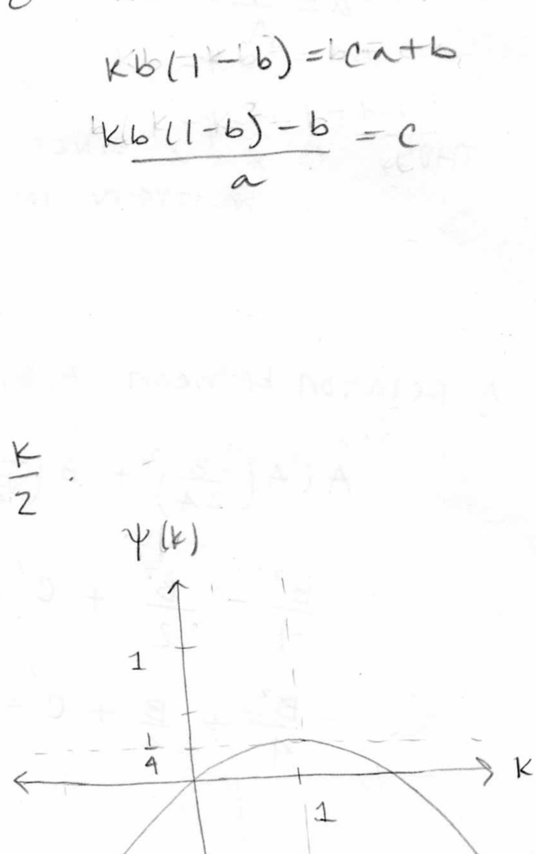
WTS: $c \in \text{RAN}(\psi)$

$\psi'(k) = -\frac{k}{2} + \frac{1}{2} = 0$

$\Rightarrow k = 1$.

so $y_{max} = \psi(1) = \frac{1}{4}$.

nice!



THUS, $\psi(k)$ IS ONLY INVERTIBLE WHEN $c \in (-\infty, \frac{1}{4}]$.

(c) ASSUME $q(x)$ HAS 2 DISTINCT FIXED POINTS

WTS: $q \underset{\phi}{\sim} \ell$. $\qquad \ell(x) = Kx(1-x)$

$$q(\phi(x)) = A(ax+b)^2 + B(ax+b) + C$$
$$= Aa^2x^2 + 2aAxb + Ab^2 + Bax + Bb + C$$

$$\phi(\ell(x)) = a(Kx - Kx^2) + b$$
$$= aKx - aKx^2 + b$$

$Aa^2 = -aK \qquad 2aAb + Ba = aK \qquad Ab^2 + Bb + C = b$

$a = \frac{-K}{A}$ $(**)$ $\qquad 2Ab + B = K$ $(*)$

$b = \frac{K-B}{2A}$

SINCE $q(x)$ HAS 2 DISTINCT FIXED POINTS AND $b = Ab^2 + Bb + C$, THEN $b, K,$ AND a HAVE 2 SOLUTIONS.

SO,

$$b = Ab^2 + Bb + C$$
$$0 = Ab^2 + b(B-1) + C, \text{ AND}$$

good!

$$b = \frac{-(B-1) \pm \sqrt{(B-1)^2 - 4AC}}{2A}, \text{ THEN}$$

$2Ab = -(B-1) \pm \sqrt{(B-1)^2 - 4AC}$ AND $2Ab + B = K$ $(*)$, SO,

$$K = 2Ab + B = B - B + 1 \pm \sqrt{(B-1)^2 - 4AC} = 1 \pm \sqrt{(B-1)^2 - 4AC}.$$