

MATH 145: HOMEWORK 2

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DUE 1/22/13

(A) GIVEN $g(x) = Ax^2 + Bx + C$, SHOW $g(x)$ IS CONJUGATE TO $f(x)$ TO $x^2 + c$, $c \in \mathbb{R}$, using $\phi(x) = ax + b$.POSSIBLE g CONJUGATE TO $x^2 + c$?RELATION BETWEEN A, B, C AND c .LET $f(x) = x^2 + c$.

NTS: $g(\phi(x)) = \phi(f(x))$

$$\begin{aligned} & A(ax+b)^2 + B(ax+b) + c' = g(\phi(x)) \\ & = Aa^2x^2 + 2Aaxb + Ab^2 + Bax + Bb + c' \end{aligned}$$

$$\begin{aligned} \text{and } \phi(f(x)) &= a(x^2 + c) + b \\ &= ax^2 + ac + b \end{aligned}$$

$$Aa^2 = a$$

$$2Aab + Ba = 0$$

$$Ab^2 + Bb + c' = ac + b$$

$$a = \frac{1}{A}$$

$$2Ab + B = 0$$

$$\frac{Ab^2 + Bb + c' - b}{a} = c$$

$$b = -\frac{B}{2A}$$

THUS, $g \not\sim f$, SINCE a, b, c CAN BE
WRITTEN IN TERMS OF A, B, C' .A relation between A, B, C' and c is

$$A\left(A\left(-\frac{B}{2A}\right)^2 + B\left(-\frac{B}{2A}\right) + C' + \frac{B}{2A}\right) = c$$

$$\frac{B^2}{4} - \frac{B^2}{2} + C' + \frac{B}{2} = c$$

$$-\frac{B^2}{4} + \frac{B}{2} + C' = c \quad \checkmark$$

$$(B) \text{ WTS: } kx(1-x) \underset{\not\phi}{\sim} x^2 + c$$

Let $\ell(x) = kx(1-x)$, $f(x) = x^2 + c$, $\phi(x) = ax + b$.

$$(\text{WTS: } \ell(\phi(x)) = \phi(f(x)))$$

$$\begin{aligned}\ell(\phi(x)) &= k(ax+b)(1-(ax+b)) \\ &= kax+kbk - k(ax+b)^2 \\ &= kax+kbk - ka^2x^2 - 2ka\cdot b - kb^2 \\ &= -ka^2x^2 - 2ka\cdot b + kax - kb^2 + bk\end{aligned}$$

$$\phi(f(x)) = a(x^2 + c) + b$$

$$\begin{aligned}\Rightarrow -ka^2 &= a & -2ka\cdot b + ka &= 0 & kb - kb^2 &= ca + b \\ a &= -\frac{1}{k} & -2b + 1 &= 0 & kb(1-b) &= ca + b \\ b &= \frac{1}{2} & b &= \frac{1}{2} & \frac{kb(1-b) - b}{a} &= c\end{aligned}$$

THUS, $\ell \underset{\not\phi}{\sim} f$,

$$\text{AND } c = \left[\frac{k}{2} \left(\frac{1}{2} \right) - \frac{1}{2} \right] \cdot (-k) = -\frac{k^2}{4} + \frac{k}{2}$$

$$\text{let } \psi(k) = -\frac{k^2}{4} + \frac{k}{2}$$

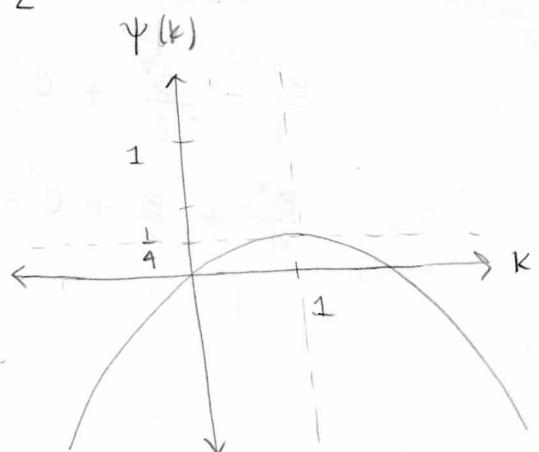
WTS: $c \in \text{RAN } (\psi)$

$$\psi'(k) = -\frac{k}{2} + \frac{1}{2} = 0$$

$$\Rightarrow k = 1.$$

$$\text{so } y_{\max} = \psi(1) = \frac{1}{4}.$$

Nice!



THUS, $\psi(k)$ IS ONLY INVERTIBLE WHEN $c \in (-\infty, \frac{1}{4}]$.

CHAPTER 3: DIFFERENTIAL EQUATIONS

LINEAR EQUATIONS

$(A^2 + B^2 + C^2) - AB + BC + CA$ shows that $g(x)$ is increasing on \mathbb{R} if and only if A, B, C are such that $A^2 + B^2 + C^2 \geq AB + BC + CA$. This means that all solutions of A, B, C

are such that any two consecutive terms of the sequence $\{a_n\}$ satisfy $a_{n+1} > a_n$. This is equivalent to

(c) ASSUME $g(x)$ HAS 2 DISTINCT FIXED POINTS.

WTS: $g(x) \not\sim l(x)$ for all x because $l(x) = kx(1-x)$

$$\begin{aligned} g(\phi(l(x))) &= A(ax+b)^2 + B(ax+b) + C \\ &= Aa^2x^2 + 2aAx + Ab^2 + Bax + Bb + C \end{aligned}$$

$$\begin{aligned} \phi(l(x)) &= a(kx - kx^2) + b \\ &= akx - akx^2 + b \end{aligned}$$

$$Aa^2 = -ak \quad 2aAb + Ba = akx \quad Ab^2 + Bb + C = b$$

$$\begin{aligned} a &= -\frac{k}{A} \quad (***) \\ b &= \frac{k-B}{2A} \end{aligned}$$

SINCE $g(x)$ HAS 2 DISTINCT FIXED POINTS AND $b = Ab^2 + Bb + C$,
THEN b, k , AND a HAVE 2 SOLUTIONS.

SO, 2 distinct solns of $g(x) = 0$,

$$\begin{aligned} b &= Ab^2 + Bb + C \\ 0 &= Ab^2 + b(B-1) + C, \text{ AND} \end{aligned}$$

good!

$$b = \frac{-(B-1) \pm \sqrt{(B-1)^2 - 4AC}}{2A}, \text{ THEN}$$

$$2Ab = -(B-1) \pm \sqrt{(B-1)^2 - 4AC} \text{ AND } 2Ab + B = k \quad (*), \text{ SO,}$$

$$k = 2Ab + B = B - B + 1 \pm \sqrt{(B-1)^2 - 4AC} = 1 \pm \sqrt{(B-1)^2 - 4AC}.$$