5.1. Design an affine differential equation having a saddle point at $(1, 1)$ with the line $x - 1 = 2(y - 1)$ as stable manifold and the line $x = 1$ as unstable manifold. Draw some phase portraits. (Affine means linear plus constant.)

5.2. The damped harmonic oscillator is given by

$$\ddot{x} = -\omega^2 x - \mu \dot{x}.$$

where ω and μ are real parameters known as the frequency and the damping. (or friction) coefficient. (i) Reexpress the oscillator as a system in the $(x, \dot{x})$ plane by putting it in 1st order form.

(ii) Find the eigenvalues of the corresponding two-by-two matrix.

(iii) Write out the general solution.

(iv) Draw several phase portraits according to several choices of (ω, μ). Include $(\omega, \mu) = (1, 0)$.

5.3. Strogatz writes a limit cycle in polar coordinates r, θ as $\dot{\theta} = 1, \dot{r} = r(1 - r^2)$. (Look up 'limit cycle' in his index.) Convert this equation to a system of 1st order differential equation in the Cartesian xy plane. (The correct answer will have a polynomials in x, y as r.h.s.)