HW 4

4.1. Recall the tent map $x \mapsto T_m(x)$. As m increases from 0 to 1 the tent gets steeper up until the point where it just starts to 'poke' through the roof $y = 1$ at the value $m = 2$. Let T be the tent map at this special value $m = 2$.

A. Graph a few iterates of T.

B. Show that T^k has 2^k periodic orbits of period k and that all of these orbits are unstable.

4.2. Consider the shift map on the bi-infinite sequence space of 0'a and 1's.

A. List the fixed points of F.

B. List the period 2 orbits which are not fixed points.

C. List the period 3 orbits which are not fixed points. Eg. $\ldots 011.011011011\ldots$ is one. Alternatively: you can use the notation $\bar{0}11$.

D. For p a prime, how many distinct period p orbits are there which are not prime?

4.3. Map the one-sided sequence space of 0's and 1's to the unit interval by thinking of a sequence as the binary expansion of a number. Thus: $0110000\ldots$ maps to the number $0 * \frac{1}{2} + 1 * \frac{1}{2^2} + 1 * \frac{1}{2^3}$. This map is onto.

A. *Prove* the map is not one-to-one by exhibiting two distinct sequences that map to the same number.

B. Recall the definition of "continuous" and "connected" . Look up or recall a basic theorem about the continuous image of a connected set. Use this theorem to prove abstractlly that the map of part A can have no continuous inverse. (Do not use that the map of A is not one-to-one.)

C. Show that the one-sided shift map on the space of one sided sequences of 0's and 1's is semi-conjugate to the doubling map $x \mapsto 2x$ on the circle $\mathbb{R}/\mathbb{Z}$ of numbers x mod 1. Hint: take as semi-conjugacy the map from 4.2 A, mod 1.

4.4. Look up the definition of "space-filling curve". Using such a curve, prove that we can map the Cantor set onto the closed unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ in the place. Generalize your assertion to the unit cube in d-dimensional space.

HW 5.

5.1 Let $x \mapsto f(x)$ be a smooth *strictly monotone increasing* map of the interval $I = [0,1]$ *onto* itself. Suppose also that whenever $f(x) = x$ for $x \in I$ we have $\frac{df}{dx}(x) \neq 1$.

A. Show that the fixed points of f in the unit interval are alternating stable and unstable. Sketch the phase portrait of such an f.

B. Prove f has only a finite number of fixed points in the unit interval.

5.2 A Suppose that F is a discrete dynamical system, i.e a map which we iterate. Suppose that p is a point which has both period 2 and period 3 for F. Prove that F is a fixed point.

B. Generalize the previous problem by replacing $(2, 3)$ by a general relatively prime pair of integers (m, n).

5.3. Construct a semi-conjugacy from the doubling map $x \mapsto 2x$ mod 1 to the tent map at the special value $m = 2$. (See HW4.1.) Hint: your semi-conjugacy to the unit interval will be a $2:1$ at all but a few points.