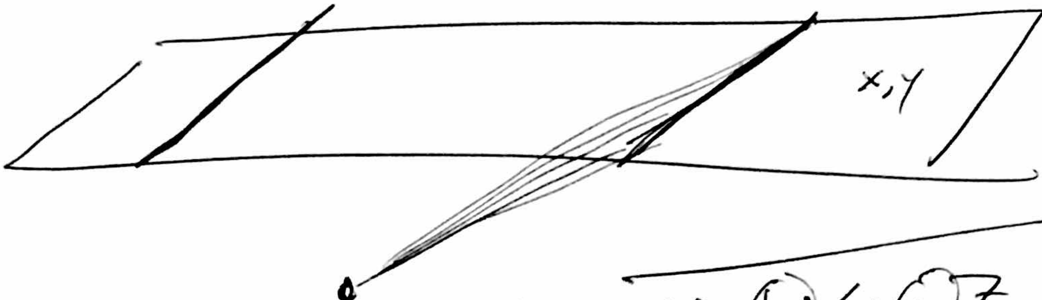
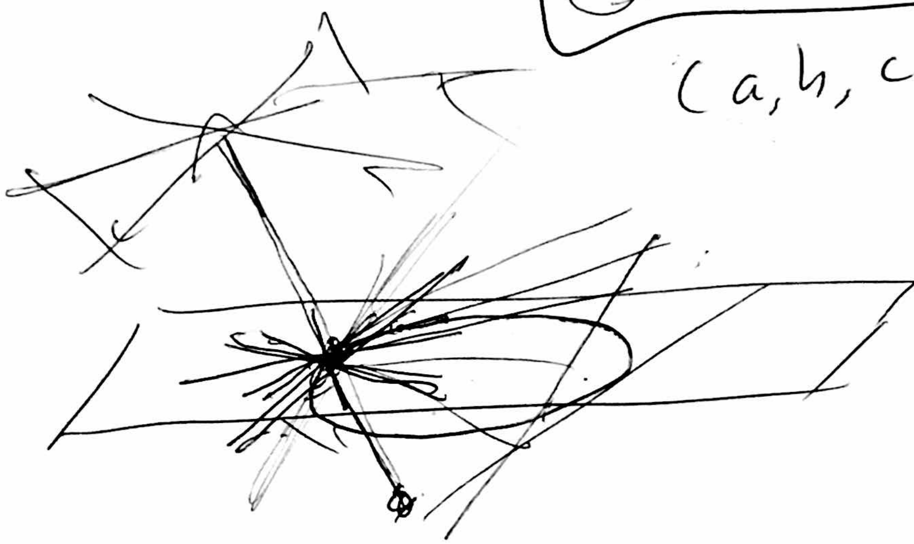
$ax+by+c=0$



$[x, y, 1]$

$\boxed{aX+bY+cZ=0}$

(a,b,c) dual to (X,Y,Z)?



P line in $\mathbb{CP}^2$ given by choice of (a,b,c) according to

but $(\lambda a, \lambda b, \lambda c)$ define same line $\lambda \neq 0$

lines $= \mathbb{P}(\mathbb{C}^{3*})$

$\mathbb{C}^3$ $\mathbb{C}^n$

all ~~of~~ nondeg quadratic forms are the same.

over $\mathbb{R}^3$ No!

index

$X^2+Y^2+Z^2$

$X^2+Y^2-Z^2$

$X^2-Y^2-Z^2$

$-Z^2=+(iZ)^2$

$(X,Y,Z)\,Q\begin{pmatrix}X\\Y\\Z\end{pmatrix}$

diag.

λ_1

λ_2.

λ_3.

$2X^2+Y^2+Z^2$

$x=\sqrt{2}X$

lines in $\mathbb{P}V \leftrightarrow$ pts in $\mathbb{P}V^*$.

$\{[v] : \alpha(v) = 0\} \leftarrow [\alpha] \in \mathbb{P}V^*$

p3

sme $\alpha \neq 0$.
$\alpha \in V^*$

$\ker \alpha = \ker(\lambda \alpha)$

$\lambda \neq 0$
$\lambda \in \mathbb{C}$

Incidence variety

$\{(p, \ell) : p \in \mathbb{P}V,\ \ell \in \mathbb{P}V^*,\ p \in \ell\}$

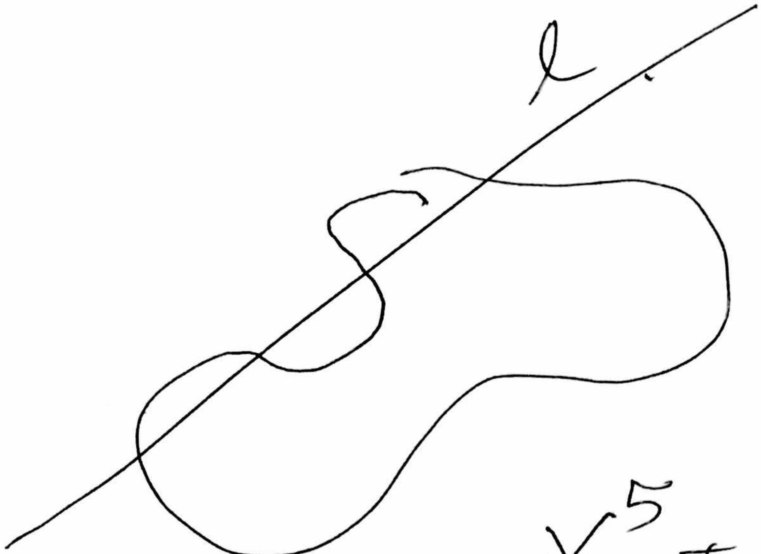
$\uparrow \quad \uparrow$

$[v] \quad [\alpha]$

$\alpha(v) = 0$.

$\alpha = a\,dX + b\,dY + c\,dZ$

$v = X\frac{\partial}{\partial X} + Y\frac{\partial}{\partial Y} + Z\frac{\partial}{\partial Z}$.

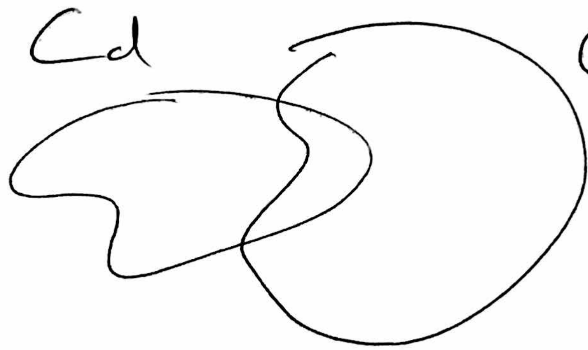


$d = 5$

$X^5 + aX^4Y^{\cancel{2}} + 12YZ^4 = 0.$

$l \cap C$ has d points.

$x^2 + y^2$



$\underline{d \cdot m}$

V 3-d cx vector space

$\mathbb{P}V \simeq \mathbb{CP}^2$. $\qquad V = \mathbb{C}^3$.

e_1, e_2, e_3.

$Xe_1 + Ye_2 + Ze_3$.

pt in $\mathbb{P}V$, $p = [v]$

$= \mathbb{C}$-span of v.

$v \neq 0$.

line in $\mathbb{P}V = \pi$(plane thru 0).

planes thru 0 are given by $\alpha \in V^*$, $\alpha \neq 0$.

$$\begin{array}{cc} V \setminus 0 & v \\ \pi \downarrow & \downarrow \\ \mathbb{P}V & [v]. \end{array}$$

$\Lambda = \ker \alpha$.

$= \{v : \alpha(v) = 0\}$.

$\alpha : V \xrightarrow{\mathbb{C}\text{-linear}} \mathbb{C}$