

FIGURE 19. The three-body problem. The forces are along the edges of the triangle formed by the masses.

$$m_1 \ddot{q}_1 = F_{21} + F_{31},$$

$$m_2 \ddot{q}_2 = F_{12} + F_{32},$$

$$m_3 \ddot{q}_3 = F_{23} + F_{13},$$

$$F_{ba} = -\frac{Gm_a m_b}{r_{ab}^2} \hat{q}_{ab}.$$

$$r_{ab} = |q_a - q_b|$$

a and b, and

$$\hat{q}_{ab} = \frac{q_a - q_b}{r_{ab}}$$

symmetries: Galilean group:

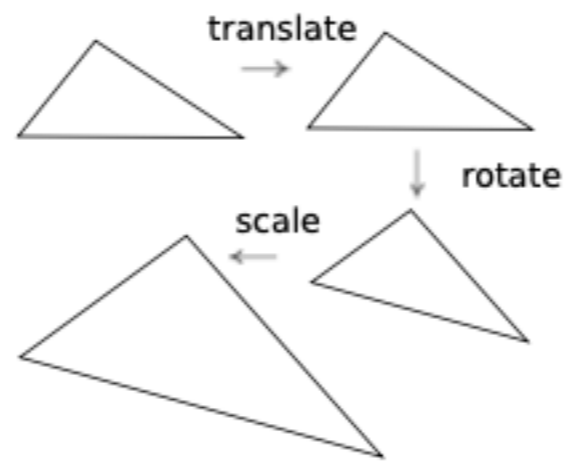
$$q_a(t) \mapsto q_a(t) + c \text{ (space translation)}$$

$$q_a(t) \mapsto R(q_a(t)) \text{ (space rotation or reflection)}$$

$$q_a(t) \mapsto q_a(t - t_0) \text{ (time translation)}$$

$$q_a(t) \mapsto q_a(-t) \text{ (time reflection)}$$

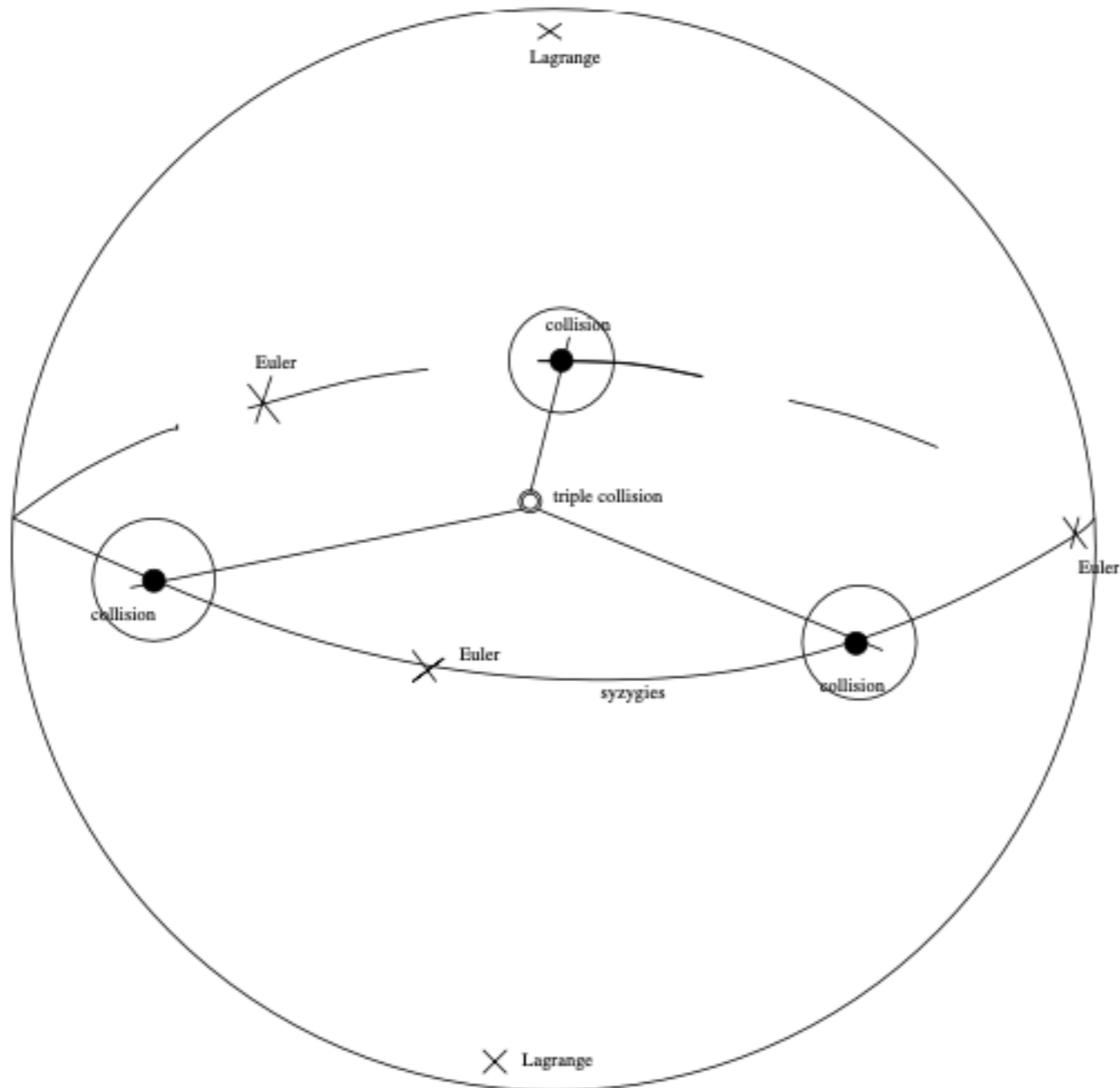
$$q_a(t) \mapsto q_a(t) + tv \text{ (boost).}$$

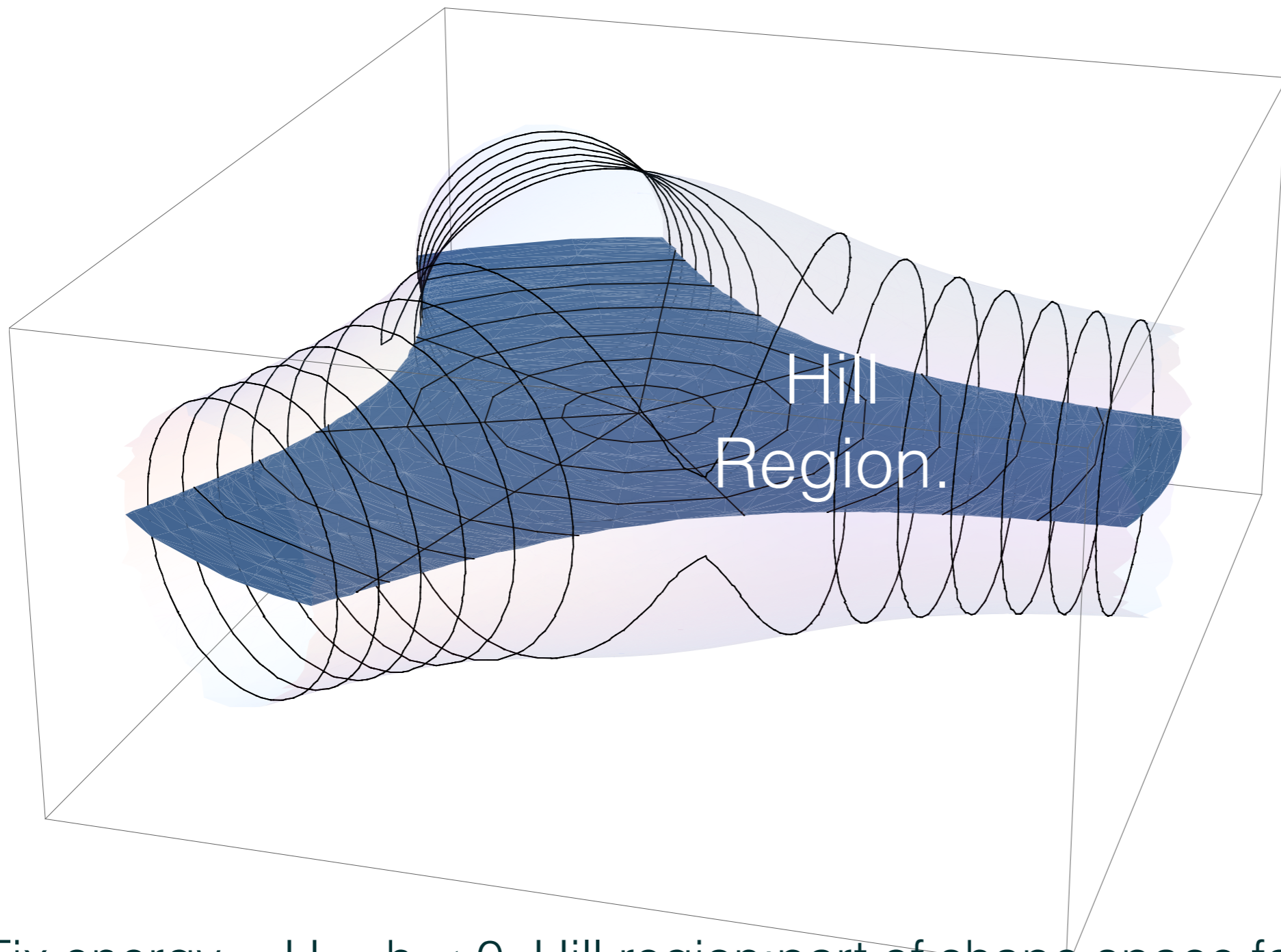


$$\begin{aligned} E(q, \dot{q}) &= K(\dot{q}) - U(q) \\ &= \frac{1}{2} \sum m_a |\dot{q}_a|^2 - \sum_{a < b} f_{ab}(r_{ab}) \\ &= \text{energy (for time translations)} \end{aligned}$$

$$P = \sum m_a \dot{q}_a = \text{linear momentum (for space translations)}$$

$$J = \sum m_a q_a \wedge \dot{q}_a = \text{angular momentum (for space rotations) .}$$





Fix energy = $H = -h < 0$. Hill region: part of shape space for which there is a v and $H(q,v) = -h$. Domain where motion occurs. Identical to region with $U(q) > +h$

