class 1

intros.

<u>def</u> What's a Riemann surface?

<u>Recall</u>: smooth 2-d manifold or "smooth surface"

Top space: X

Covered by open sets U_α called "charts": w/

$$\psi_\alpha : U_\alpha \xrightarrow{\sim} V_\alpha \subset \mathbb{R}^2, \quad V_\alpha \text{ open}.$$

s.t.

$$\underbrace{\psi_\beta \circ \psi_\alpha^{-1}}_{} : V_\alpha \dashrightarrow V_\beta \quad \text{is smooth,}$$

invertible

"overlaps"

discuss broken arrow notation.

Identify $\mathbb{R}^2 \simeq \mathbb{C}$

$$(x,y) \to x+iy = z.$$

Insist: overlaps are holo.

$$V_\alpha \dashrightarrow V_\beta$$

Then X is a Riem surface

Examples

1. $\mathbb{C}$.

2. $\mathbb{C} \setminus \{p_1, p_2, \ldots, p_N\}$

3. $\mathbb{CP}^1 = \mathbb{C} \cup \{\infty\}$
two charts, one about zero, say z
one about ∞, say w.
overlap: $w = \frac{1}{z}$

Alternatively $\mathbb{CP}^1$ = cx lines thru $(0,0)$ in $\mathbb{C}^2$.

$[z, w] = [\frac{z}{w}, 1]$ or $[1, \frac{w}{z}]$

4. let $p(z)$ be any polyn in z

$w = \sqrt{p(z)}$

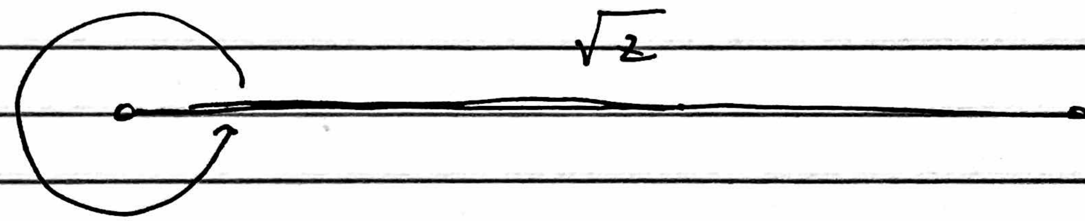
is a R.S.

"R.S" of $\sqrt{p(z)}$.

eg

Alternatively: $w^2 = p(z) \subset \mathbb{C}^2_{(z,w)}$

eg $p(z) = z$.



Exer What's the topology of "$\sqrt{z}$"

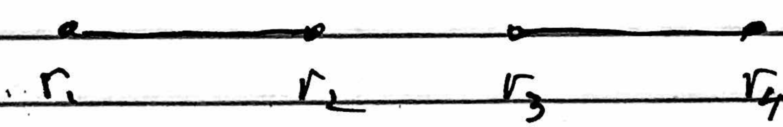
of "$\sqrt{p(z)}$"

= Commercial break =

Holomorphic Implicit Fn Thm.

To understand $w^2 = p(z)$

Eg:



quartic $p(z)$ w/ distinct real roots

$\to$ elliptic curve.

generalizing:
$$\{F(z,w)=0\} \subseteq \mathbb{C}^2.$$

Exer: Show $zw=0$ is not a R.S. by showing it is not a topological surface.

IFT: requires $dF \neq 0$ whenever $F=0$.

ie $\left(\frac{\partial F}{\partial z}(z,w), \frac{\partial F}{\partial w}(z,w)\right) \neq (0,0)$

provided $F(z,w)=0$.

Eg "Fermat surfaces"

$$\cancel{z}^n + \cancel{w}^n = 1.$$

homogenize : ~~z = x~~

$$x^n + y^n = z^n \quad \text{in } \mathbb{CP}^2.$$

This is a cpt R.S.