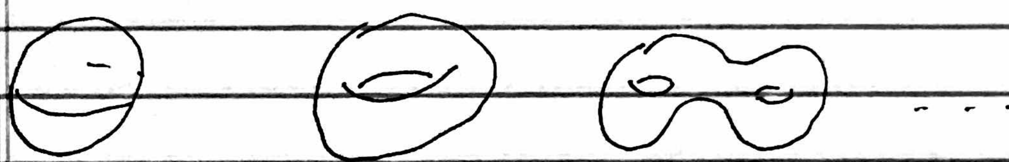
Genus, g.

X cpt oriented surface w/out ∂.

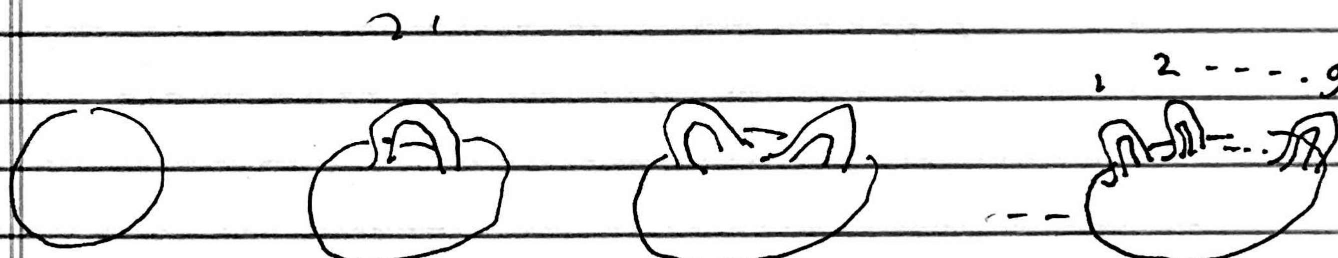
Thm X is completely characterizes by a single #, $g \in \mathbb{N}$.

That is: $g(X) = g(Y) \Leftrightarrow X \underset{\text{diffeo.}}{\simeq} Y$

egs.



$g=0$ $g=1$ $g=2$ ...



0 handles 1-handle 2-handles g handles

Characterizing g ...

genus g.

1. $\chi(X_g) = 2 - 2g$.

where $\chi(X) = V - E + F$
$$= b_0(X) - b_1(X) + b_2(X)$$

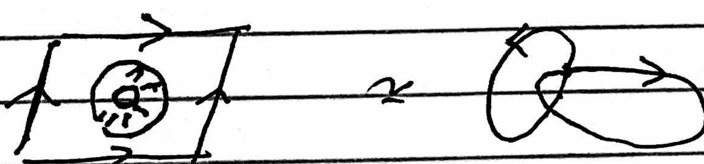
$$b_i(X) = \dim H^i(X, \mathbb{R}).$$

2. $b_0(X) = b_2(X) = 1$ so.

$$\begin{aligned} 2g &= \dim H^1(X, \mathbb{R}) \\ &= \dim H_1(X, \mathbb{R}) \\ &= \operatorname{rank} H_1(X, \mathbb{Z}) \end{aligned}$$

3. Puncture a hole in X.
(Remove 2-skeleton)
The result deformation retracts onto the 1-skeleton. This 1-skeleton is the wedge of $2g$ circles.

Eg $g = 1$



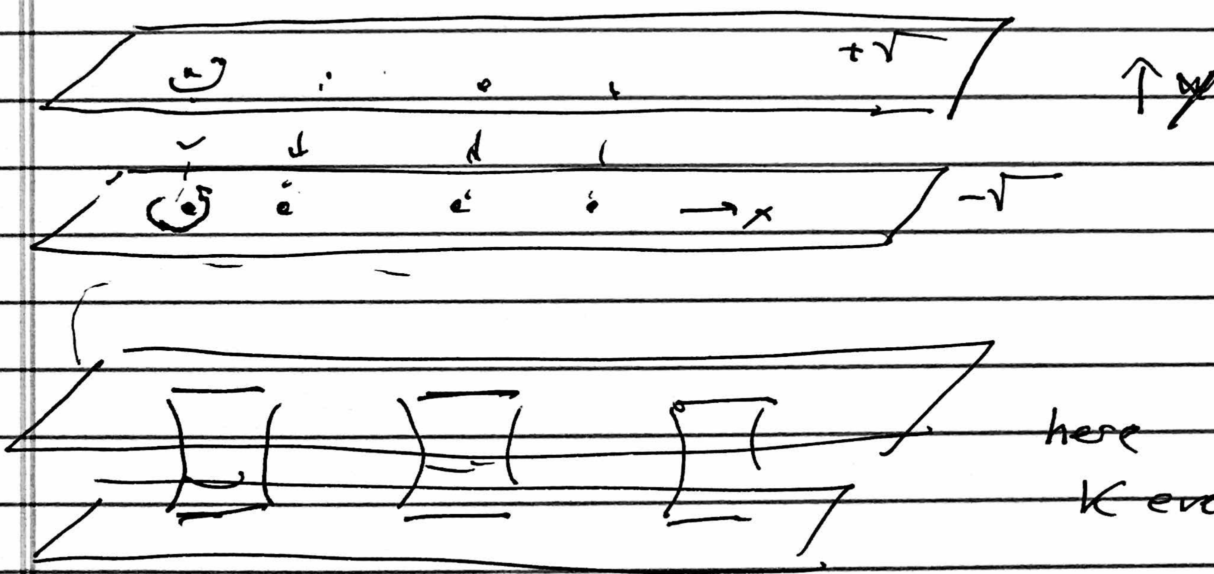
$S^1 \vee S^1$

<u>Example</u>

genus of a hyperelliptic curve

$\sqrt{p(x)}$, p deg K polyn no double roots.

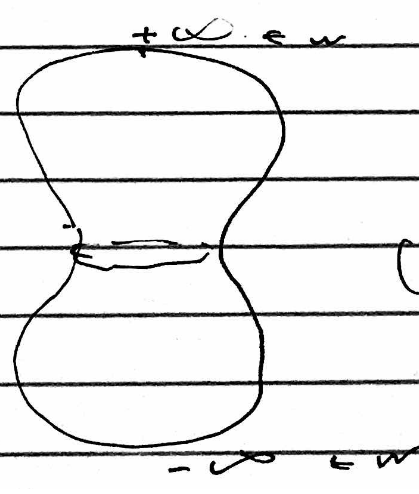
$y^2 = p(x)$



here K even

eg: $K = 2$ $\quad \sqrt{x(x-1)}$

$\sim$



$(\pm\infty)^2 = \infty$

$g = \cancel{\lfloor \frac{k}{2} \rfloor} - \begin{cases} 0, & k = 1, 2 \\ 1, & k = 3, 4 \\ 2, & k = 5, 6 \ldots \end{cases}$

so $g = \lfloor \frac{k-1}{2} \rfloor$

Compactifying $y^2 = p(x)$

or

$$w^2 = p(z)$$

cf FK p 102.

$\cdot\ w \to \infty \iff z \to \infty$

how many values for w @ ∞? one or two? more? Ans

$$\cancel{\frac{1}{w^2} = \frac{1}{p(z)} = \frac{1}{z^n + a_{n-1} z^{n-1} + \ldots}}$$

set $V = \frac{1}{w}$, $U = \frac{1}{z}$

$$\cancel{w^2 = \frac{1}{z^n(1 + a_{n-1}\frac{1}{z} + a_{n-1}\frac{1}{z^n} \ldots}}$$

$$\cancel{= U^n \frac{1}{(1 + a_{n-1}U + \ldots}}$$

$\begin{cases} 2, & k \text{ even} \\ 1, & k \text{ odd} \end{cases}$

¿ Why...??? — later!