Structure of class,
for now.

some lecture
some worked HW or other problems
some student presentation.
always open for questions

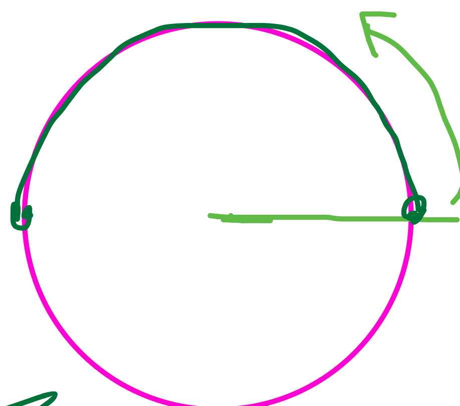
main texts : Flanders
Guillemin Pollack
Sjamaar.

alternates: Burke I & II
Lee.
etc etc.

oh. weekly HW:
try to do it all
hand in one at least every Tu
be ready to present one " week.

postings: my web site. for class.
HW: in a Dropbox.

$t \longleftrightarrow \tau$



$x = \cos t$

$y = \sin t$

$x = t$

$y = \sqrt{1-t^2}$.

$\tau = \cos t$

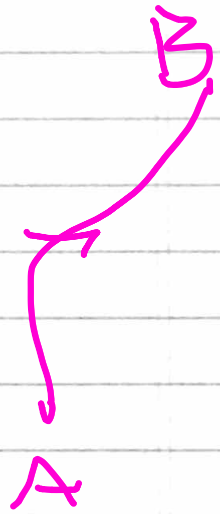
$\sin t = \pm\sqrt{1-\tau^2}$.

1. <u>1-forms</u>

a one-form on the plane is an expression of the form

$$\alpha = P(x,y)\,dx + Q(x,y)\,dy$$

<u>meaning</u>: integrand for line integrals.



$c : I \to \mathbb{R}^2$. $c(t) = (x(t), y(t))$.

$$c^*\alpha = \underbrace{P(x(t),y(t))\,\frac{dx}{dt}\,dt + Q(x(t),y(t))\,\frac{dy}{dt}\,dt}_{f(t)\,dt}$$

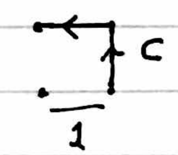
$$\int_c \alpha = \int_I c^*\alpha.$$

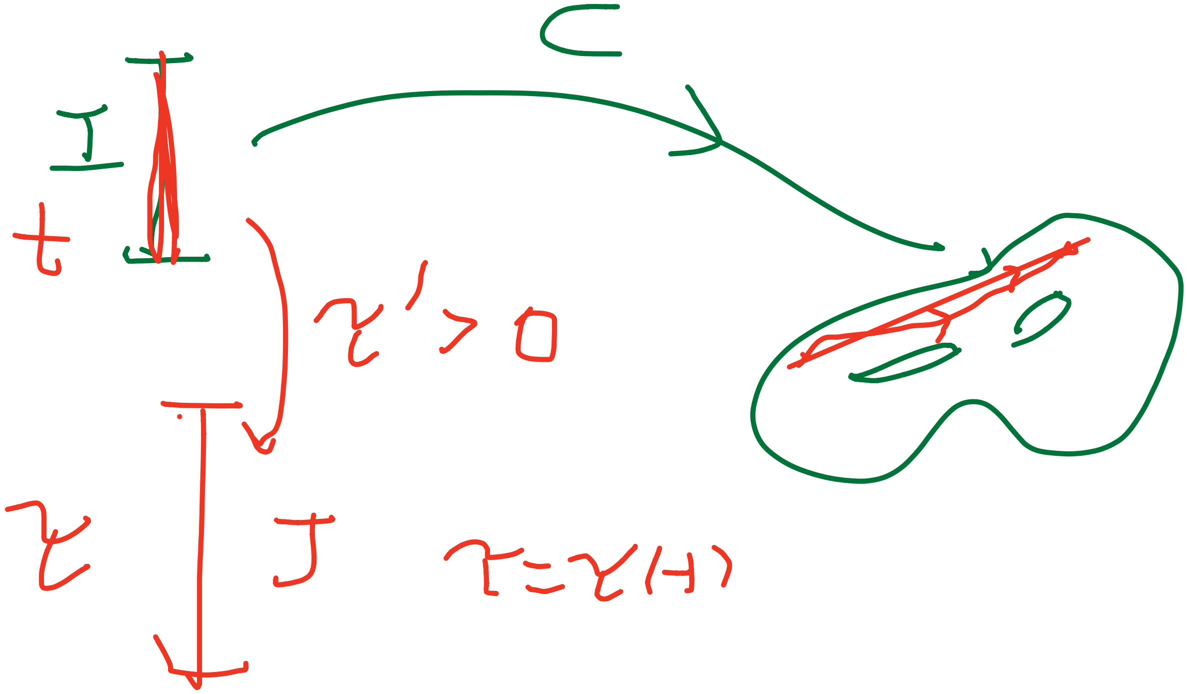
I an interval.

<u>indep of how c is parameterized</u>.

Someone: tell me: what that means?

eg: $\int x\,dy = ?$





I
t
τ' > 0
τ
J
τ = τ(t)

Basic properties

$$\int_c \alpha = -\int_{-c} \alpha$$

who's $-c$?

if $I = [0,1]$ $\quad -c(t) = c(1-t)$.

homotopy

$$\int_{c_1 * c_2} \alpha = \int_{c_1} \alpha + \int_{c_2} \alpha.$$

$$\int_c A\alpha + B\beta = A\int_c \alpha + B\int_c \beta.$$

$A, B \in \mathbb{R}$

What's $c_1 * c_2$?

$A\alpha + B\beta$?

$$3x\,dy + 5(x^2dx + dy)$$

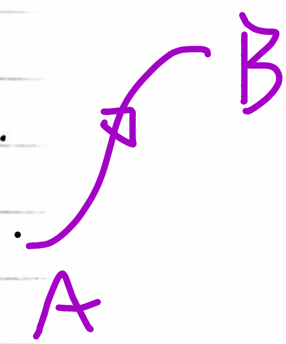
$$= (3x + 5x^2)\,dx + 5\,dy.$$

$f(x, y)$

1. $$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

$$\left(df = \sum \frac{\partial f}{\partial x^i} dx^i \right)$$

F Thm. Calc.



<u>Prop</u> $\int_c df = f(B) - f(A)$

if $c : [a,b] \to \mathbb{R}^2$

$c(a) = A$, $c(b) = B$.

<u>Pf</u> $c^* df = \frac{d}{dt}(f \circ c)(t)\, dt$.

Now use F Thm Calc

<u>Cor.</u> if c is closed: $A = B$,

$\oint_c df = 0$.

write $\oint$ for closed curves integrals

<u>Prop</u> If $\int_c \alpha = 0$ $\forall$ closed curves c then $\alpha = df$ for some $f : \mathbb{R}^2 \to \mathbb{R}$.

$$d(P\,dx + Q\,dy)$$

$$= \underbrace{\left(\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy\right)}\wedge dx$$

$$+ \underbrace{\left(\frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial y}dy\right)}\wedge dy$$

$$= dP\wedge dx + dQ\wedge dy$$

$$\left(dx\wedge dx = 0,\ dy\wedge \cancel{dy = -dy}\wedge dy = 0\right)$$

$$= \frac{\partial P}{\partial y}dy\wedge dx + \frac{\partial Q}{\partial x}dx\wedge dy$$

$$= \left(-\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}\right) dx \wedge dy$$

Pf...?

$d : \Omega^k \to \Omega^{k+1}$.

Ω^0 = fns functions

Ω^1 = 1-forms

Ω^2 = 2-forms

= expressions of forms $g(x,y)\,dx \wedge dy$.

extrin dlg.

rule: $dx \wedge dy = -dy \wedge dx$.

= integrands for integrals over regions.

Computation $ddf = 0$.

Poincare lemma The following are equiv. for $\alpha \in \Omega^1(\mathbb{R}^2)$.

1) $\forall$ closed curves c $\int_c \alpha = 0$.

2) $d\alpha = 0$

3) $\exists f \in \Omega^0$ $\alpha = df$.