L17 ∇ 2

B.

Prop (Q, ds^2). Then

$\exists!$ metric compatible, torsion free affine connection on Q

It is called the Levi-Civita connection.

It is given by the previous formula* for $\dim Q = 2$.

where $ds^2 = (\theta^1)^2 + (\theta^2)^2$.

$$\nabla_X e_1 = -\omega(X) e_2; \quad \nabla_X e_2 = +\omega(X) e_1$$

where

<u>Metric compatible</u>:

$$X\langle Y, Z\rangle = \langle \nabla_X Y, Z\rangle + \langle Y, \nabla_X Z\rangle$$

<u>Torsion-free</u>: $\nabla_X Y - \nabla_Y X = [X, Y]$

In case $(Q, ds^2) = (\mathbb{R}^n, Euc)$

The $\nabla = D$: $\nabla_X Y = DY \cdot X$

Jac matrix

where $Y : \mathbb{R}^n \to \mathbb{R}^n$!

L17 . ∇ 1

(Meaning; B')

what is a connection?
How is it related to geodesics?
what is curvature?

Affine connection on Q.

$$\mathfrak{X}(Q) \times \mathfrak{X}(Q) \longrightarrow \mathfrak{X}(Q)$$
$$X, Y \longmapsto \nabla_X Y.$$

bilinear in both slots
Liebnitz over $C^\infty(Q)$ in Y.
linear " $C^\infty(Q)$ in X:

$$\nabla_{fX}(gY_1 + Y_2) = f\nabla_X gY_1 + f\nabla_X Y_2.$$
$$= f(X[g]Y_1) + fg\nabla_X Y_1$$
$$+ f\nabla_X Y_2.$$

$f, g \in C^\infty(Q)$.

L17 ∇3

If Y defined in a nbhd of a pt p & $v \in T_pQ$

or if Y defined along a curve tangent to v at p

Then $(\nabla_v Y)(p)$ is well defnd:

Take any v-fld X with $X(p) = v$.

set $(\nabla_v Y)(p) = (\nabla_X Y)(p)$

result well defined.

geod eqns:

$$\boxed{\nabla_{\dot\gamma}\dot\gamma = 0}$$