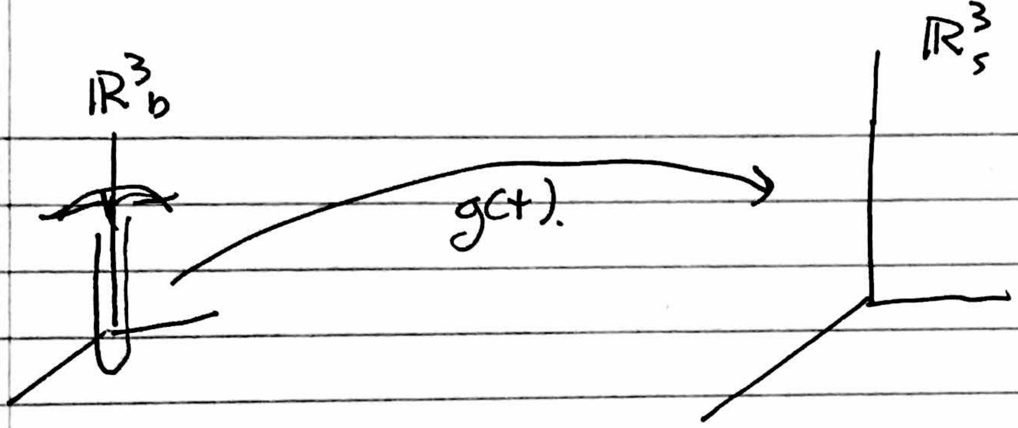

Config space: $\mathrm{Isom}_0(\mathbb{R}^3_b, \mathbb{R}^3_s) \simeq \mathbb{R}^3 \times SO(3)$

orient preserving, or: one component

representation of: Choose a single isometric embedding i_0 of ~~the~~ body in space.

all others: $g \circ i_0$

where $g : \mathbb{R}^3_s \longrightarrow \mathbb{R}^3_s$.

Use: any isometric map $\mathbb{R}^3_s \to \mathbb{R}^3_s$ of the form

$$\vec{x} \to R\vec{x} + \vec{b} \qquad \vec{b} \in \mathbb{R}^3 \text{ translation}$$

$$R \in SO(3) \quad \text{rotation.}$$

Metric, moment of inertia.

$$X \in \mathbb{R}^3_b \quad , \quad x \in \mathbb{R}^3_s$$

$$x = gX$$

Now

$$x(t) = g(t)X$$

$$\text{K.E.: } \frac{1}{2}\int \rho(X) \left\| \frac{dx}{dt} \right\|^2 d^3X$$

$$= \frac{1}{2}\int \rho(X) \left\langle \frac{dg}{dt}X, \frac{dg}{dt}X \right\rangle d^3X$$

<u>Manifestly</u> <u>left invariant</u>: under all of <u>SE(3)</u>

if $h: \mathbb{R}^3_s \longrightarrow \mathbb{R}^3_s$ a rotation

$$= \frac{1}{2}\int \rho(X) \left\langle h\frac{dg}{dt}X, h\frac{dg}{dt} \right\rangle dt$$

We can write:

$$\dot{\vec{x}} = \vec{\omega}_s \times \vec{x} + \vec{b}$$

$$= g(\vec{\omega}_b \times \vec{X}) + b$$

where

$$\vec{\omega}_b = (g^{-1}\dot{g})_{rot}$$ converted to a vector

also if

$$\cancel{x}g(t) \longrightarrow g(t) + \overset{\text{const translation}}{Tr}.$$

ie:

$$x(t) = g(t)\cancel{X} \longrightarrow g(t)X + Tr.$$

Then

$$\dot{x} = \dot{g}\cancel{X} \longrightarrow \dot{g}\cancel{X}$$

so

$$KE(\dot{g}) = KE(\dot{g} + Tr).$$

$\Rightarrow$ a metric on $SE(3)$
$\simeq Isom(\mathbb{R}^3_b, \mathbb{R}^3_s)$

which is ~~l~~ invariant under the action of $Isom(\mathbb{R}^3_s)$

general form m $TSE(3) = so(3) \times \mathbb{R}^3$
ang rel "in the body frame" $= \mathbb{R}^3 \times \mathbb{R}^3$.

$$K(\omega, v) = \langle \mathbb{I}\omega, \omega \rangle + \|v\|^2.$$

in appropriate coordinates

$\mathbb{I} = 3 \times 3$ sym +def matrix

Metric becomes :

a left invariant metric
on $SO(3)$

cf.