Same type of computating

$$\delta A = 0 \iff \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^\mu}\right) = \frac{\partial L}{\partial q^\mu}.$$

$\mu = 1, \dots n.$

if L of $K - V$ type
of a nat'l mech sys.

$$\iff \nabla_{\dot{q}}\, \dot{q} = -\nabla V(q)$$

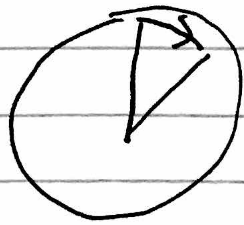
{ Levi-Civita connection of metric

eg: $V \equiv 0.$

$$\nabla_{\dot{q}}\, \dot{q} = 0.$$

eg. geodesics on S^2.

$$ds^2 = d\varphi^2 + \sin^2\varphi\, d\theta^2$$



so

$$L = \frac{1}{2}(\dot{\varphi}^2 + \sin^2\varphi\, \dot{\theta})^2$$

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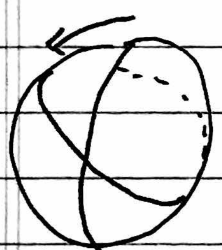
geod eqns:

$$\cancel{\frac{d}{dt}}\left(\frac{\partial L}{\partial \dot\varphi}\right. = \dot\varphi$$

$$\frac{\partial L}{\partial \dot\theta} = \sin^2\varphi\,\dot\theta$$

$$\Rightarrow \frac{d}{dt}\left(\sin^2\varphi\,\dot\theta\right) = 0 \quad \left(= \frac{\partial L}{\partial \theta}.\right.$$

$$\frac{d}{dt}\dot\varphi = 2\sin\varphi\cos\varphi(\dot\theta)^2 \quad \left(= \frac{\partial L}{\partial \varphi}\right)$$



planes: $ax+by+cz=0$.

Special solns: $\theta = \text{const}$

$\dot\theta = 0.$

$\varphi = t$. planes thru N pole

<u>Exer</u>: Using the EL eqns, show

each geodesic is the intersection of S^2 w/ a plane thru O.

$\mathbb{F}L$ = Leg. transf.

Rec. reading LC Young.
Optimal Control & the Calc. of variations.

$L : TQ \longrightarrow \mathbb{R}$.

each T_qQ is a vector space.

curves tangent to T_qQ

$(q, v) \mapsto (q, v + tw)$, $v, w \in T_qQ$.

set

$$\mathbb{F}L(q,v)(w) = \frac{d}{dt}\Big|_{t=0} L(q, v+tw)$$

verify : in coord:

$$(\mathbb{F}L(q,v))_i = p_i$$

where $p_i dq^i \in T^*_q Q$

sat.

$$p_i = \frac{\partial L}{\partial \dot{q}^i} := \frac{\partial L}{\partial v^i}$$

Here coord q^i on Q
induce vector bundle coord
v^i on TQ
p_i on T^*Q

By: $v = \sum v^i \frac{\partial}{\partial q^i}$

$$p = \sum p_i dq^i$$

Set

$$\tilde{H} = p(v) - \mathbb{F}L(q,v)$$

restricted to the graph of $\mathbb{F}L$:

$$TQ \longrightarrow T^*Q$$

so graph $\mathbb{F}L \subset TQ \oplus T^*Q$.

if $\mathbb{F}L$ is a diffeo on each fiber then can solve:

$p = \mathbb{F}L(q\,v)$ for $v = \mathbb{F}L_q^{-1}(p)$

&

$$\tilde{H}(q,p,v(p)) = \cancel{pv} = H(q,p)$$

primary eg.

Nat'l mech. system:

$$p_\mu = g_{\mu\nu}(q)\dot{q}^\nu.$$

$$s_\nu = g_{\mu\nu} v^\nu$$

$$v^\nu = g^{\nu\beta} p_\beta.$$

Compute

$$H(q,p) = \frac{1}{2}(p,p)^{\#}_q + V(q)$$

where

$$L(q,v) = \frac{1}{2}\langle v,v\rangle_q - V(q),$$

& where $(\ ,\ )^{\#} : T^*_q Q \times T^*_q Q \to \mathbb{R}$

is the metric dual to

$$\langle\ ,\ \rangle : T_q Q \times T_q Q \to \mathbb{R}$$

Hamiltonian version of
EL eqns.

1) Compute $H = H(q,p)$

$$\begin{cases} \dot{q}^i = \dfrac{\partial H}{\partial p_i} \\ \dot{p}_i = -\dfrac{\partial H}{\partial q^i} \end{cases}$$

Hamiltons. eqns

$\Leftrightarrow \frac{d}{dt}(q,p) = X_H(q,p)$

$= \text{sgrad}\, H.$

sgrad : $\omega(X_H, \cdot) = dH.$

or $i_{X_H}\omega = dH$

where $\omega = \sum dp_i \wedge dq^i$

L14 B

__On__ to :

Canonical one-form
two-form.

Symp. geometry.